Optimal Control for Battery Storage Using Nonlinear Models

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***Abstract*- Battery storage systems (BSSs) have become increasingly popular for grid applications due to the growing need for flexibility and reserve in power systems with rapidly developed renewable generation. Successful assessment and deployment of a BSS require optimizing its operation, and thereby, maximizing the potential benefits. In existing studies on economic assessment and optimal scheduling of a BSS, modeling of charging/discharging operation and the corresponding impacts on state-of-charge (SOC) is over simplified, which could result in inaccurate assessment results and even infeasible operation schedules. This paper proposes a general model to capture varying SOC change rate as a nonlinear function of charging/discharging power and SOC level. An optimal control is developed for the BSS based on the proposed nonlinear model. The optimal control using a nonlinear model is compared with a representative linear optimization method using a simplified model through a real-world energy storage evaluation project to show the significance of the proposed method.**

***Keywords*- battery, energy storage system, dynamic programming, nonlinear model, optimization.**

NOMENCLATURE

$E_{\max}$	Battery energy capacity.
K	Number of time periods in optimization time window.
p_k	Power exchange between BSS and grid (measured at the grid connection point) during time period *k*, which is positive when injecting power into grid, i.e., using generator convention.
p_k^{batt}	Rate-of-change of energy stored in the battery at the end of time period *k*, which is positive when the battery is discharged.
$p_{\max}^{+}, p_{\max}^{-}$	Maximum power (i.e., measured at the grid connection point) that can be injected and withdrawn into/from grid, respectively.
$r_{\text{ch}}, r_{\text{disch}}$	State-of-charge (SOC) change rate per 100 kW for charging and discharging, respectively.
$\underline{S}_k, \overline{S}_k$	Lower and upper bounds of SOC during time period *k*, respectively.
s_k	Battery SOC at the end of time period *k*.
ΔT	Time step size.
η^{+}, η^{-}	Discharging and charging efficiency of the battery storage, respectively, including components such as conductor, power electronics, and battery.
$\eta(\cdot)$	Marginal round-trip efficiency as a function of SOC.
λ_k	Energy price of time period *k*.

I. INTRODUCTION

Operation of the electric power sector requires flexibility to realize instantaneous balance between generation and constantly changing demand. Energy storage has been a candidate for meeting such a flexibility requirement for years. With the rapid growth of renewable energy, the inherent uncertainty and variability present difficulties and challenges to power system operators. Recent developments and advances in energy storage and power electronics technologies are making their application a viable solution for grid problems. As many countries place greater emphasis on renewable generation, energy storage is becoming increasingly important and holds substantial promise for transforming the electric power industry.

Many studies have been devoted to optimization and evaluation of BSSs for various grid applications. Studies [1] and [2] are dedicated to various battery technologies and methods of assessing their economic viability and impacts on power systems. In 2007 [3], the authors evaluate the economic performance of NaS batteries for energy arbitrage and flywheels for regulation services, based on fixed utilization factors for NYISO and PJM systems. Reference [4] incorporates realistic CAISO regulation signals and battery responses to yield more granular results. In [5], the authors investigate the application of BSS to relive transmission line thermal constraints, and thereby, increase the transfer capability. Based on a case study, an economic analysis of benefits and costs is provided. Reference [6] presents a distributed algorithm to optimally coordinate energy storage with distributed generators. In [7], an evaluation framework and co-optimization are proposed to assess BSS economic performance considering multiple grid applications simultaneously, including energy arbitrage, balancing services, capacity value, distribution upgrade deferral, and outage mitigation. Reference [8] considers an application bundle, including energy and demand charge reduction. A programming-based method is developed for economic assessment and optimal sizing of behind-the-meter BSSs. In [9], the authors develop a peak-shaving control algorithm to determine battery charging and discharging operation, and then calculate the economic benefits in demand charge reduction. In

[10], an analytical optimal sizing method is proposed based on objective quantitative analysis of costs and benefits for customer-side BSS, which could identify key factors that affect optimal sizing.

These studies optimize the charging/discharging operation to best utilize the limited power and energy capacity of BSS and then assess the economic performance accordingly. Nevertheless, the methods that are used to determine the optimal BSS charging/discharging schedule are not capable of accurately modeling BSS operation. For example, most of these studies (e.g., [5] and [9]) simply use a constant round-trip efficiency (RTE) to capture BSS losses. However, the same RTE with different one-way charging/discharging efficiencies may yield different optimal operating schedules. More importantly, due to the inability to represent one-way efficiencies in optimization, one cannot accurately estimate the SOC during charging (or discharging), and therefore, could obtain an infeasible operating schedule. While the optimal control methods in some existing studies such as [7] are based on one-way efficiencies, they can handle constant efficiencies, but not varying efficiencies. In [11], energy storage is studied for energy and demand charge reduction using a circuit model that expresses battery terminal voltage and current as nonlinear functions of the SOC. The circuit model assumes constant internal voltage and requires significant efforts to identity battery rated energy rated capacity and model parameters. It may not be able to provide required accuracy and is difficult to implement in practice. Furthermore, it is not capable of modeling varying charging/discharging capabilities at different SOC levels. To overcome these limitations, this paper proposes a general BSS model that expresses the SOC change rate as a nonlinear function of charging/discharging power and SOC level. An optimal control method is then developed to utilize the nonlinear model to determine an optimal schedule of BSS operation and evaluate the corresponding benefits.

The rest of this paper is organized as follows: Section II reviews a typical existing method using constant efficiencies to determine the optimal control of a BSS. In Section III, we first discuss the shortcomings and limitations of existing methods with constant efficiency models, and propose a nonlinear model that can more accurately capture the impacts of SOC and operating power on BSS operation. Then, we propose an innovative optimal control method that is capable of incorporating the proposed general nonlinear model for BSS scheduling and evaluation. In Section IV, the proposed and existing methods are used for energy arbitrage analysis in a real-world energy storage project as an example to show the significance of the proposed method. Finally, concluding remarks are offered in Section V.

II. Existing Methods with Constant Efficiency Model

In this section, we review a representative method using a BSS model with constant efficiency to determine optimal battery control for economic assessment and operational scheduling. Because the amount of energy stored in a BSS is limited, the charging/discharging operations at different time periods are interdependent. For example, injecting more energy into the grid in one hour increases the benefits for that hour, but leaves less energy for future use, and therefore may reduce the overall economic benefits. Therefore, the optimal scheduling must be performed over multiple time periods. A BSS also has charging/discharging power capacity, for which different grid services may compete against each other. For example, increasing discharging power for energy arbitrage service decreases the battery's ability to provide other services. Moreover, there are losses associated with a BSS charging/discharging operation, which must be modeled and considered in the optimal scheduling formulation in order to obtain a profitable and effective operating plan.

In the case of energy arbitrage, the objective function is the net benefits of battery charging/discharging for given hourly energy prices over a look-ahead time horizon, as expressed in (1)

$$\sum_{k=1}^{K} \lambda_k p_k \Delta T \,, \tag{1}$$

where p_k is the power exchange between BSS and the grid (measured at the grid connection point) during time period *k*, which is positive when injecting power into grid, K is the number of time periods in the optimization time window, λ_k is the energy price of time period k, and ΔT is time step size. The charging/discharging power must be within the operating range considering both battery and energy conversion system power rating,

$$-p_{\max}^{-} \le p_k \le p_{\max}^{+} \,, \tag{2}$$

Where $p_{\max}^{-}$ and $p_{\max}^{+}$ are the maximum charging and discharging power of the BSS, respectively. The rate of change of energy stored in BSS p_k^{batt} is related to charging/discharging power at grid coupling point p_k using the charging/discharging efficiencies as

$$p_k^{\text{batt}} = \begin{cases} p_k/\eta^{+} & \text{if } p_k \ge 0 \text{ (discharging)}, \\ p_k \eta^{-} & \text{if } p_k < 0 \text{ (charging)}, \end{cases} \tag{3}$$

where η^{+} and η^{-} are the discharging and charging efficiencies, respectively. The change in SOC can be calculated as

$$\Delta s_k = p_k^{\text{batt}} \Delta T / E_{\max} \,, \tag{4}$$

where $E_{\max}$ is the rated energy capacity of the BSS. Finally,

the dynamics of SOC can be expressed as

$$s_{k+1} = s_k - \Delta s_k \,, \tag{5}$$

where s_k is the SOC level of the BSS at the end of time period *k*, and Δs_k expresses the change in SOC during time period k. The SOC level needs to be restricted to be between its lower and upper bounds as expressed in (6), either for safe operation of the BSS or to meet user specifications.

$$\underline{S}_k \le s_k \le \overline{S}_k \,. \tag{6}$$

With the objective function and various constraints, we are now ready to present the optimization problem formulation to determine the optimal charging/discharging operation for energy arbitrage application as follows.

$$\mathbf{P}_1 : \max_{p_k, p_k^{\text{batt}}, \Delta s_k, s_k} \sum_{k=1}^{K} \lambda_k p_k \Delta T \,, \tag{7}$$

subject to constraints from (2) to (6). It has been shown in [7] and [8] that optimization tricks can be applied to convert optimization problem $\mathbf{P}_1$ to a standard linear programming problem, which is then solved to determine the optimal charging/discharging operation.

III. Proposed Optimal Control Method with Nonlinear Model

This section first discusses the shortcomings and limitations of existing methods using constant efficiency and rated discharging/charging power. A general nonlinear model is then proposed to better represent varying charging/discharging power capabilities and efficiencies at different SOC and output power levels. Finally, an optimal control is developed to utilize the proposed general nonlinear battery models for optimal scheduling and economic assessment. The same energy arbitrage application presented in the previous section is again used as an example to better explain and compare the proposed method with the existing method.

In the method presented in Section II, the change in SOC with different charging/discharging power is estimated using battery-rated capacity and constant charging/discharging efficiencies. Such a method is subject to several disadvantages and limitations:

- The energy that can be discharged or charged to the battery depends on discharging/charging power. Using a single rated value $E_{\max}$ for different power operation cannot accurately model the capability of a BSS.
- The feasible charging/discharging power also depends on the SOC. The BSS may not be able to operate at any value within $[-p_{\min}, \; p_{\max}]$ for some SOC.
- The overall one-way efficiencies of the BSS need to be estimated based on battery efficiency, inverter efficiency, power for auxiliaries, and other factors. The estimation of each of these components requires approximation and introduces error that can be compounded.
- The charging/discharging efficiency also varies with a BSS operation, which cannot be accurately modeled using constant efficiencies.

A general nonlinear model is proposed to address these limitations, as shown in (8) and (9),

$$p \in \mathcal{P}_s \tag{8}$$

$$\Delta s = f(p, s) \tag{9}$$

where *p* denotes the charging/discharging power from a BSS, $\mathcal{P}_s$ denotes feasible set of *p* for an SOC level of *s*, and Δs denotes SOC change rate, which is a function of *p* and *s*.

Such a nonlinear model can be obtained by experimenting and operating a BSS under various conditions such as operating mode, power, SOC, and temperature. The entire process of constructing the nonlinear BSS model can be automated by programming experiment and using a script to process the recorded data. The SOC level is measured and recorded for different charging/discharging power outputs. With the outputs, we can easily determine the feasible operating power ($\mathcal{P}_s$) at different SOC (*s*), and determine change of SOC (Δs) as a function of charging/discharging power (*p*) and SOC (*s*). As an example, a 1 MW/3.2 MWh vanadium redox BSS is evaluated for an array of charging/discharging power. The corresponding Δs function in (9) is plotted in Fig. 1.

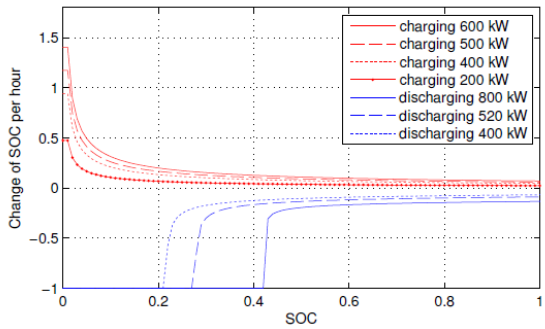

Fig. 1. SOC change rate versus SOC level for different charging/ discharging power levels.

As can be seen, the change in SOC rate varies with the SOC level. At the same SOC level, different charging and discharging power levels also affect the change in the SOC rate. In addition, one can identify the feasible operating power set at different SOC levels in (8). For example, when $s \ge 43\%$, the BSS can be operated on all 7 charging/discharging power levels. When $43 \ge s \ge 28\%$, the BSS cannot be discharged at

800 kW, and Ps only contains the remaining 6 operating power levels.

Based on the nonlinear model, optimal control of a BSS is developed as follows.

For any time period *k*, with 1) the feasible operating power range for different s_k and 2) expression of Δs (analytical or as look-up tables), we can relate the change in SOC in each time period *k* to the operating power and SOC,

$$p_k \in \mathcal{P}_{s_k}\,, \tag{10}$$

$$\Delta s_k = f(p_k, s_k)\,. \tag{11}$$

Note that the constant efficiency model represented by constraints from (2) to (4) can also be converted to the same form as (10) and (11). Therefore, we can formulate a more simplified but general optimization problem as follows:

$$\mathbf{P}_2: \max_{p_k, \Delta s_k, s_k} \sum_{k=1}^{K} \lambda_k p_k \Delta T\,, \tag{12}$$

subject to constraints (10), (11), (5), and (6). Such a method removes the need to estimate the rated energy capacity and discharging/charging efficiencies and improves the modeling accuracy related to how different charging/discharging operations affect the SOC. Note that both $\mathbf{P}_1$ and $\mathbf{P}_2$ are deterministic optimization and do not explicitly address the uncertainty associated with the prices. For operation scheduling, one can take the expected value of prices as input and the optimization maximizes the expected benefit. The receding horizon control helps to mitigate the impacts of price uncertainty on the optimal solution.

Compared with $\mathbf{P}_1$, the formulation in $\mathbf{P}_2$ better models the BSS operation, but is more challenging to solve because it is generally a nonlinear and nonconvex optimization problem. The simplest solution strategy is the enumeration method, but this method is generally computationally prohibitive. For example, with a 24-hour look-ahead window and 15-minute time step size, there are 96 time periods the BSS operation needs to be explored. If we discretize the feasible SOC range at each period into 100 values, the number of possible charging/discharging operation combinations is 10,096. The charging/discharging power limits can eliminate some infeasible combinations. Nevertheless, this still leaves us with a possible solution space with extremely high dimensionality. The dynamic programming (DP) method has many advantages over the enumeration scheme. The most important one is the reduction in the dimensionality of the problem. With DP, infeasible combinations can be detected *a priori*, and information about previously investigated combinations can be used to eliminate inferior combinations. This will significantly improve efficiency. A DP algorithm is developed in [11] to minimize the customer electricity bill based on a circuit model. Herein, a DP algorithm is proposed solve the optimization in $\mathbf{P}_2$ using the proposed nonlinear battery model to maximize the revenue from energy arbitrage. The scheduling problem is first divided into stages, and each stage *k* represents a scheduling period (e.g., 15 minutes). Each stage is divided into states $\{s_k\}$. A state represents the SOC level and encompasses the information (including the state trajectory previous to current stage and the corresponding benefit) required to move from one state in a stage to another state in the next stage. At each stage *k*,

1) all feasible operations are first explored.
   a) The feasible operating power at the grid coupling point is determined based on (10).
   b) The corresponding change of the SOC is calculated for different feasible power levels based on (11).
   c) The corresponding SOC is evaluated using (5) and checked against (6) to eliminate any infeasible operation.
   d) The corresponding cost/revenue is calculated for all feasible operating power levels based on the objective function in (12).
2) the maximum arbitrage value in stage *k* with state *J* is then calculated as

$$R(k,J) = \max_I \left[U(k-1,I;k,J) + R(k-1,I)\right] \tag{13}$$

where $R(k,J)$ is the maximal arbitrage value at state (k,J), and $U(k-1,I;k,J)$ is the energy revenue/cost associated with power discharging/charging operation that transits from state $(k-1,I)$ to (k,J).

IV. Case Study

The Washington State Clean Energy Fund (CEF) focuses on deployment and demonstration of energy storage in an effort to explore its role in Washington State and to assess its value to Washington State's utilities and citizens [12]. To maximize the value of the CEF, Pacific Northwest National Laboratory has worked with Washington State and three winning teams, including Avista Utilities, Snohomish PUD, and Puget Sound Energy, to demonstrate and assess a diverse scope of applications for energy storage, such as energy arbitrage, regulation and load following services, Volt/Var control, load-shaping, outage mitigation, and deferment of distribution system upgrade. The evaluation framework together with these demonstration projects will inform and empower other utilities in Washington State and in the region, storage technology developers, and state regulators to prudently and confidently pursue the deployment of energy storage. In this paper, as an example, energy arbitrage assessment is performed for a UniEnergy Technologies battery system to show the significance of the proposed method. The battery system has

been placed and tested at Turner substation in Pullman, Washington. The economic evaluation is performed for a BSS that contains two identical vanadium-flow battery assemblies with total combined ratings of 2 MW/6.4 MWh. While the BSS is capable of providing 6.4 MWh from fully charged to fully discharged, about 10.7 MWh is required to recharge the BSS, resulting in an average RTE equal to 0.6. The SOC change rate versus SOC with different charging/discharging levels for a single assembly (1 MW/3.2 MWh) is plotted in Fig. 1. The Mid-Columbia prices from 2011 to 2015 have been obtained from Powerdex [13] and used for arbitrage analysis in this work.

The optimal charging/discharging operations are determined using 1) optimization $\mathbf{P}_1$ with constant efficiency and discharging/charging power capability (existing method), and 2) optimization $\mathbf{P}_2$ with the nonlinear model (proposed method). The corresponding annual benefits are plotted in Fig 2. As can be seen, the two methods generate very different results. The estimated benefits using the proposed method are much higher than those of the existing method in all 5 years, and the difference is as much as 80% of the annual benefits from the existing method.

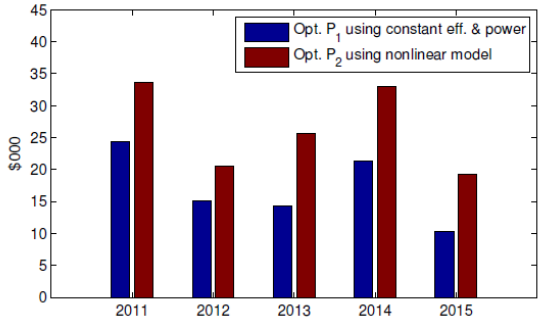

Fig. 2. Annual benefits in energy arbitrage.

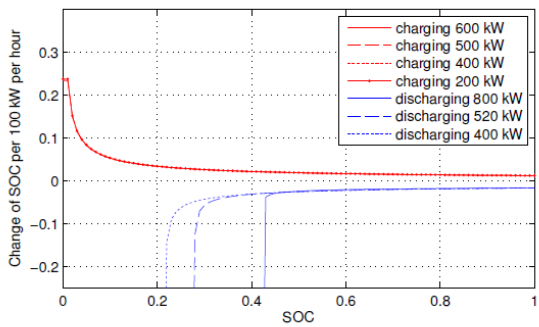

Fig. 3. SOC change rate per 100 kW versus SOC for different charging/discharging power levels.

To understand cause of the difference, the characteristics of a BSS are further explored. The SOC change rate per 100 kW versus the SOC for different charging/discharging power levels is plotted in Fig. 3. This can be understood as an indicator that is equivalent to charging/discharging efficiency; it also shows how much the SOC is reduced (or increased) to obtain 100 kW discharging (or charging) power for per unit time. It is interesting to see that for the MESA 2 BSS, charging (or discharging) at different power levels results in the same efficiency, which only varies with the SOC. When discharging, the operable power capability also varies with the SOC. The marginal RTE at a different SOC can be calculated as

$$\eta(s) = \frac{r_{\text{ch}}(s)}{r_{\text{disch}}(s)} \tag{14}$$

where s denotes SOC level, and $r_{\text{ch}}(\cdot)$ and $r_{\text{disch}}(\cdot)$ are the SOC change rate per 100 kW as a function in SOC for charging and discharging, respectively. The marginal RTE is plotted in Fig. 4.

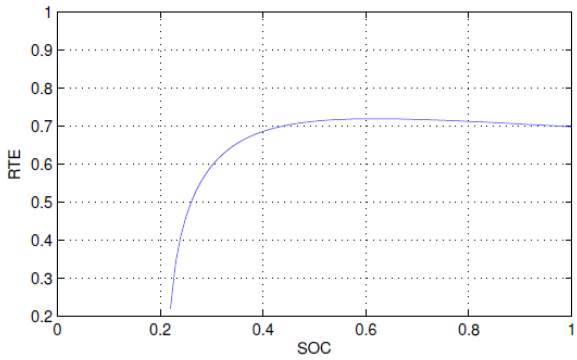

Fig. 4.Marginal round-trip efficiency versus SOC.

As can be seen, as SOC increases from 20% to 100%, marginal RTE increases rapidly before 40%, reaches the maximum around 50%- 60%, and then decreases slightly. It is interesting to note that although cycling a BSS from full to empty and then to full results in an average RTE equal to 60%, a BSS can be operated with a better efficiency when the SOC is above 30%. Therefore, the optimization $\mathbf{P}_1$ using a constant RTE of 60% underestimates the efficiency of a BSS for many possible operations and leaves the BSS on standby for many time periods when arbitrage could be profitable.

To better show this, the energy prices, charging/discharging operations, and SOC from both methods are plotted for two days, in 2015, in Fig. 5.

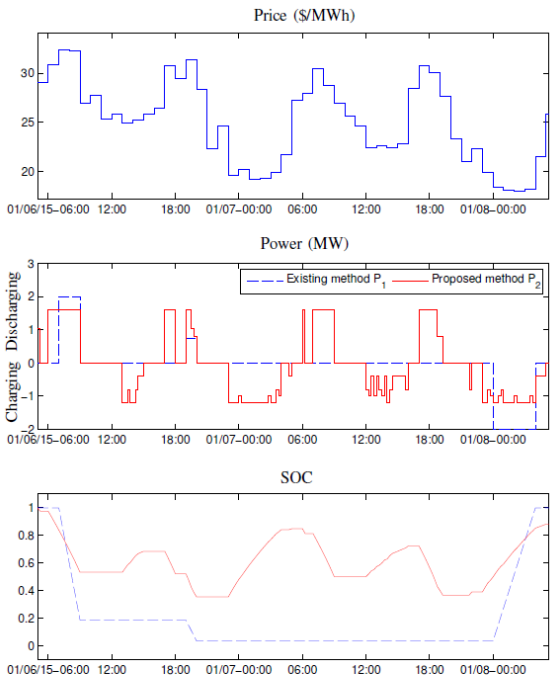

Fig. 5. Charging/discharging operation and SOC in sample days.

The hours at the beginning and end of the sample period correspond to very high and low prices, respectively. Both methods generate similar battery discharging and charging operation at the beginning and end of the sample period because the price difference is big enough compared with RTE and energy arbitrage using BSS is profitable. However, existing method $\mathbf{P}_1$ outputs some infeasible operation. For example, the BSS is discharged at 2 MW from 7 to 9 a.m. and the SOC decreases from 100% to 20%. In fact, the BSS can only be discharged at this full power output within a very limited SOC range. For the other hours, existing method $\mathbf{P}_1$ leaves the BSS in standby most of the time because, using a constant RTE of 60%, the price difference is not big enough to recover 40% losses in energy arbitrage. On the other hand, the proposed method $\mathbf{P}_2$ with a nonlinear model is capable of accurately exploring the BSS operating space at different operating power and SOC levels, takes into account the varying losses, finds profitable operation, and operates the BSS at a higher efficiency region to maximize the benefits from energy arbitrage.

V. Conclusion and Future Work

This paper presents a novel nonlinear battery model and optimal control method for evaluation and operational scheduling of BSS. Compared with existing methods, the proposed method can better capture varying charging/discharging efficiencies and charging/discharging power capabilities, and therefore can generate more realistic and optimal operation of a BSS for grid applications. The case study using a commercial BSS shows that failure to incorporate accurate nonlinear models in optimal scheduling could result in significant errors in assessment of economic benefits and an infeasible operation schedule. In future work, we plan to apply the proposed method with a nonlinear BSS model for other grid applications such as regulation service, distribution deferral, and demand charge reduction.

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