

# Estimating Potential Revenue from Electrical Energy Storage in PJM

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**Abstract**—FERC order 755 and FERC order 784 provide pay-for-performance requirements and direct utilities and independent system operators to consider speed and accuracy when purchasing frequency regulation. Independent System Operators (ISOs) have differing implementations of pay-for-performance. This paper focuses on the PJM implementation. PJM is a regional transmission organization in the northeastern United States that serves 13 states and the District of Columbia. PJM’s implementation employs a two part payment based on the Regulation Market Capability Clearing price (RMCCP) and the Regulation Market Performance Clearing Price (RMPCP). The performance credit includes a mileage ratio. Both the RMCCP and RMPCP employ an actual performance score. Using the PJM remuneration model, this paper outlines the calculations required to estimate the maximum potential revenue from participation in arbitrage and regulation in day-ahead markets using linear programming. Historical PJM data from 2014 and 2015 was then used to evaluate the maximum potential revenue from a 5 MWh, 20 MW system based on the Beacon Power Hazle Township flywheel plant. Finally, a heuristic trading algorithm that does not require perfect foresight was evaluated against the results of the optimization algorithm.

## I. INTRODUCTION

In deregulated electricity markets storage is ultimately only as valuable as the revenue stream generated by the storage device, regardless of the application or benefit. This revenue stream comes from participating in markets for energy and ancillary services (e.g., frequency regulation, operating and contingency reserves) [1]. In regulated regions, vertically integrated utilities must invest in technologies that provide reliable electricity to the consumer at the lowest cost. In this scenario, electricity storage must be compared to the cost of competitive technologies that provide the capabilities required by the utility. An additional source of revenue is government incentives designed to guide future investment decisions based on the public good.

The two potential revenue streams considered in this paper are energy arbitrage and participation in the regulation market. Arbitrage involves purchasing (charging) energy when prices are low, e.g., during times of low demand, and selling (discharging) energy when prices are high, e.g., during times of peak demand.

Regulation up (RegUp) and down (RegDown) are ancillary services designed to maintain frequency stability. Sometimes they are combined into a single regulation product. If the load increases while generation is held constant, the frequency will drop. In order to maintain tight tolerances on the frequency,

generation must be constantly dithered so that load and generation are equal. Depending on the market, a balancing authority or vertically integrated utility will control generation on a second by second basis to track the load. The balancing authority must reserve enough regulation capacity to meet expected variations in load.

Regulation up is the ability to provide additional generation on command. Regulation down is the ability to reduce generation, or store power, on demand. Until recently, the practice was to reimburse regulation providers based mainly on capacity reserved along with compensation for any electricity that is purchased or sold. This approach did not compensate fast-responding systems for more accurately following commanded regulation signals and the increased benefit provided compared to slower resources. FERC order 755 and FERC order 784 provide pay-for-performance requirements and direct utilities and independent system operators to consider speed and accuracy when purchasing frequency regulation [2], [3]. Independent System Operators (ISOs) have differing implementations of pay-for-performance. This paper focuses on the PJM implementation.

A framework is outlined in this paper for calculating the maximum revenue from an electricity storage system that participates in a day-ahead market, i.e., energy arbitrage, and in a regulation market. The approach is designed to calculate the best-case scenario using historical data to simulate operation with perfect day-ahead energy and reserve price forecasts. This best-case scenario calculation is critical because it provides an upper bound on the revenue that can be collected by a storage facility and can be used to score other trading strategies. Hence, it is useful in estimating an upper bound for the value of a storage facility. Cost data is required to perform a cost-benefit analysis for a particular system and location. Information on the capital and operational costs of different energy storage technologies may be found in [4]. It should also be noted that this approach is only valid for scenarios where the size of the storage is such that it does not impact market prices. For large systems that might impact the market, a production cost modeling approach must be implemented.

The approach in this paper formulates the revenue maximization problem as a linear program. The energy storage model and optimization formulation builds on the results in [5], where the authors present a stochastic framework for the valuation of electricity storage. Revenue from energy arbitrage and the regulation ancillary services market are only two of

the potential benefits of electricity storage devices. A complete review of potential revenue streams is outlined in [6], [7]. An early summary of potential arbitrage revenue in various markets is found in [8].

Previous results using a similar approach (without pay-for-performance) were presented in [9], [10], [11]. The algorithm, results for CAISO data (including a sensitivity analysis for each parameter), and results for several implementable trading algorithms appear in [9]. ERCOT results for a single node, two years of data, and implementable trading algorithms are presented in [10]. All nodes in ERCOT were analyzed over a three year period to look at the impact of location and to identify longer term trends in [11]. This paper extends the optimization approach to include pay-for-performance as implemented by PJM, and presents results for a system modeled after the Beacon Hazle Township flywheel plant [12].

This report is organized as follows: Section II provides an overview of the PJM pay-for-performance implementation. Section III presents the energy storage model that is used throughout this paper. Section IV provides the revenue maximization problem formulation. Section V presents results for a 5 MWh, 20 MW energy storage system modeled after the Beacon plant. Concluding remarks are found in Section VI.

## II. PJM PAY-FOR-PERFORMANCE

Motivated by FERC order 755 [2], the industry is evolving towards pay-for-performance where compensation is based on the amount of work performed by a device and the payment must reflect the device's accuracy when following a regulation signal. A good example of this model is the PJM Interconnection, which is a regional transmission organization (RTO) that coordinates the movement of wholesale electricity in all or parts of Delaware, Illinois, Indiana, Kentucky, Maryland, Michigan, New Jersey, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia, West Virginia and the District of Columbia [13]. Remuneration is based on the capability offered as well as the performance provided [14]. Both payments are weighted by a performance score. The performance score is calculated for each hourly interval from three components: the delay score, the correlation score, and the precision score [15]. At this time, all three are weighted equally. The capability component, described in equation (1), is a function of the Regulation Market Capability Clearing Price (RMCCP) [14]. The capability credit is the hourly-integrated regulation,  $REG_t$ , times the actual performance score for the hourly period,  $\eta_t$ , times the RMCCP.

$$\text{RMCCP credit} = REG_t \times \eta_t \times \text{RMCCP}_t \quad (1)$$

For fast responding resources, the performance component described in (2) is a function of the Regulation Market Performance Clearing Price (RMPCP) [14]. The performance credit is the hourly-integrated regulation,  $REG_t$ , times the actual performance score for the hourly period,  $\eta_t$  times the mileage ratio,  $\beta_t^M$ , times the RMPCP.

$$\text{RMPCP credit} = REG_t \times \eta_t \times \beta_t^M \times \text{RMPCP}_t \quad (2)$$

PJM offers two different regulation signals:  $RegA$  and  $RegD$ .  $RegA$  is a low pass filtered area control error (ACE) signal designed for traditional regulating resources.  $RegD$  is a high pass filtered ACE signal for faster responding resources like energy storage. For  $RegD$  systems, the PJM mileage ratio,  $\beta_t^M$ , is defined as:

$$\beta_t^M = \frac{\text{RegD Mileage}}{\text{RegA Mileage}} \quad (3)$$

Mileage is simply defined as the movement requested by the regulation control signal. For example, the  $RegD$  mileage is defined as:

$$\text{RegD Mileage} = \sum_{i=1}^N |RegD_i - RegD_{i-1}| \quad (4)$$

over the one hour time period. The PJM mileage ratio increases the compensation for faster responding resources. The increased mileage results from following a signal with higher frequency content. The total compensation for a plant providing regulation services is the sum of the RMCCP credit and the RMPCP credit.

The next section covers the model of an energy storage system.

## III. ELECTRICITY STORAGE MODEL

The key parameters that characterize a storage device are:

- 1) *Power Rating*: [MW] The maximum power of the storage device (charge and discharge).
- 2) *Energy Capacity*: [Joules or MWh] The amount of energy that can be stored.
- 3) *Efficiency*: [%] Efficiency can be broken down into two components: conversion efficiency,  $\gamma_c$ , and storage efficiency,  $\gamma_s$ . Conversion efficiency describes the losses encountered when input power is stored in the system. Storage efficiency describes the time-based losses in a storage system.
- 4) *Ramp Rate*: [MW/min] the ramp rate describes how quickly the storage device can change its power level.

For the analysis in this paper, we are concerned with the quantity of energy charged or discharged during each time period for each potential activity (e.g., arbitrage or regulation). For arbitrage, the device will maintain a constant output power over each time period. For regulation, it is assumed that the device is capable of tracking the regulation signal. We also assume the ramping time is negligible (i.e., energy storage ramp rates are high). If the ramp rate is slow compared to the time period, this approximation does not hold and a model that incorporates ramp rate must be employed.

The parameters in Table I are those involved in storage system constraints. Thus, the maximum quantity that can be sold/discharged in a single period is equivalent to:

$$\bar{q}^D = (\text{Maximum discharge power level}) \times \tau \quad (5)$$

Likewise, the maximum quantity that can be bought/recharged in a single period is equivalent to:

$$\bar{q}^R = (\text{Maximum recharge power level}) \times \tau \quad (6)$$

TABLE I  
STORAGE PARAMETERS

Symbol	Storage Parameter
$\tau$	Time period length (e.g., one hour).
$T$	Number of time periods in optimization.
$\bar{q}^D$	Maximum energy sold in a single period (MWh).
$\bar{q}^R$	Maximum energy bought in a single period (MWh).
$\bar{S}$	Maximum energy storage capacity (MWh).
$\gamma_s$	Storage efficiency over one period (%).
$\gamma_c$	Conversion efficiency (%).

For a storage device that provides only one service, there are two decision variables in the optimization: the energy sold  $q_t^D$  (discharged) at time  $t$ , and the energy purchased  $q_t^R$  (recharged) at time  $t$  in MWh. They are assumed to be non-negative quantities. In this case, the state of charge (SOC)  $S_t$  at any time  $t$  is given by:

$$S_t = \gamma_s S_{t-1} + \gamma_c q_t^R - q_t^D \quad \forall t \in T \quad (7)$$

which states that the SOC at time  $t$  is the SOC at time  $t-1$  adjusted for storage losses plus any net charging (adjusted for conversion losses) minus the quantity discharged during  $t$ . Additional constraints include:

$$0 \leq S_t \leq \bar{S}, \quad \forall t \in T \quad (8)$$

$$0 \leq q_t^R \leq \bar{q}^R, \quad \forall t \in T \quad (9)$$

$$0 \leq q_t^D \leq \bar{q}^D, \quad \forall t \in T \quad (10)$$

For a device that is participating in arbitrage and the regulation market, a few additional parameters must be added into the storage device model. An additional decision variable must be added to capture the quantity bid in to the regulation market,  $q_t^{REG}$ . For this analysis, it is assumed that the assigned quantity is equal to the bid quantity ( $q_t^{REG} = REG_t$ ). This decision variable is assumed to be a non-negative quantity. In regulation markets, there is no guarantee that the capacity reserved will actually be deployed. A representative PJM *RegD* regulation command signal is shown in Figure 1.

In order to quantify the change in SOC from participation in the regulation market, it is useful to define the RegUp efficiency  $\gamma_t^{RU}$  as the fraction of the RegUp reserve capacity that is actually deployed at time  $t$ . Similarly, the RegDown efficiency  $\gamma_t^{RD}$  is the fraction of the RegDown reserve capacity that is actually deployed at time  $t$ . In the actual operation of a storage system,  $\gamma_t^{RU}$  and  $\gamma_t^{RD}$  will vary over each time interval. To formulate the problem as an LP optimization, a known value must be employed. Fortunately, PJM provides historical regulation signals so it is possible to calculate  $\gamma_t^{RU}$  and  $\gamma_t^{RD}$  at each time step. Thus, the SOC at time  $t$  for a device participating in arbitrage and regulation is given by:

$$S_t = \gamma_s S_{t-1} + \gamma_c q_t^R - q_t^D + \gamma_c \gamma_t^{RD} q_t^{REG} - \gamma_t^{RU} q_t^{REG} \quad (11)$$

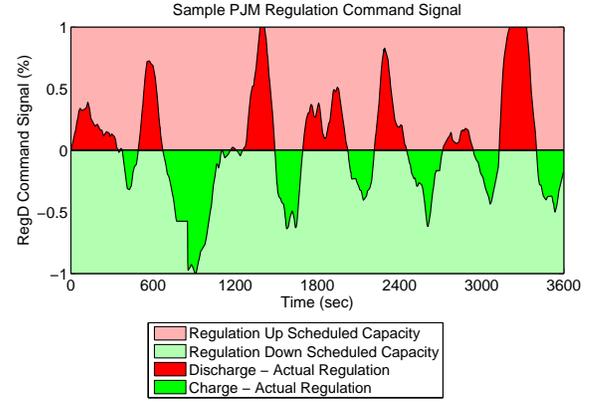


Fig. 1. Representative PJM *RegD* regulation command signal (June 1, 2014) [16].

And it is complemented by the following constraints:

$$0 \leq S_t \leq \bar{S}, \quad \forall t \in T \quad (12)$$

$$0 \leq q_t^R + q_t^{REG} \leq \bar{q}^R, \quad \forall t \in T \quad (13)$$

$$0 \leq q_t^D + q_t^{REG} \leq \bar{q}^D, \quad \forall t \in T \quad (14)$$

The quantity allocated to regulation,  $q_t^{REG}$ , reduces the maximum potential quantities allocated to arbitrage subject to the charge/discharge constraints of the device.

#### IV. MAXIMIZING STORAGE REVENUE

The problem of maximizing revenue from an energy storage device is naturally formulated as an LP optimization problem [17]. The energy storage model presented above is combined with a cost function to maximize the revenue for arbitrage combined with participation in the regulation market. The objective function when the storage device participates in arbitrage and regulation is given by:

$$\max \sum_{t=1}^T [(P_t - C_d)q_t^D - (P_t + C_r)q_t^R + q_t^{REG} \eta_t (\beta_t^M RMPCP_t + RMCCP_t)] e^{-rt} \quad (15)$$

where each term is defined as:

$P_t$	LMP for energy at time $t$ (\$/MWh)
$C_d$	Cost for discharging (\$/MWh)
$C_r$	Cost for recharging (\$/MWh)
$q_t^D$	Energy discharged at time $t$ (MWh)
$q_t^R$	Energy charged at time $t$ (MWh)
$q_t^{REG}$	Regulation capability at time $t$ (MWh)
$\eta_t$	Performance score at time $t$ (%)
$\beta_t^M$	Mileage ratio at time $t$
$RMPCP_t$	Regulation Market Performance Clearing Price (\$/MWh)
$RMCCP_t$	Regulation Market Capability Clearing Price (\$/MWh)
$e^{-rt}$	Discounting term (time value of money)

In many areas, the net energy for regulation is settled at the real-time price. This provides an additional arbitrage opportunity between the day ahead price and the real-time price. For this analysis, the price  $P_t$  was assumed to represents

both. While this does not reflect the actual settlement process, it keeps the optimization from incorporating any arbitrage between the day ahead and the real-time market.

Constraints shown in (11)-(14) complete the optimization problem for maximizing revenue from arbitrage and regulation. The solution is the energy bought and sold at each time step as well as the amount offered into the regulation market that maximizes the storage unit revenue. The PJM model employs a single market for regulation (RegUp and RegDown).

The next section applies these optimization techniques to estimate the maximum potential revenue for a 5 MWh, 20 MW energy storage device modeled after the Beacon Power Hazle Township, Pennsylvania flywheel plant.

### V. CASE STUDY

This section presents results for a representative energy storage system located in PJM, the Beacon Power Hazle Township flywheel plant [12]. The Beacon facility was developed, built, and commissioned through the DOE Smart Grid Demonstration Program. It comprises 200 flywheels that can source or sink 20 MW for 15 minutes. The facility, owned and operated by Hazle Spindle LLC (a subsidiary of Rockland Power Partners, LP), sells frequency regulation services to PJM. An aerial photo of the plant is shown in Figure 2.



Fig. 2. Beacon Power plant at Hazle Township [18].

Historical financial and regulation signal data for the analysis was obtained from the PJM website [16]. One year of data was considered, spanning from June 2014 to May 2015. Day ahead energy prices for the HAZLETON 1-4 node (PnodeID 11473605) were used. The RMPCP, RMCCP, and mileage ratio were available directly from the PJM website. The values for  $\gamma_t^{RU}$  and  $\gamma_t^{RD}$  were calculated using the 2-second regulation signal available on the PJM website (using trapezoidal integration). The RegD regulation data on the PJM website is normalized from -1 to 1. Assuming that the regulation bid is assigned, the commanded regulation signal is calculated by multiplying the RegD signal times the bid quantity. The parameters for the Beacon plant are shown in Table II. For this analysis, a discount rate of  $r = 0$  was employed. The Pyomo optimization modeling language was used to arrive at the results [19], and the optimization was run on monthly data.

The arbitrage and regulation results for a representation of the Beacon Hazle Township flywheel plant using perfect

TABLE II  
ENERGY STORAGE SYSTEM PARAMETERS.

Parameter	Value
$\bar{q}^D$	20 MWh
$\bar{q}^R$	20 MWh
$\bar{S}$	5 MWh
$\gamma_s$	0.98
$\gamma_c$	0.85
$\eta_t$	0.95

knowledge for June 2014 to May 2015 data are summarized in Table III. The columns  $\% q^R$ ,  $\% q^D$ , and  $\% q^{REG}$  represent the fraction of time spent performing each activity. As expected, the optimal policy is to participate in the frequency regulation market the majority of the time. Given the characteristics of a flywheel plant, i.e., high power and low energy, the Beacon plant is well suited for the characteristics of the PJM market compared to a plant that is designed primarily for energy time shifting (arbitrage). For the time frame considered, there is considerable variation in the maximum monthly potential revenue, ranging from \$341,281.46 to \$998,392.65. The revenue from the capacity credit, mileage payment, and arbitrage is presented in Table IV. This data shows the impact of the mileage payment on total remuneration. The mileage payment accounts for approximately 24% of total revenue. While the arbitrage credit is positive for the year, it is negative for most months. This is likely the result of procuring energy to maintain a state of charge required to participate in frequency regulation.

TABLE III  
ARBITRAGE AND REGULATION OPTIMIZATION RESULTS USING PERFECT KNOWLEDGE, JUNE 2014-MAY 2015.

Month	$\% q^R$	$\% q^D$	$\% q^{REG}$	Revenue
06/14	0.65	0.41	98.67	\$487,185.94
07/14	1.22	0.38	98.06	\$484,494.90
08/14	1.20	0.38	98.06	\$354,411.61
09/14	1.23	0.52	97.73	\$401,076.97
10/14	1.30	0.38	97.85	\$535,293.84
11/14	1.71	0.58	96.43	\$431,106.41
12/14	1.07	0.50	96.92	\$341,281.46
01/15	0.80	1.10	97.34	\$443,436.10
02/15	1.03	1.37	96.59	\$998,392.65
03/15	0.87	0.71	98.41	\$723,692.29
04/15	0.90	0.20	98.76	\$527,436.11
05/15	1.02	0.37	98.62	\$666,290.70
			<b>Total</b>	\$6,394,098.97

In order to estimate the potential revenue without perfect knowledge (e.g., knowing the future), the following strategy was tested:

- Bid 20 MW into the regulation market every hour
- For hours that result in state of charge violations, stop following the signal and receive no remuneration

TABLE IV  
ARBITRAGE AND REGULATION OPTIMIZATION RESULTS  
USING PERFECT KNOWLEDGE, JUNE 2014-MAY 2015.  
COMPARISON OF REVENUE STREAMS.

Month	RMCCP Credit	RMPCP Credit	Arbitrage Credit	Total Revenue
06/14	\$356,412.73	\$130,286.06	\$487.16	\$487,185.94
07/14	\$351,131.53	\$135,123.18	-\$1,759.82	\$484,494.90
08/14	\$231,708.06	\$124,760.87	-\$2,057.32	\$354,411.61
09/14	\$280,496.49	\$121,979.31	-\$1,398.84	\$401,076.97
10/14	\$389,520.38	\$148,445.40	-\$2,671.94	\$535,293.84
11/14	\$315,773.83	\$117,698.79	-\$2,366.21	\$431,106.41
12/14	\$250,525.71	\$92,077.48	-\$1,321.73	\$341,281.46
01/15	\$335,093.93	\$102,707.75	\$5,634.43	\$443,436.10
02/15	\$837,537.28	\$141,229.67	\$19,625.70	\$998,392.65
03/15	\$561,451.79	\$160,354.43	\$1,886.07	\$723,692.29
04/15	\$373,388.33	\$155,942.07	-\$1,894.29	\$527,436.11
05/15	\$537,115.47	\$129,786.70	-\$611.47	\$666,290.70
<b>Total</b>	<b>\$4,820,155.53</b> <b>75.38%</b>	<b>\$1,560,391.71</b> <b>24.40%</b>	<b>\$13,551.74</b> <b>0.21%</b>	<b>\$6,394,098.97</b> <b>100%</b>

The state of charge was assumed to be 50% at the beginning of each time period. A state of charge violation occurred if the state of charge exceeded the range of 0-100%. Using this strategy, there were 453 hours over the course of the year where the plant would not have the capacity to follow the RegD signal (approximately 5.2% of the time). If zero revenue is assumed for these hours, the total revenue for the year comes to \$5,936,912.40, or 92.5% of the maximum calculated from the optimization. A slightly more sophisticated algorithm would likely produce better results.

## VI. CONCLUSION

In this paper, a linear programming optimization approach was outlined for estimating the maximum potential revenue from an energy storage system participating in arbitrage and the regulation market with pay-for-performance. The approach was tailored towards PJM's pay-for-performance implementation. If cost data is available, the same methodology can be used to estimate net revenue. Maximum potential revenue using perfect foresight is a key metric for evaluating implementable trading strategies. Using 2014-2015 price data and a model of the Beacon Power Hazle Township flywheel plant, the maximum potential revenue from arbitrage and frequency regulation was estimated assuming perfect foresight. The optimal policy was to engage primarily in frequency regulation, which is well suited for the Beacon Power business model. A trading strategy that does not require perfect foresight was also evaluated, and it captured 92.5% of the maximum revenue calculated from the optimization. Future research will focus on developing energy storage control algorithms that maximize revenue without relying on perfect prediction.

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