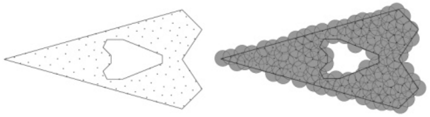
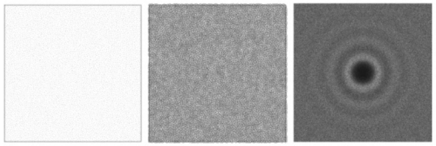
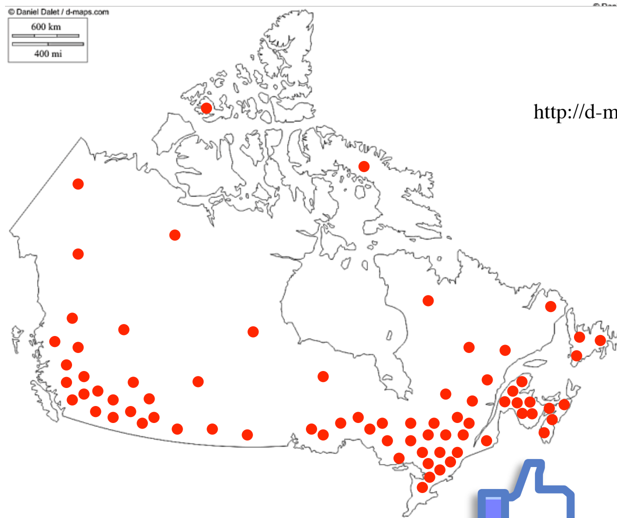
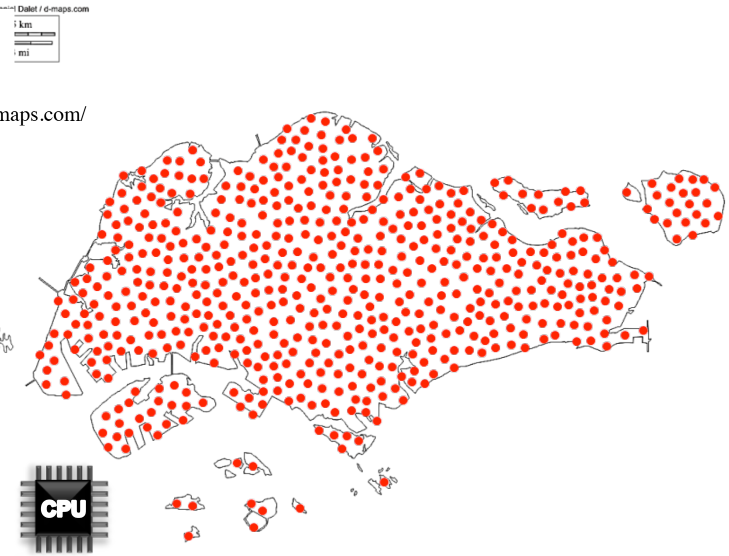
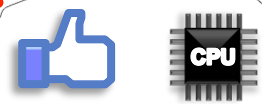










Efficient Maximal Poisson-Disk Sampling

Mohamed S. Ebeida, Anjul Patney, Scott A. Mitchell, Andrew A. Davidson, Patrick M. Knupp, John D. Owens

Sandia National Laboratories, University of California, Davis

Scott - presenter
SIGGRAPH2011

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company,
for the United States Department of Energy's National Nuclear Security Administration
under contract DE-AC04-94AL85000.

Maximal Poisson-Disk Sampling

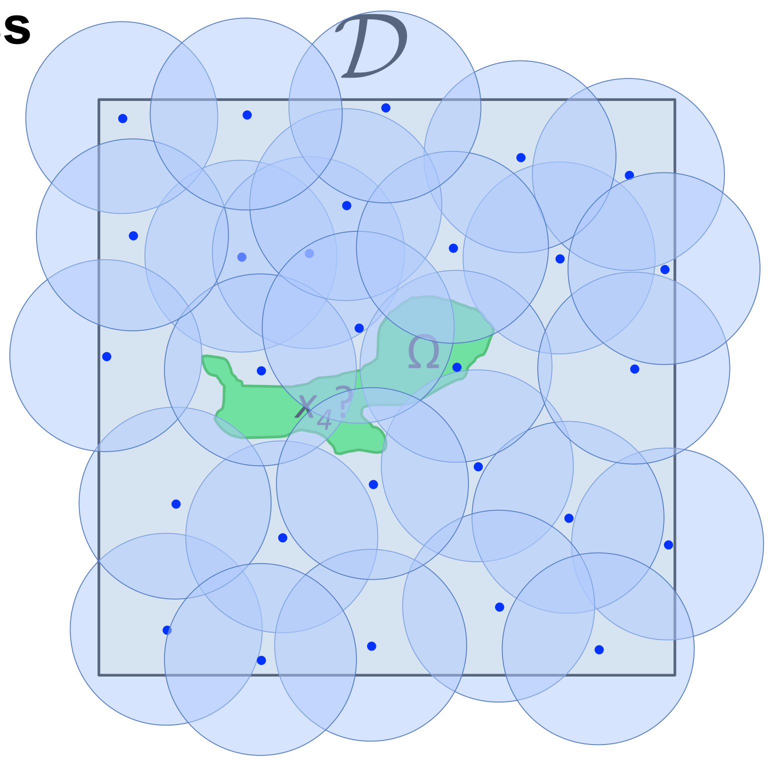
- **What is MPS?**
  - **Dart-throwing**
  - **Insert random points into a domain, build set X**
    - **With the “Poisson” process**

Empty disk: $\forall x_i, x_j \in X, x_i \neq x_j : ||x_i - x_j|| \geq r$

Bias-free: $\forall x_i \in X, \forall \Omega \subset \mathcal{D}_{i-1} :$

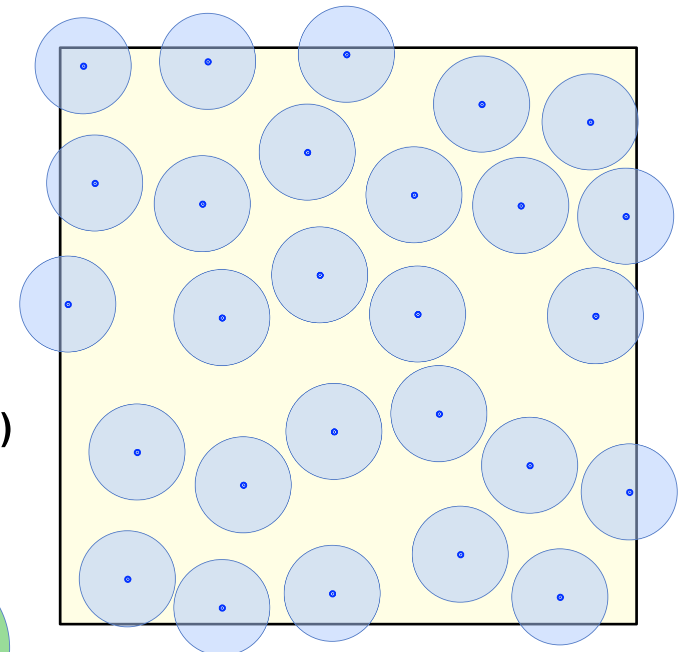
$$P\left(x_i \in \Omega\right) = \frac{\text{Area}(\Omega)}{\text{Area}(\mathcal{D}_{i-1})}$$

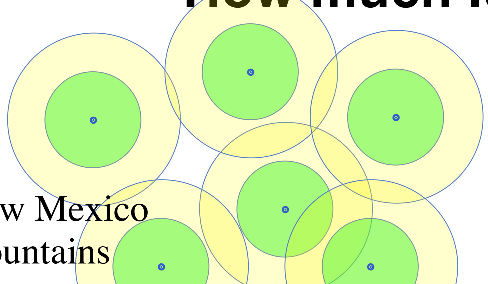
Maximal: $\forall x \in \mathcal{D}, \exists x_i \in X : ||x - x_i|| < r$



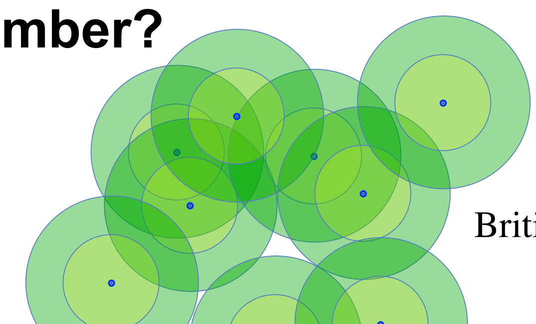
MPS a.k.a.

- Statistical processes
  - Hard-core Strauss disc processes
    - Non-overlap: inhibition distance r_1
    - cover domain: disc radius r_2
- Nature
  - Trees in a forest
    - Variable disk diameter = tree size
    - Points are tree trunks
    - Disks are tree leaves or roots
  - Given satellite pictures (non-maximal)
    - How many trees are there?
    - How much lumber?
- Random sphere packing
  - Non-overlapping r/2 disks
  - Atoms in a liquid, crystal





New Mexico mountains



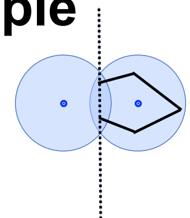
British Columbia

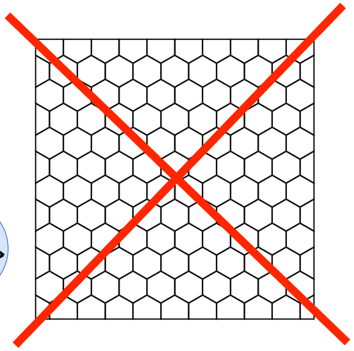
What is MPS good for?

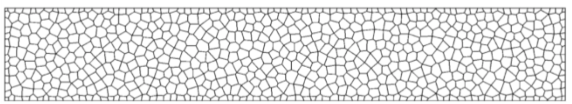
- Graphics – sample points for texture synthesis
  - Generate blue noise distributions for anti-aliasing
  - Without Moire and other visible patterns
- Unbiased process leads to points with
  - No visible patterns between distant points.
    - pairwise distance spectrum close to truncated blue noise powerlaw
  - Our eyes sensitive to patterns
  - Randomness hides imperfections
    - stare at dry-wall in your house sometime, try to find the seams

What is MPS good for?

- Physics simulations – why SNL paid for my trip ☺
  - Voronoi mesh, cell = points closest to a sample
  - Fractures occur on Voronoi cell boundaries
    - Mesh variation ⊂ material strength variation
    - CVT, regular lattices give unrealistic cracks
      - Unbiased sampling gives realistic cracks
  - Ensembles of simulations
  - Domains: non-convex, internal boundaries

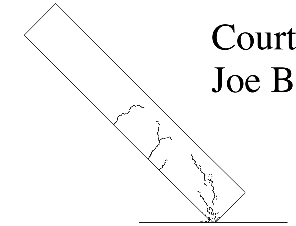


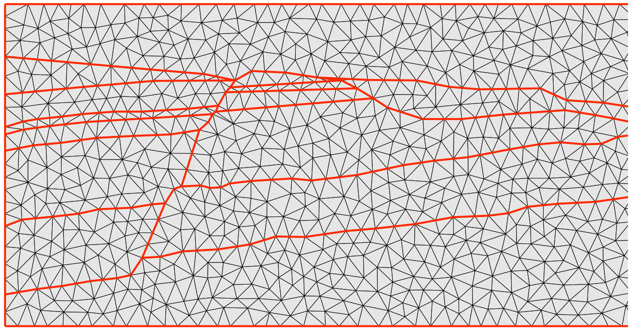




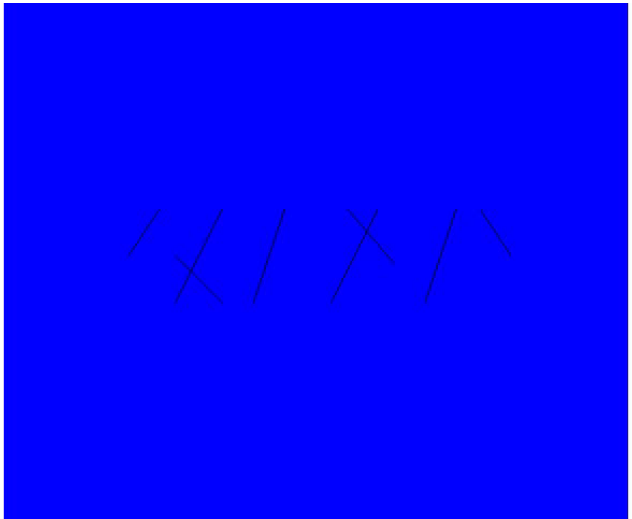
Fracture Simulations

Courtesy of
Joe Bishop (SNL)



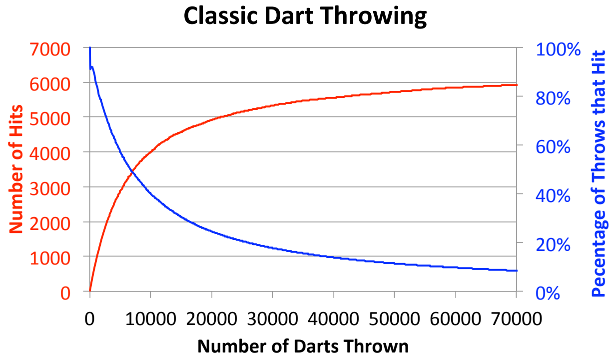


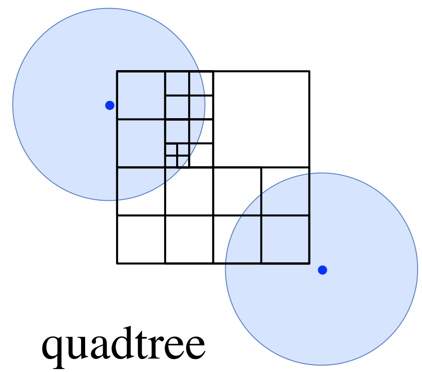
Seismic Simulations
maximal helps Δ quality

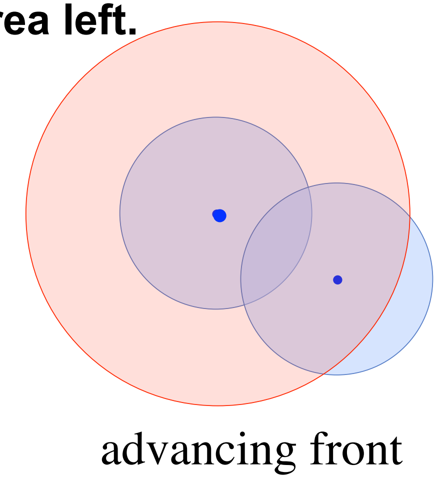


Algorithm for MPS

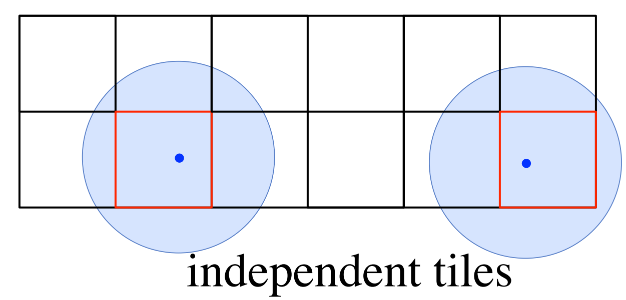
- **Classic algorithm**
  - **Throw a point, check if disk overlaps, keep/reject**
  - **Fast at first, but slows due to small uncovered area left. Can't get maximal.**





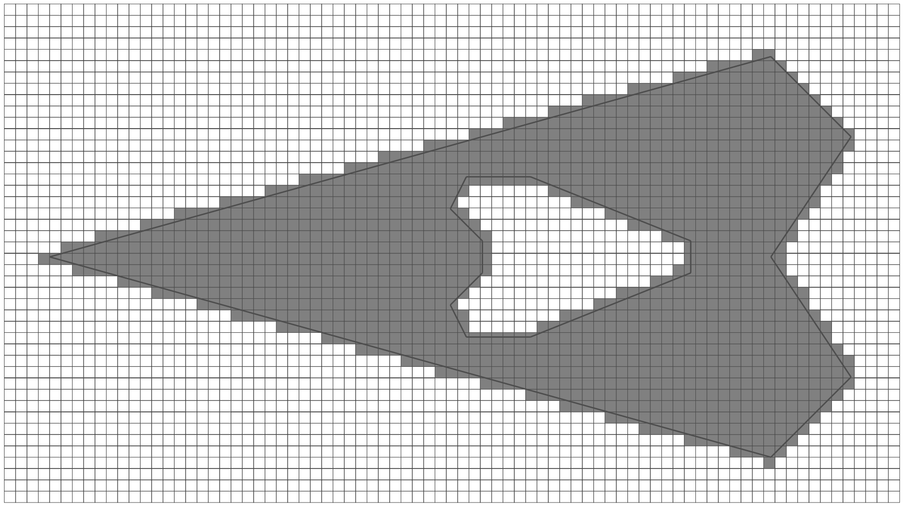


- **Speedup by targeting just the uncovered area**
  - **Others use quadtrees to approximate the uncovered area**
  - **Others use advancing front to sample locally**
  - **Others use tiles to aid parallelism**
- **Common issues**
  - **Not strictly "unbiased" process**
    - **Outcome may be indistinguishable from an unbiased process's outcome**
  - **Not maximal: dependent on finite precision**
  - **Memory or run-time complexity**
  - **Ours is first provably bias-free, maximal, E(n log n) time O(n) space**

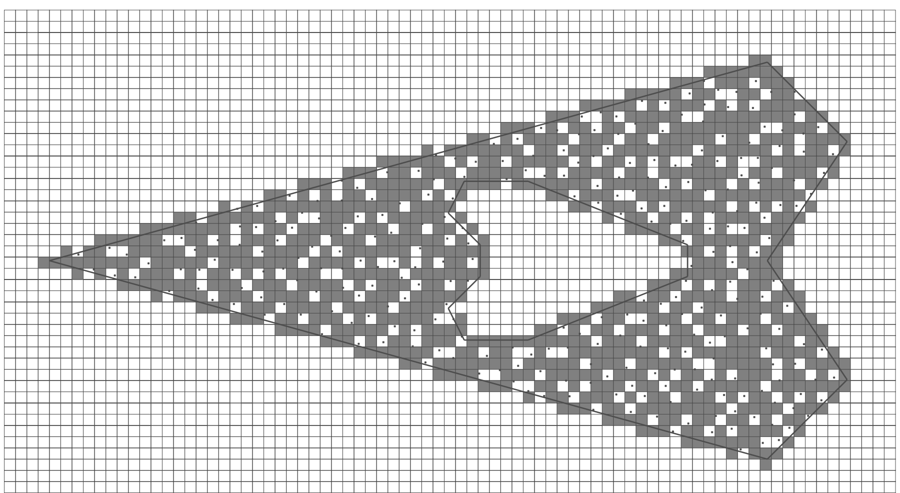


Sandia National Laboratories

Algorithm

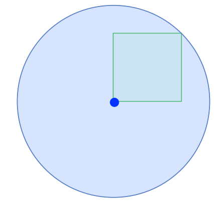


Initial Pool C



End of Phase I: white cells with a point

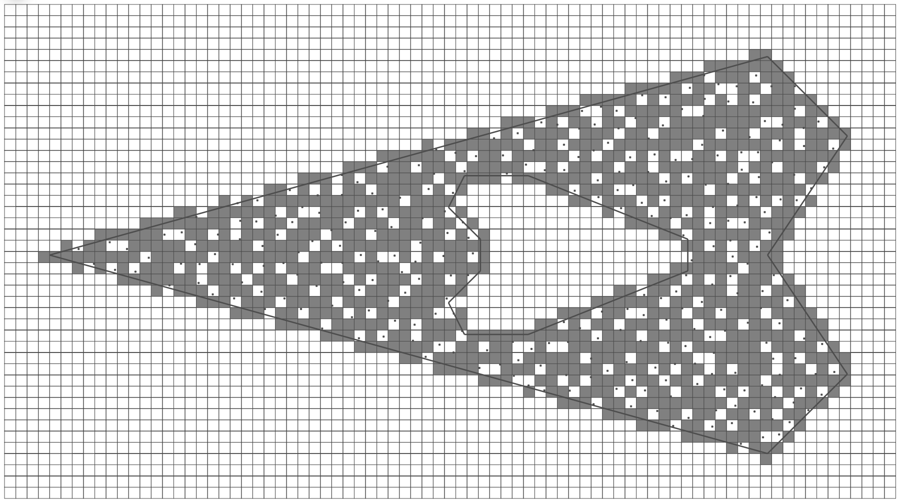
- **Background square grid**
  - **Square diagonal = r**



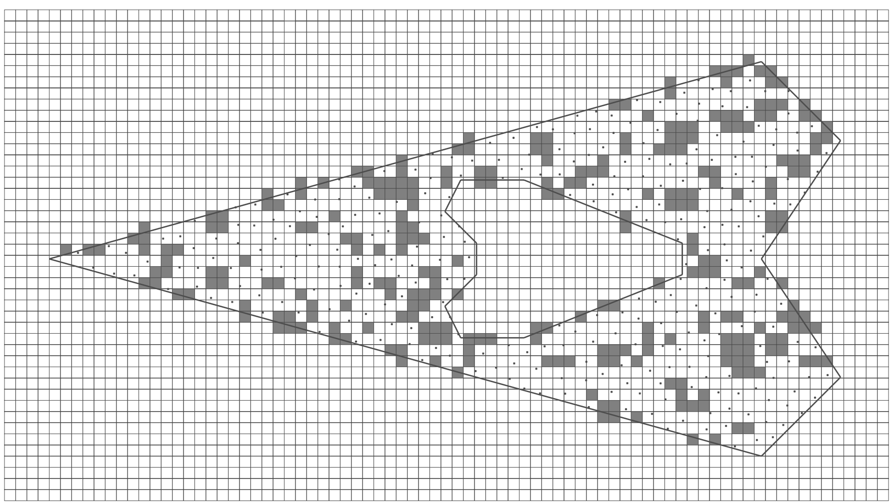
- **Flood fill**
  - **Build pool of cells C : not-exterior to domain**
- **Phase I: quickly cover most of the domain**
  - **Pick a square from pool**
  - **Pick point in square**
  - **If point uncovered (likely)**
    - **Keep point**
    - **Remove square from pool**
  - **Repeat $a|C|$ times**

Sandia National Laboratories

Algorithm

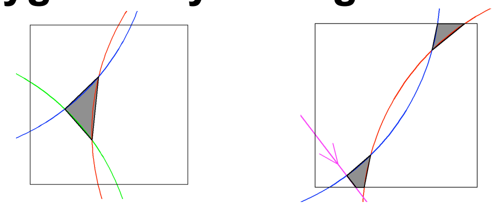


End of Phase I: white cells with a point



Start of Phase II: dark cells not-covered

- Target remaining uncovered area
- Construct square \ disks
  - Polygon easy surrogate for arc-gon



- Replace pool of squares by polygons
- Phase II: repeat
  - Pick polygon from pool
    - Weighted by its area (only log n step)
  - Pick point in polygon
  - If uncovered
    - Keep point
    - Remove polygon from pool
    - Update nearby polygons
- **Works well because**
  - Voids are scattered
  - Small arc-gons are well approximated by polygons

Algorithm Nuance - Phase II stages

- **"Algorithm is simple,… in a good way" - Reviewer**
- **Lazy update of polygons' areas and pool, in "stages"**
  - **More simple datastructures**
  - **No tree needed, flat array for pool, fewer pointers**
  - **Run-time proof gets more complicated**

Prior slide

```
Phase II: repeat
        Pick polygon from pool
                Weighted by its area (only log n step)
        Pick point in polygon
        If uncovered
                Keep point
                Remove polygon from pool
                Update nearby polygons
```

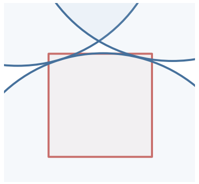
→

Lazy update

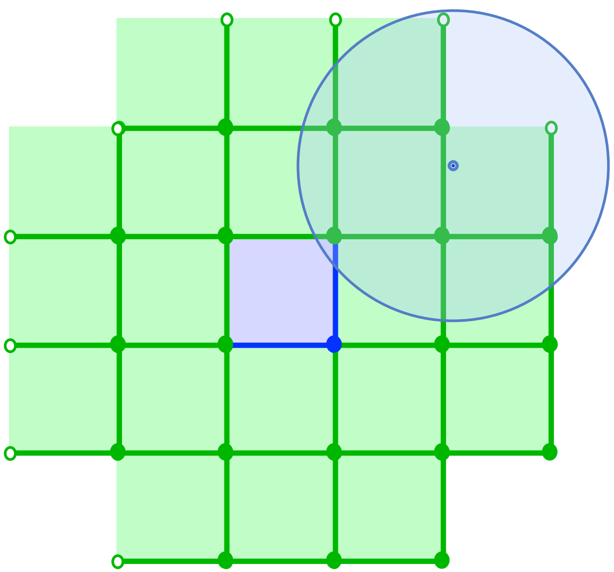
```
Phase II: repeat
    Repeat c|Pool| times
        Pick polygon from pool
            Weighted by its area (only log n step)
        Pick point in polygon
        If uncovered
            Keep point
    New stage - update all polygons
    Rebuild pool and weights
```

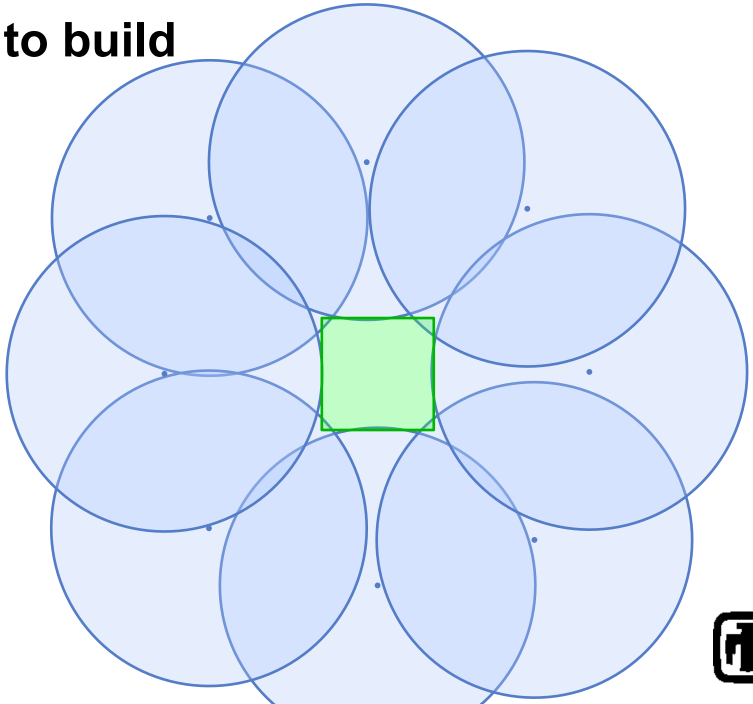
Complexity Proofs Sketch

- WTS constant time & space per point
  - Everything is local, and constant size
- #squares = θ(#points_in_sample)
- Sid Meier Civilization template
  - 21 nearby squares, 0 or 1 disks per square
    - By geometry, ≤ 4 voids per cell
    - By geometry, ≤ 9 (8?) disks bounding a void
- Constant time to check if point is uncovered
- Polygons are constant size, time to build



Four voids





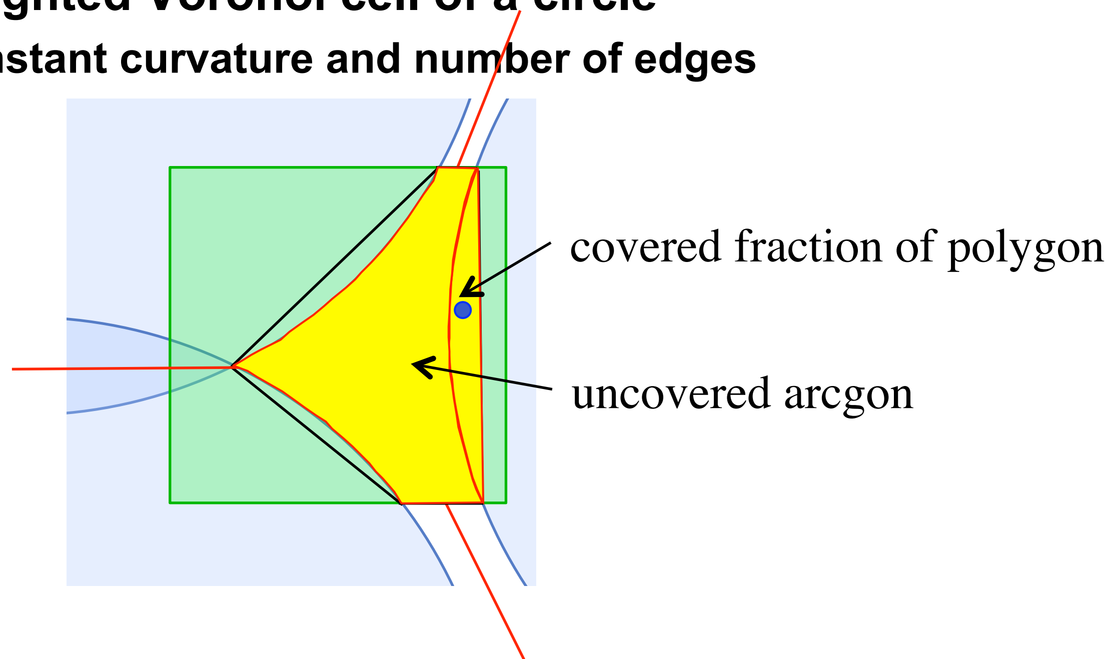
Complexity Proofs Sketch

- **Constant work per generated point, but what about the rejected (covered) points?**
  - **Phase I, O(|C|) throws**
  - **Phase II**

$$\text{Area(arcgon)} > c\,\text{Area(polygon)} \Leftrightarrow P(x_i : \text{uncovered}) > c$$

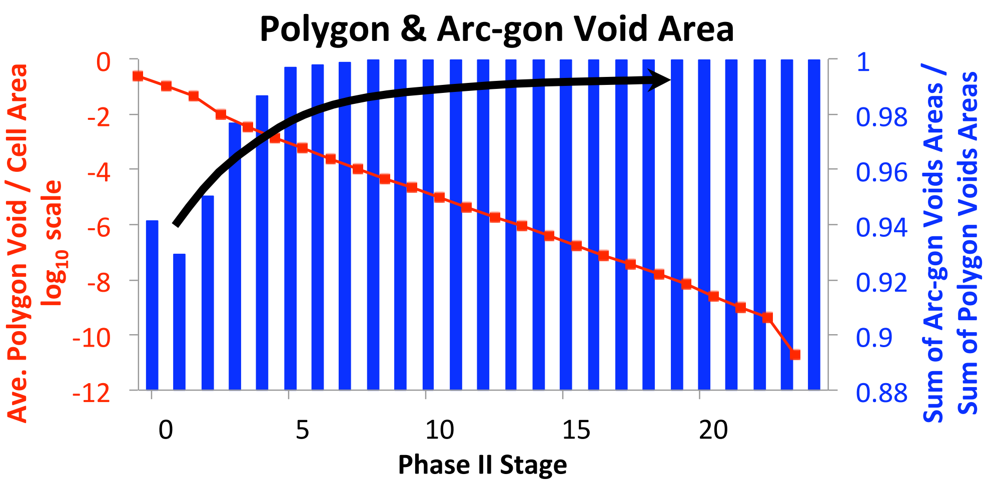
$$\Leftrightarrow \#\,\text{accepted} > c_2\,\#\,\text{rejected}$$

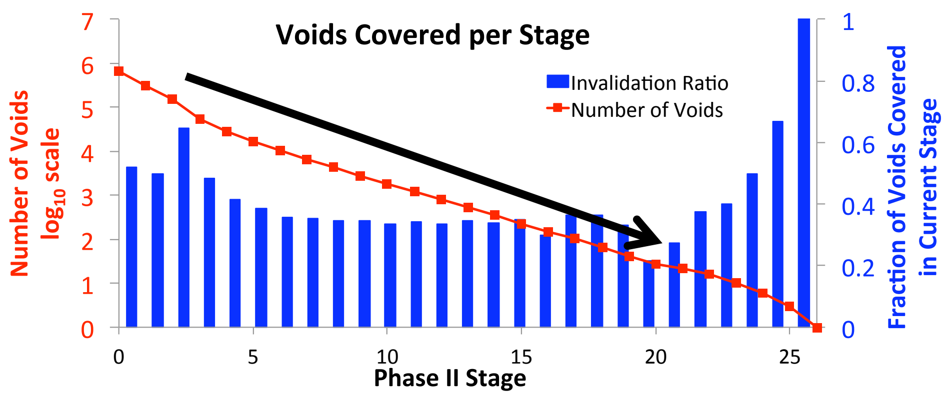
  - **Via weighted Voronoi cell of a circle**
    - **Constant curvature and number of edges**

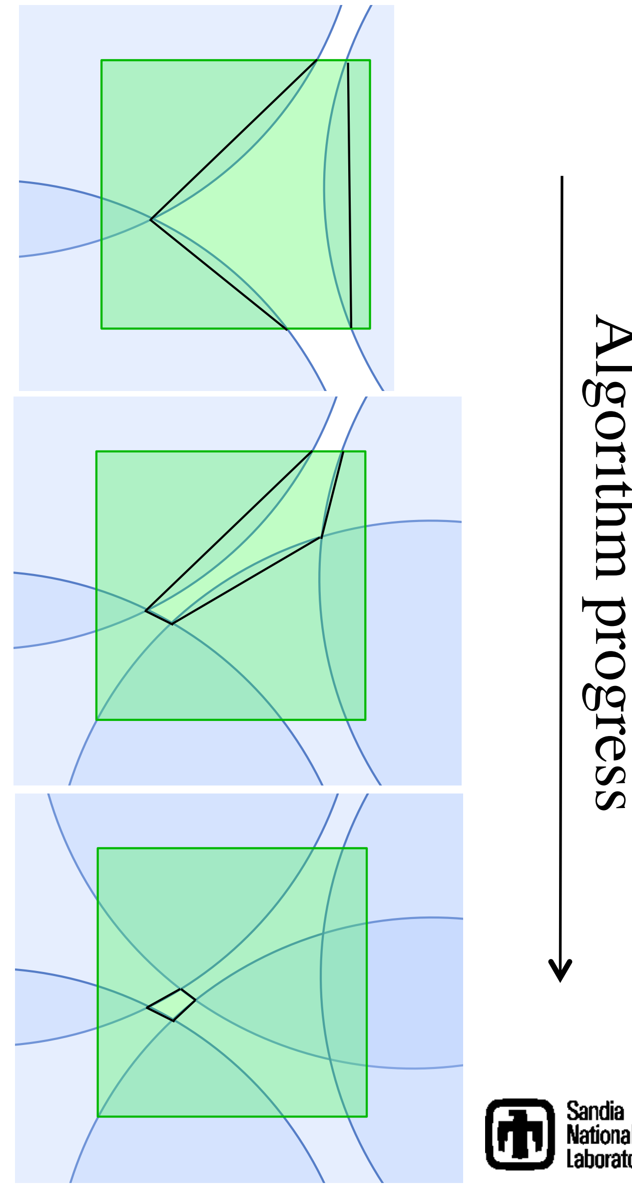


Fewer Rejected Points Later

- Polygons ➔ arcgon as voids get smaller
  - We get more efficient (contrast)







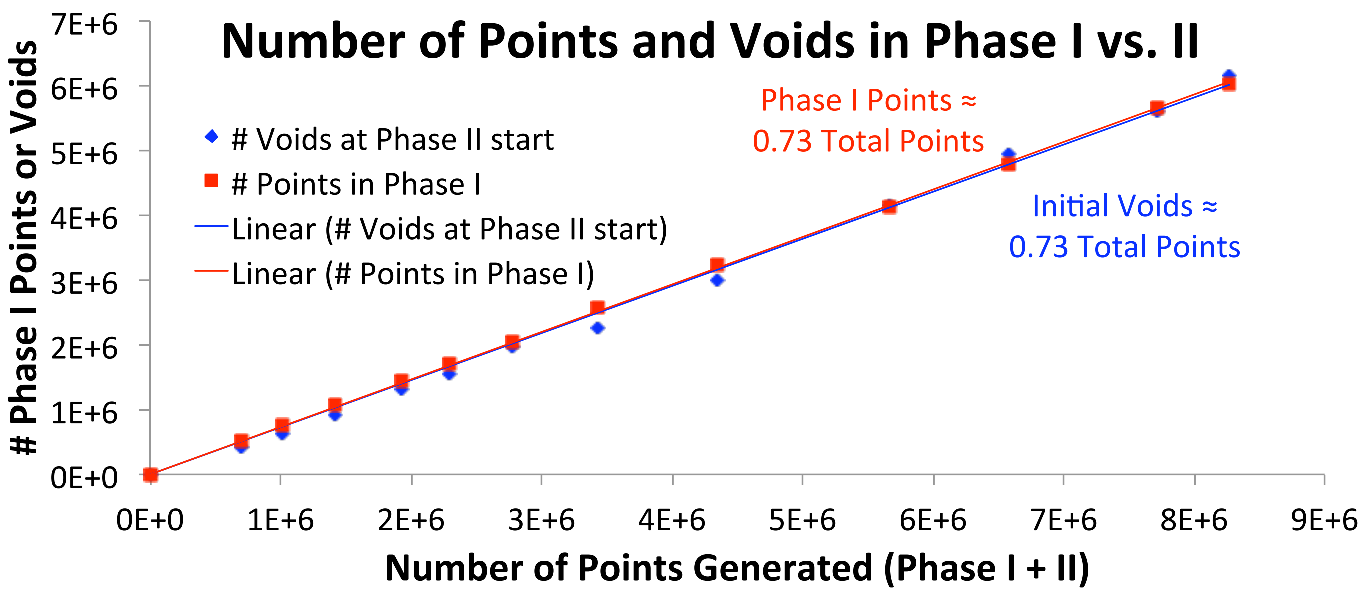
Complexity

- Complexity – everything is local, all steps constant time
  - except log(n) to select a polygon, weighted by area
  - that is a relatively inexpensive step
  - constructing geometric primitives is the expensive part
- Constant fraction of generated points are output points

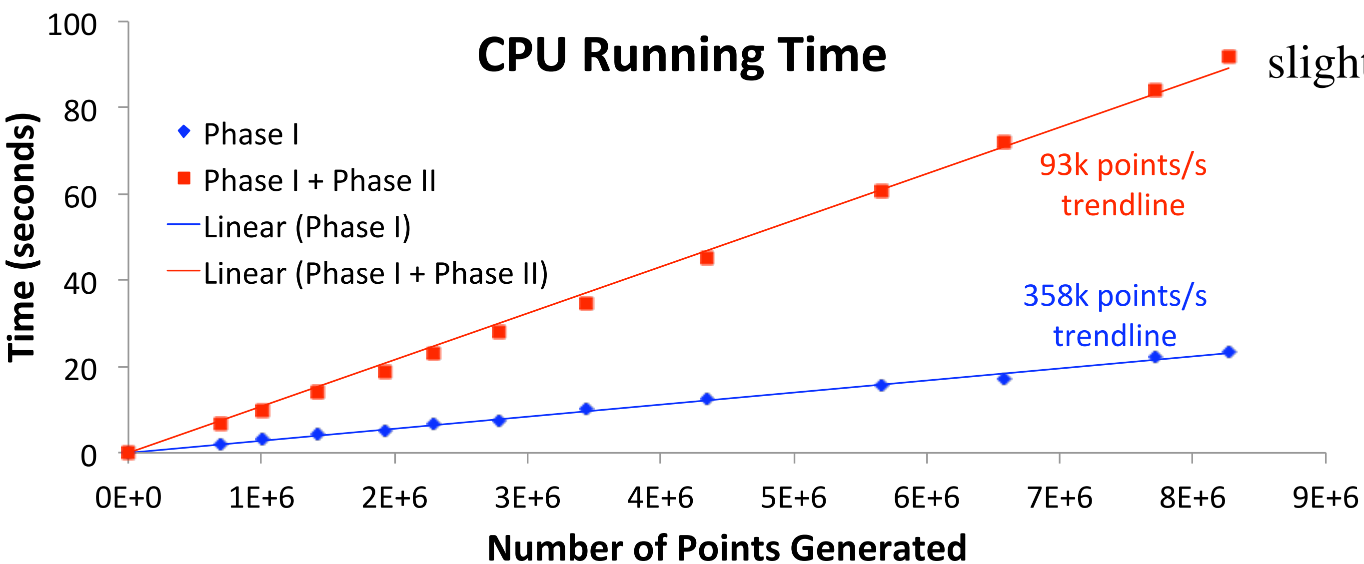
$$\mathrm{Time} = \mathrm{E}(Cn + c\, n \log n)$$

$$\mathrm{Space} = O(n)$$

Runtime – Why we do Phase I



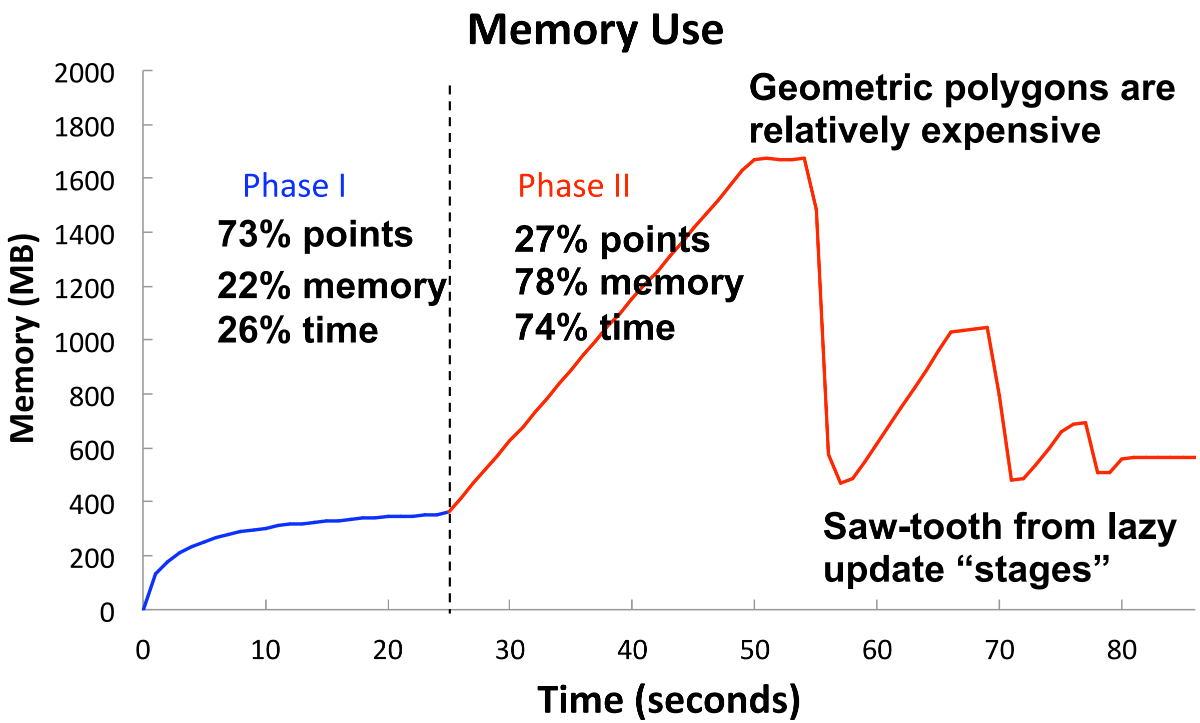
- **Phase I**
  - **73% of points**
  - **26% of runtime**



slight uptick from log

Sandia National Laboratories

Serial Memory Use

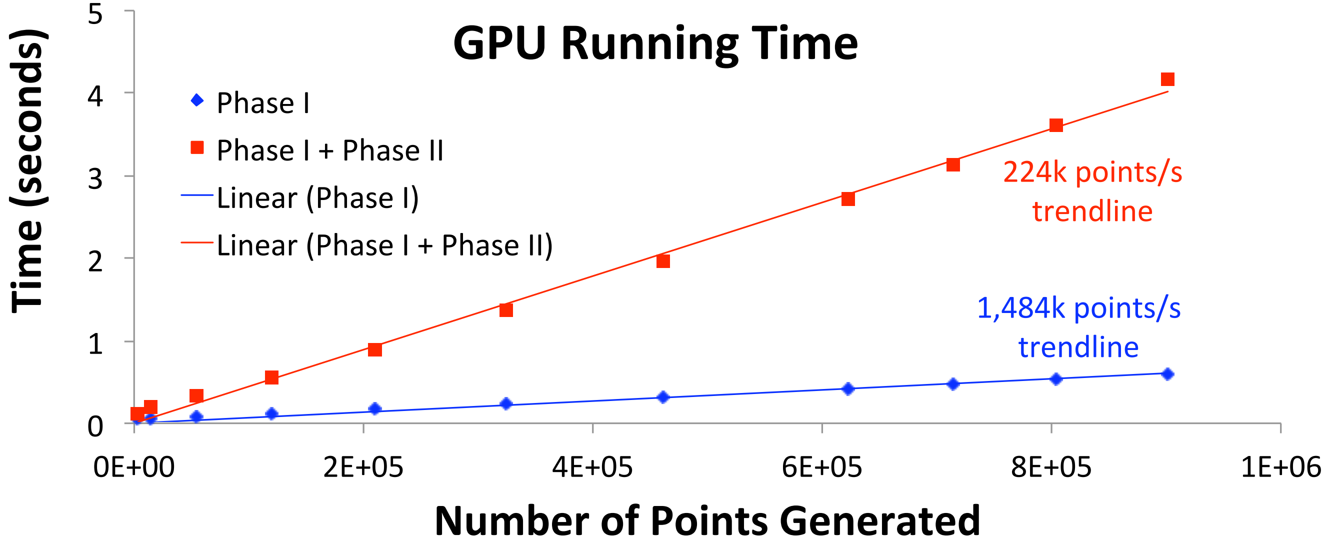


GPU Algorithm

Points generated in parallel, conflicts resolved in an unbiased way

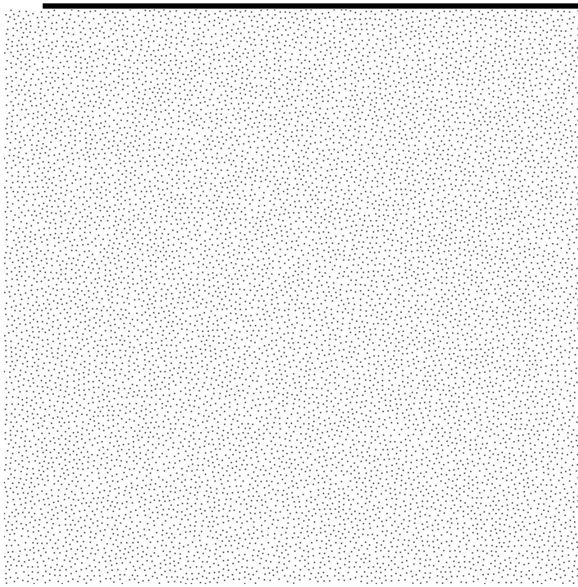
- **Point buffers: candidate and final**
- **Phase I**
  - **Iterate: synchronize at start of iteration**
    - **Generate $|\mathcal{C}|/5$ candidate points**
    - **Square states: empty, test, accepted, done**
      - **Done = Point from prior iterations**
      - **Test = Point doesn't conflict with nearby "done" points, compute in parallel**
      - **Accepted = Point is earlier (id) than conflicting "test" points, compute in parallel**
    - **Migrate accepted points to done, otherwise remove**
- **Phase II**
  - **Construct polygons, compute in parallel**
    - **Squares "rejected" if covered by prior disks, has no polygon, no work to do**
    - **Split polygons into triangles**
  - **Proceed as Phase I, with triangles playing role of squares**

GPU Performance

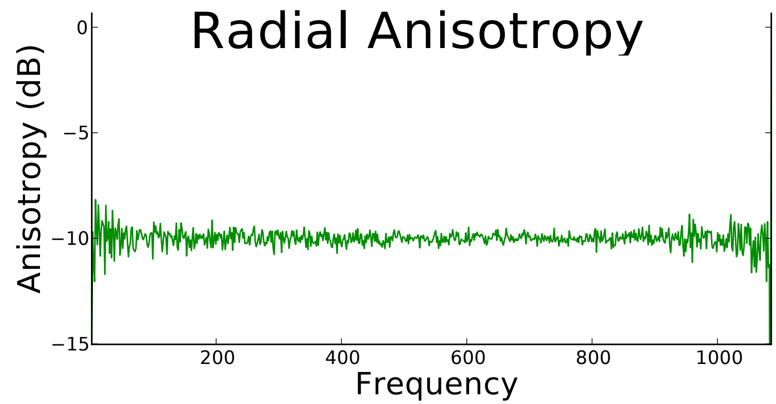


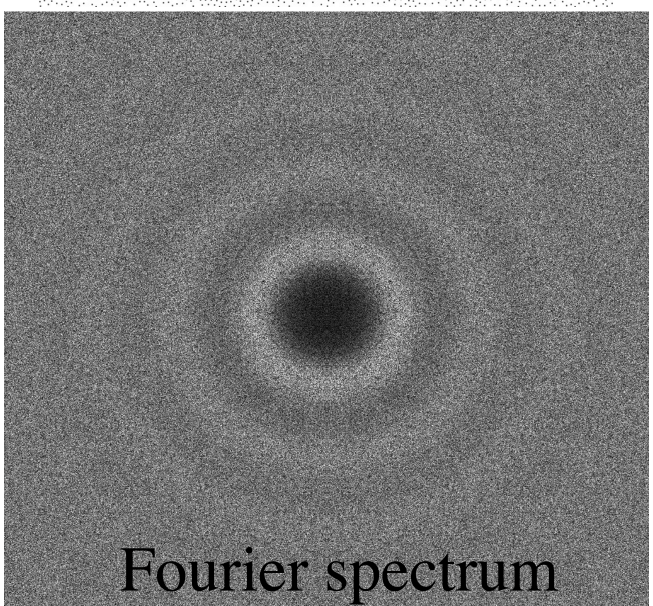
NVIDIA GTX 460
2.4x speedup over serial (6.7x memory bandwidth)
1 million points in 1 GB RAM

Unbiased Parallel Sample



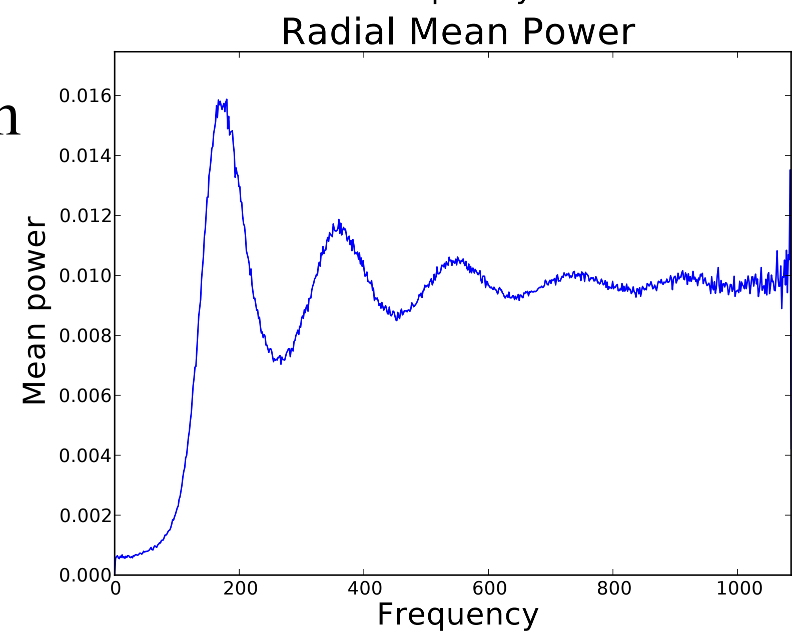
10k pts





Fourier spectrum

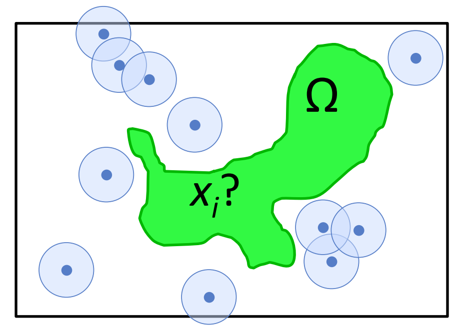
Rings from inhibition radius



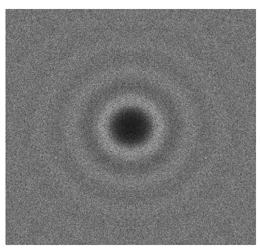
"Unbiased" Opinion

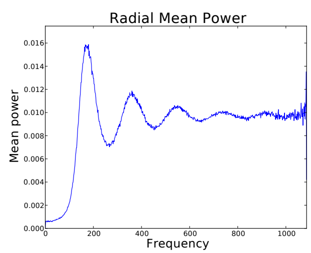
- **Unbiased as a description of (serial) process**
  - **insertion probability independent of location**

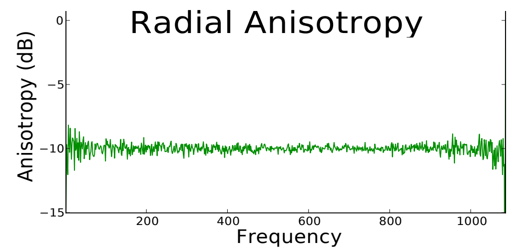
$$P(x_i \in \Omega) \propto \mathrm{Area}(\Omega)$$



- **Unbaised as a description of outcome**
  - **pairwise distance spectra, blue noise**







- **Unbiased process leads to unbiased outcome, but so might other processes**
  - **Opinion: need something beyond "viewgraph norm"**
  - **Need metrics for "how unbiased is it"**
    - **Define spectrum S that is the limit distribution of unbiased sampling, and standard deviations.**
    - **Our process generated S', and |S-S'| < 0.4 std dev (S)**

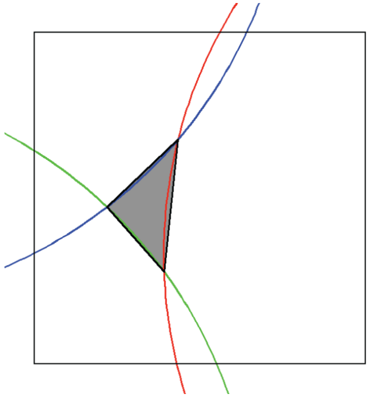
Sandia National Laboratories

Synopsis of Contribution

- **Poisson-disk distributions**
  - **Simple, efficient implementation**
  - **Provable guarantees**
    - **Maximal**
    - **Unbiased**
    - **O(n) space**
    - $E(Cn + {}_c n \log n)$ **time**
- **Domains**
  - **2d**
  - **Polygons with holes, non-convex**
- **Algorithmic innovations**
  - **Two phases**

    I. fast to cover most of domain

    II. careful to cover remainder
  - **Approximate uncovered "voids", square $\cap$ circles, with polygons. Careful weighting and selection**



Sandia National Laboratories

Future

- Extensions
  - Could do away with polygonal approximation and weight and sample directly – every dart is a hit! (w/ Thouis Ray Jones)
- Higher dimensions
  - geometric primitives unappealing
  - prefer just use hypercubes

- Thouis Ray Jones, jgt accepted paper
  - model explicit time-of-arrival for each point
  - synchronize locally as needed
  - vs. unbiased by one dart at a time, inherently serial