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Title: VoroCrust Geometry: 3D Polyhedral Meshing with True Voronoi Cells Conforming to Prescribed Surface Points

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We introduce VoroCrust for creating polyhedral meshes of 3D solids enclosed by 2D surfaces. VoroCrust is the first algorithm for 3D Voronoi meshes that naturally *conform* to prescribed surface points. Conformality is distinguished from the usual *clipping* of Voronoi cells by the surface, which always results in extra surface vertices beyond the original samples, and may result in non-planar, non-convex, or even non-star-shaped cells. VoroCrust creates cell seeds such that the original points on the surface manifold are vertices of the 3D cells, and the only surface vertices. This avoids shrinkage and other changes. All cells are true Voronoi cells; the surface does not restrict or constrain the Voronoi cells, rather the cells are geometrically placed to reconstruct the surface, by the cell facets separating inside and outside seeds. These facets are well-shaped and usually triangles. Mesh polyhedra enjoy all the nice properties of Voronoi cells, such as being convex with planar facets. Cell aspect ratios and dihedral angles are bounded. We have not yet addressed the issues of small edges, sharp edge angles, and small area faces, which may be important for some types of simulations. In contrast to the well-known “power crust” surface reconstruction algorithm, VoroCrust fills the volume with tunable 3D cells with good shape, and its 2D manifold reconstruction is from an *unweighted* Voronoi diagram. The VoroCrust algorithm starts with sample points capturing small and sharp surface features, and are dense compared to the local thinness and curvature. We can create these by Maximal Poisson-disk Sampling (MPS), or the points may be given as input. In either case we create a sphere around each sample. Some triples of overlapping spheres define a pair of intersection points, mirrored on each side of the manifold. Pairs outside all other spheres are analogous to weighted triangulation circumcenters, lifted above (and sunk below) the manifold according to its weighted distance. These are Voronoi seeds. We add well-spaced seeds in the far interior; these may be chosen to create a hex-dominant mesh. We generate the 3D Voronoi tessellation of all these seeds. The manifold is reconstructed by the Voronoi facets separating the inside and outside cells, the lifted and sunk seed pairs. This talk describes the geometry of VoroCrust, the primal-dual-dual-primal dance and the power center lifting. Mohamed Ebeida’s talk in session MS711, “Polygonal and Polyhedral Discretizations in Computational Mechanics,” describes the engineering aspects of VoroCrust.