VoroCrust Geometry:
3D polyhedral meshing
with true Voronoi cells
conforming to prescribed surface points.

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(speaker instead of Scott)
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Related Prior USNCCM Talk

- Mohamed’s talk covered algorithm, reconstruction properties, implementation, and example meshes
- This talk covers geometric primal-dual math, algorithmic correctness

VOROCRUST
Volumetric Voronoi Meshing that Conforms to Surface Samples

- Mohamed Ebeida
- Optimization and Uncertainty Quantification Department
- Sandia National Laboratories
VoroCrust

ALGORITHM OVERVIEW
‘Classic’ VoroCrust Primal-Dual-Primal Dance

Input: primal mesh of the input surface, points and edges in 2d and triangles in 3d
Primal-Dual-Primal Dance

Create weighted Voronoi balls around primal points

Properties:
- Balls overlap if points share an edge
- Balls protect points, as no seeds will be inside balls
Create dual unweighted seed pairs at weighted Voronoi sphere intersections

- red = exterior
- blue = interior

2D: 2 circles intersect at two points
3D: 3 spheres intersect at two points

Require: both intersection points lie outside all other balls

Want seeds sufficient to reconstruct surface
- surface mesh nodes are Voronoi vertices
- surface mesh edges are Voronoi edges
- surface mesh triangles are Voronoi facets
Primal-Dual-Primal Dance

**Property:** Unweighted Voronoi diagram reconstructs the surface mesh

Want seeds sufficient to reconstruct surface
surface mesh nodes are Voronoi vertices
surface mesh edges are Voronoi edges
surface mesh triangles are Voronoi facets
Enhancement

**Freedom:** Additional seeds outside balls, added for mesh quality.
E.g. could use a (graded) disk packing or lattice points for a hex-dominant mesh.
**Enhancement**

**Freedom:** Additional seeds outside balls, added for mesh quality. E.g. could use a (graded) disk packing or lattice points for a hex-dominant mesh.
Challenge

• Problem:
  – Sometimes one intersection point is covered, and the other in the pair is uncovered

• Solution:
  – “Basic” algorithm adds additional seeds
    • reconstructs vertices, edes, facets one by one
    • reconstructs a refinement of the input mesh
Basic VoroCrust Algorithm

• Step (0): input mesh
Basic VoroCrust Algorithm

• Step (1): Spheres around each mesh node
Basic VoroCrust Algorithm

• Step (2): Circles around each mesh edge, on sphere
Basic VoroCrust Algorithm

• Step (3): Four seeds on each circle
Basic VoroCrust Correctness

- Reconstruction Properties: Vertices
- By Construction ...
Basic VoroCrust Correctness

• Reconstruction Properties: Edges

Claim 1. \( \forall p \in \overline{ab}, \forall v \notin \{a, b\}, \min\{d(p, a), d(p, b)\} < d(p, v) \).
Basic VoroCrust Correctness

• Reconstruction Properties: Faces

Claim 2. \( \forall p \in \triangle abc, \forall v \notin \{a, b, c\}, \min\{d(p, a), d(p, b), d(p, c)\} < d(p, v). \)
Conclusion

• Work in progress
• Status
• Open
• backup slides
tweaked: