Motivation for Concurrence Multiscale Coupling

- **Large scale** structural failure frequently originates from small scale phenomena such as defects, microcracks, inhomogeneities and more, which grow quickly in unstable manner.
- Failure occurs due to **tightly coupled interaction** between small scale (stresses, microcracks, material instabilities, cracks, etc.) and large scale (vibration, impact, high loads and other perturbations).

Schwarz Alternating Method for Domain Decomposition

- Proposed in 1870 by H. Schwarz for solving Laplace PDE on irregular domains.
- Simple idea: if the solution is known in regularly shaped domains, use those as pieces to iteratively build a solution for the more complex domain.
- Basic Schwarz Algorithm
  - Initialize: Solve PDE by any method on $\Omega_1$ w/ initial guess for Dirichlet BCs on $\Gamma_t$.
  - Iterate until convergence: Solve PDE by any method (can be different than for $\Omega_1$) on $\Omega_2$ w/ Dirichlet BCs on $\Gamma_t$ that are the values just obtained for $\Omega_1$.
- Schwarz alternating method most commonly used as a preconditioner for Krylov iterative methods to solve linear algebraic equations.
- Novel idea: using the Schwarz alternating as a discretization method for solving multi-scale partial differential equations (PDVs).

Schwarz Alternating Method for Multiscale Coupling

- In the literature, Schwarz method is applied to dynamics by using space-time discretizations – unfeasible given design of our current codes and size of simulations.
- Our extension of Schawrzs coupling to dynamics uses a governing time stepping algorithm that controls time integrators within each domain.
- Can use different integrators (e.g., implicit, explicit) with different time steps in each domain.
- Simplifies the task of meshing complex geometries for the different scales.

Proof of Convergence for Finite-Deformation Solid Mechanics

- S. L. Sobolev (1936) posed the Schwarz method for linear elasticity in variational form and proved method’s convergence by proposing a convergent sequence of energy functionals.
- S. G. Mikhailin (1951) proved convergence of Schwarz method for general linear elliptic PDEs.
- A. Mota, I. Tezaur, C. Alleman (2013)*: derived a proof of convergence of the alternating Schwarz method for the finite deformation quasistatic nonlinear PDEs (assumming the energy functional $F(\phi)$ defined below is quasi-convex), and determined a geometric convergence rate for the finite deformation quasi-static problem.

\[ F(\phi) = \int_\Omega W(\mathbf{F}, \mathbf{X}, \mathbf{T}) \, dv \ - \int_{\Gamma_T} B \cdot \mathbf{p} \, d\Gamma_t + \lambda_1 \int_{\Omega_1} \mathbf{d} \cdot \mathbf{d}^T \, d\Omega_1 \]


Four Variants of Schwarz Alternating Method for Quasistatic

- Full Schwarz
- Inexact Schwarz
- Modified Schwarz
- Monolithic Schwarz

Schwarz Alternating Method for Dynamics

- Component-based design for rapid development of capabilities.
- Extensive use of libraries from the open-source Trilinos project.
- Use of the Phalanx package to decompose complex problem into simpler problems with managed dependencies.
- Use of the Sacado package for automatic differentiation.
- Use of Telco package for block preconditioning.
- Performance portability to GPUs and Xeon Phi via Kokkos.
- Parallel implementation of Schwarz uses the Data Transfer Kit (DTK).

Implementation within Albany Finite Element Code

- Linear elastic clamped beam with Gaussian initial condition for the 2-d simulation.

Numerical Results: Quasistatics

- Foulk’s Singular Bar
  - 10 bar with area proportional to square root of length with strong singularity on left end of bar and simple hyperelastic material model with no damage.
  - Test case goal: explore viability of 4 variants of Schwarz alternating method, test convergence (expect faster convergence in less iterations w/ increased overlap).

- Cuboid
  - Coupling of two cuboids with square base w/ NeoHookean type material model.
  - Schwarz alternating method converges linearly.
  - There is faster linear convergence with increasing overlap volume fraction.

- Bolted Joint
  - Multi-scale problem of practical scale: coupling 85K composite tet 10 element mesh (bolts, $\Omega_1$) with 56K hex element mesh (parts, $\Omega_2$).
  - NeoHookean material model: steel bolts and base, remaining parts are aluminum.
  - Lateral displacement load applied at top: applies compression to 2 bolts, tension to remaining 2.

- Torison
  - Nonlinear elastic bar subjected to high degree of torsion.
  - Dynamic Schwarz method is used to couple two regions of the bar using different mesh resolutions, different element types, and different time integration schemes, once more without introducing any dynamic artifacts.

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