BOUNDS OF SUPERCAPACITOR OPEN-CIRCUIT VOLTAGE CHANGE AFTER CONSTANT POWER EXPERIMENTS

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This paper examines the lower and upper bounds of the supercapacitor open-circuit voltage change after a constant power experiment, which is mainly due to the charge redistribution process and the equivalent series resistance (ESR). To derive the bounds, a simplified equivalent circuit model is developed based on the supercapacitor datasheet. Comprehensive constant power experiments are performed to verify the bounds. Results show that the bounds are valid for supercapacitor samples with different rated capacitance and voltage. This paper provides a tool to quickly estimate the supercapacitor open-circuit voltage change after a constant power process.

Keywords: supercapacitor modeling, charge redistribution, constant power experiment, equivalent series resistance (ESR), open-circuit voltage.

INTRODUCTION

Supercapacitors, also known as ultracapacitors, electric double layer capacitors (EDLCs), or electrochemical capacitors, are capacitors with large capacitances so that they can be used as secondary power sources. Supercapacitor-based energy storage systems have been employed by a variety of applications including electric and hybrid vehicles [1], smart grid [2], wireless sensor networks [3], and biomedical devices [4] due to the attractive characteristics of supercapacitors [5] such as long cycle life and high power density. To better utilize this energy storage technology, many aspects of supercapacitors have been investigated and several examples are system sizing [1], impedance characteristics [6], aging diagnosis [7], and state of charge (SOC) estimation [8]. Supercapacitor voltage is a vital parameter, which can be used an indicator of the supercapacitor SOC. Supercapacitor voltage is also utilized in cell balancing circuits. Multiple supercapacitor cells need to be connected in series to boost the voltage for microgrid applications. The individual cell voltage usually differs because of the manufacturing tolerance. To ensure safe and long-time operation of the supercapacitor bank, the cell voltages need to be equalized. When it comes to the supercapacitor voltage behavior, self-discharge [9-11] has been of great interest because it causes voltage drop and energy loss. While this characteristic is responsible for the long-term decay of the supercapacitor open-circuit voltage, the short-term behavior of the supercapacitor open-circuit voltage [12] is mainly determined by the charge redistribution process and the equivalent series resistance (ESR). A detailed study of the supercapacitor voltage behavior during charge redistribution [12-14] has been conducted using the supercapacitor variable leakage resistance (VLR) model [10, 11]. The impact of supercapacitor charge redistribution on power management in wireless sensor networks [15-17] has been illustrated by considering the task scheduling problem.

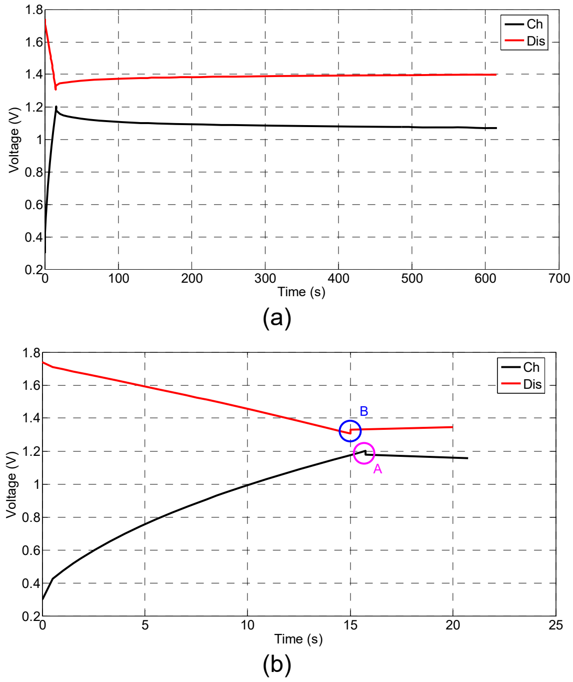
While the supercapacitor VLR model is a powerful tool for studying the open-circuit voltage behavior in detail, an estimate of the supercapacitor voltage change bounds also reveals critical information. On the other hand, the VLR model requires a relatively complex process to determine its parameter values. However, in some occasions the manufacturer datasheet is the only available resource to characterize the supercapacitor. Therefore, this paper aims to provide explicit formulas to estimate the bounds of the supercapacitor open-circuit voltage change after a constant power process based on information extracted from the supercapacitor datasheet.

The remainder of this paper first illustrates the supercapacitor open-circuit voltage change during charge redistribution and introduces a simplified supercapacitor model based on the manufacturer datasheet, then derives the lower and upper bounds of the supercapacitor open-circuit voltage change after a constant power action. After that, it presents the design and results of constant power experiments for four supercapacitor samples and concludes that the derived bounds apply to these samples.

BOUNDS OF SUPERCAPACITOR OPEN-CIRCUIT VOLTAGE CHANGE

Supercapacitor Charge Redistribution

To examine the supercapacitor open-circuit voltage change after a constant power process, Fig. 1 shows two charge redistribution experiments using a 10 F supercapacitor sample (manufacturer: Maxwell, model: BCAP0010, and rated voltage: 2.7 V): the "Ch" curve represents a charging process followed by charge redistribution and the "Dis" curve describes charge redistribution after a discharging action. In the "Ch" experiment, the supercapacitor is charged from 0.3004 to 1.2002 V by a constant power source of 0.4 W. At the end of the charging phase (t=15.735 s), the charging power is removed and the supercapacitor undergoes charge redistribution during the following 600 s, as shown in Fig. 1(a). The voltage change caused by the ESR is shown in Fig. 1(b). As denoted by circle A, the supercapacitor first experiences a sharp voltage drop immediately following the removal of the charging power because of the ESR: the voltage drops from 1.2002 (t=15.735 s) to 1.1765 V (t=15.740 s). The 0.005 s delay is the time resolution of the supercapacitor tester. Beginning at t=15.740 s, the supercapacitor experiences charge redistribution during the next 600 s and its voltage decreases to 1.0708 V at t=615.740 s. Taking the supercapacitor voltage (1.2002 V) at t=15.735 s as the initial value and the voltage (1.0708 V) at t=615.740 s as the final value, the voltage change is therefore -0.1294 V, which is composed of a drop of 0.0237 V (from 1.2002 to 1.1765 V) because of the ESR and a drop of 0.1057 V (from 1.1765 to 1.0708 V) because of charge redistribution.



(a)

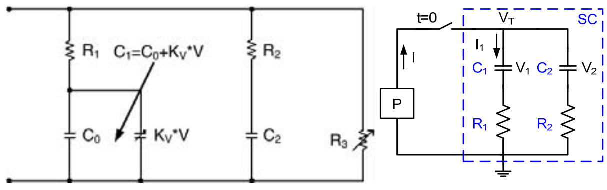
(b)

Fig. 1. Supercapacitor charge redistribution experiments. (a) Overview. (b) Voltage change due to ESR.

As for the "Dis" experiment, the supercapacitor is discharged from 1.7413 to 1.3049 V by a constant power source of 0.4 W in 15 s. After that, the discharging power is removed and the supercapacitor first experiences a sudden voltage boost because of the ESR, as denoted by circle B in Fig. 1(b): the voltage increases from 1.3049 (t=15 s) to 1.3290 V (t=15.005 s). The supercapacitor then undergoes charge redistribution during the following 600 s and the voltage increases to 1.3993 V at t=615.005 s, as shown in Fig. 1(a), which results in a voltage change of 0.0944 V after the discharging power is removed: a boost of 0.0241 V (from 1.3049 to 1.3290 V) because of the ESR and a boost of 0.0703 V (from 1.3290 to 1.3993 V) because of charge redistribution.

A Simplified Supercapacitor Model

While the VLR model shown in Fig. 2(a) is good for analyzing the detailed supercapacitor voltage behavior during charge redistribution, it is not well-suited for deriving explicit formulas to estimate the bounds of the supercapacitor open-circuit voltage change. To provide a better tool for predicting the supercapacitor voltage change because of charge redistribution and the ESR, this paper modifies the VLR model. As shown in Fig. 2(b), the simplified model in the dashed-line block consists of two RC branches: R_1 and C_1 for the first branch; R_2 and C_2 for the second branch. This model modifies the VLR model in four aspects. First, the variable leakage resistor R_3 is removed because the effect of self-discharge on the supercapacitor voltage change in the short term is insignificant [10, 11]. Second, the first branch capacitor C_1 is modeled as a constant capacitor. This is a reasonable assumption because charge redistribution takes place between the two RC branches [12-14] and the charge stored in each branch capacitor determines the supercapacitor voltage change. Therefore, the simplified model does not differentiate the charge stored in the first branch constant capacitor and the voltage-dependent capacitor. Third, the capacitances of the two branch capacitors are related by $C_2=\alpha C_1$, where α is a parameter typically ranging between 0.11 and 0.25. This assumption is based on [18] in which it is revealed that the slow branch capacitance (C_2) of a supercapacitor is a significant percentage of the total capacitance (C_1+C_2) and for most supercapacitor samples this percentage is between 0.1 and 0.2. A simple conversion gives the range of α: $0.11 \le \alpha \le 0.25$. Fourth, the resistance of the first branch resistor R_1 is represented by the ESR value specified in the supercapacitor datasheet because the experimentally determined value of R_1 is close to the ESR value [19, 20].



(a) (b)

Fig. 2. Supercapacitor models. (a) VLR model. (b) Simplified model.

Bounds of Supercapacitor Voltage Change

The simplified supercapacitor model shown in Fig. 2(b) is used to derive the bounds of the supercapacitor open-circuit voltage change, which is defined as the difference between the final voltage and the initial voltage. The initial voltage is the supercapacitor terminal voltage right before the charging/discharging power is removed. The final voltage is the supercapacitor terminal voltage when the charge redistribution process terminates. As shown in Fig. 2(b), the supercapacitor is connected to the power source P for t<0. At t=0, the switch is opened and the power source is disconnected. The supercapacitor terminal voltage V_T right before the switch is opened is denoted as $V_T(0-)$, which is written as

$$V_T(0-) = V_1(0-) + I_1(0-)R_1 \tag{1}$$

where $V_1(0-)$ is the voltage across the capacitor C_1 and $I_1(0-)$ is the current through the first branch. The charging/discharging power P is related to $V_T(0-)$ and I by

$$P = V_T(0-)I \tag{2}$$

Since the first branch is the main branch of the supercapacitor and the majority of the current flows through this branch [12, 13], it is then assumed that $I_1(0-)=I$. Additionally, based on the fact that the R_1 value is close to the ESR value, it is further assumed that $R_1=R$, where R is the ESR value extracted from the supercapacitor datasheet. Therefore, (1) is rewritten as

$$V_T(0-) = V_1(0-) + IR \quad (3)$$

The supercapacitor final voltage is evaluated when the charge redistribution process terminates, which means that the two branch capacitor voltages are equal and no current flows through the two branches. At this time, the supercapacitor terminal voltage equals the two branch capacitor voltages: $V_T(\infty)=V_1(\infty)=V_2(\infty)$. Assuming that the supercapacitor charge is conserved during the charge redistribution process, the final voltage $V_T(\infty)$ is then written as

$$V_1(0-)C_1 + V_2(0-)C_2 = V_T(\infty)(C_1 + C_2) \quad (4)$$

Given that $C_2=\alpha C_1$, the final voltage is determined as

$$V_T(\infty) = \big(V_1(0-) + \alpha V_2(0-)\big)/(1+\alpha) \quad (5)$$

The supercapacitor voltage change is therefore

$$\Delta V_T = V_T(\infty) - V_T(0-) \quad (6)$$

To estimate the bounds of the supercapacitor voltage change, the key is to evaluate $V_1(0\text{-})$ and $V_2(0\text{-})$, which cannot be experimentally measured by the supercapacitor tester. Therefore, it is necessary to relate them to the measurable supercapacitor terminal voltage right before the charging/discharging power is removed, which is denoted as V_M for clarity:

$$V_T(0-) = V_M \quad (7)$$

Together with (3), $V_1(0\text{-})$ is then written as

$$V_1(0-) = V_M - IR \quad (8)$$

As for $V_2(0\text{-})$, it can only be assumed that it is between 0 and the supercapacitor rated voltage V_R because its exact value is dependent on the previous charging/discharging process and cannot be readily related to the supercapacitor terminal voltage:

$$0 \le V_2(0-) \le V_R \quad (9)$$

Combining (2), (6), (8), and (9), the lower bound of the supercapacitor voltage change is

$$\Delta V_{TL} = (-\alpha V_M - PR/V_M)/(1+\alpha) \quad (10)$$

and the upper bound is

$$\Delta V_{TU} = (\alpha(V_R - V_M) - PR/V_M)/(1+\alpha) \quad (11)$$

As shown in (10) and (11), the following information is needed to estimate the bounds of the supercapacitor voltage change: (1) V_R and R: the rated voltage and ESR value extracted from the supercapacitor datasheet, (2) V_M and P: the measured supercapacitor terminal voltage and power right before the charging/discharging power is disconnected, and (3) α: a parameter typically ranging between 0.11 and 0.25 for most supercapacitors. Although the exact value of α cannot be determined without characterizing the supercapacitor, its range still provides useful information. The effects of the α value on the bounds can be illustrated using the simplified model. Physically, a higher α value means a larger C_2 value, a larger portion of charge to be transferred during the charge redistribution process, and ultimately a more significant voltage change.

SUPERCAPACITOR CHARGE REDISTRIBUTION EXPERIMENTS AND RESULTS

Experiments

To verify the supercapacitor voltage change bounds, the constant power experiments presented in [14] are analyzed, which are performed using the four supercapacitor samples listed in Table 1. The rated capacitance (C_R) varies from 0.1 to 100 F with a scale factor of 10. The rated voltage (V_R) includes two values: 2.7 and 5 V.

Table 1. Supercapacitor samples.

Sample	1	2	3	4
Manufacturer	Cooper Bussmann	Maxwell	Maxwell	Maxwell
Model	PB-5R0V104-R	BCAP0001	BCAP0010	BCAP0100
C_R (F)	0.1	1	10	100
V_R (V)	5	2.7	2.7	2.7

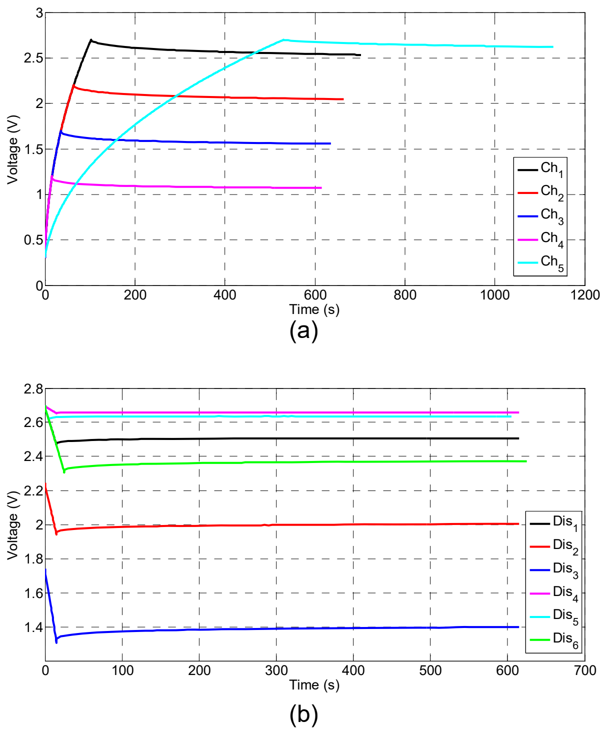


Fig. 3. Charge redistribution experiments for 10 F supercapacitor sample. (a) Constant power charge experiments. (b) Constant power discharge experiments.

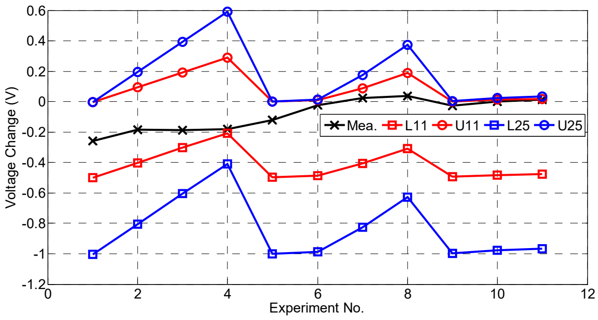
The design of constant power experiments is similar for all samples. Take the 10 F sample for instance. Fig. 3 shows the charge redistribution experiments: Fig. 3(a) for five constant power charge experiments and Fig. 3(b) for six constant power discharge experiments. Depending on the parameter swept, the experiments shown in Fig. 3(a) are divided into two groups: group 1 includes Ch_1 through

Ch4 and group 2 consists of Ch_1 and Ch_5. In group 1, the charge termination voltage is swept. The supercapacitor is charged by the same constant power of 0.4 W from the same initial voltage of 0.3 V to different final voltages: 2.7, 2.2, 1.7, and 1.2 V. The group 2 experiments are performed to study the impact of power while fixing the initial voltage of 0.3 V and termination voltage of 2.7 V: 0.4 W for Ch_1 and 0.08 W for Ch_5. The supercapacitor charge redistribution phase is 600 s for all experiments.

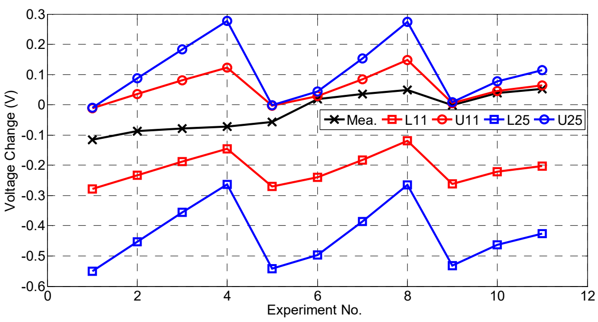
As shown in Fig. 3(b), six constant power discharge experiments are performed to sweep three parameters: discharge beginning voltage, power, and time, which are examined by three groups, respectively. Group 1 includes experiments Dis_1 through Dis_3. At the beginning of each experiment, the supercapacitor is conditioned to different initial voltages: 2.7, 2.2, and 1.7 V for experiments Dis_1 through Dis_3, respectively. Then the supercapacitor is discharged by a constant power of 0.4 W for 15 s. After that, the power is removed and the supercapacitor experiences charge redistribution during the following 600 s. Experiments Dis_1 and Dis_4 form group 2. The discharge beginning voltage is fixed at 2.7 V for both experiments. The discharging power for Dis_4 is 0.08 W. The discharge time and charge redistribution duration remain the same. The group 3 experiments Dis_1, Dis_5, and Dis_6 sweep the discharge time, which is 15, 5, and 25 s, respectively. The discharge beginning voltage (2.7 V), discharging power (0.4 W), and charge redistribution duration (600 s) apply to these three experiments.

Results

The supercapacitor voltage change bounds calculated using (10) and (11) as well as the measured values are shown in Fig. 4. The measurements are labeled as "Mea.". The lower and upper bounds calculated using α=0.11 are denoted by "L11" and "U11", respectively. Similarly, "L25" and "U25" are for α=0.25.



(a)



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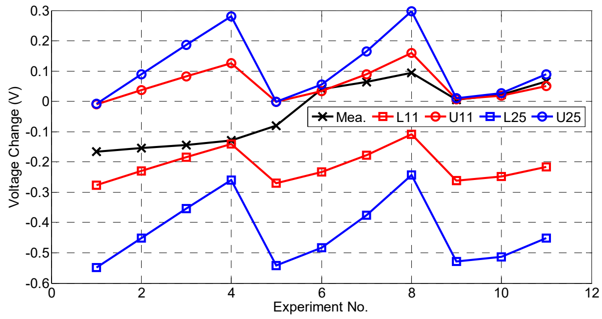

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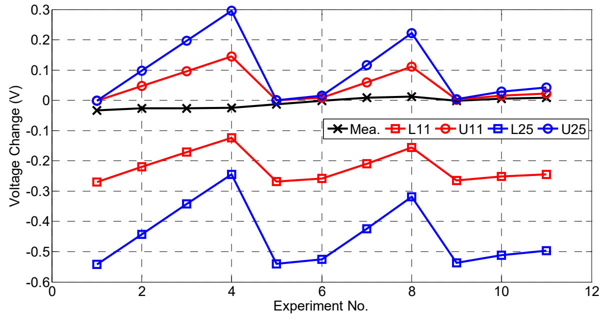

(d)

Fig. 4. Measured and estimated supercapacitor voltage changes. (a) 0.1 F. (b) 1 F. (c) 10 F. (d) 100 F.

Two observations can be made based on the results shown in Fig. 4. First, for the 0.1, 1, and 100 F samples, the measured supercapacitor voltage changes are within the bounds calculated using α=0.11, which is the minimum value of the typical range of the parameter α. Second, for the 10 F sample, although the measured values for experiments no. 6, 10, and 11 are not confined by the bounds when α=0.11, they are within the bounds when α =0.25, which is the maximum value of the typical range of α. For the remaining experiments, the measured values are within the bounds when α=0.11. These observations lead to the following conclusions. First, the supercapacitor open-circuit voltage change bounds derived based on the simplified supercapacitor model are valid for the supercapacitor samples and constant power experiments examined in this paper. Therefore, the bounds can be used to provide a quick estimate of the supercapacitor voltage behavior without going through the complex procedure to characterize the supercapacitor by performing and analyzing multiple experiments. Second, while all the other information needed for estimating the bounds can be extracted from the supercapacitor datasheet and the constant power experiment setup, the parameter α is only specified by a typical range between 0.11 and 0.25 and there is no obvious approach to relate this parameter to the supercapacitor specifications. In fact, even for samples with the same model number from the same manufacturer, a certain level of variation exists [18]. Therefore, more work needs to be conducted to further study this parameter.

CONCLUSION

This paper investigates the bounds of the supercapacitor open-circuit voltage change after a constant power process, which is mainly resulted from the charge redistribution process and the ESR. Explicit mathematical formulas for the lower and upper bounds are derived using a simplified supercapacitor model based on the information extracted from the manufacturer datasheet. Multiple constant power experiments are analyzed to verify the bounds. Results show that the bounds are valid for supercapacitor samples with different rated capacitance and voltage. Therefore, the derived formulas can be used to estimate the supercapacitor open-circuit voltage change bounds.

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