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NOMAD XIII FINAL PRESENTATION

Chaotic Vibration Response of Nonlinear Components under Forced Vibration

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Org.# 1556

Presented at the NOMAD XIII Final Presentations on August 4, 2026

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AGENDA

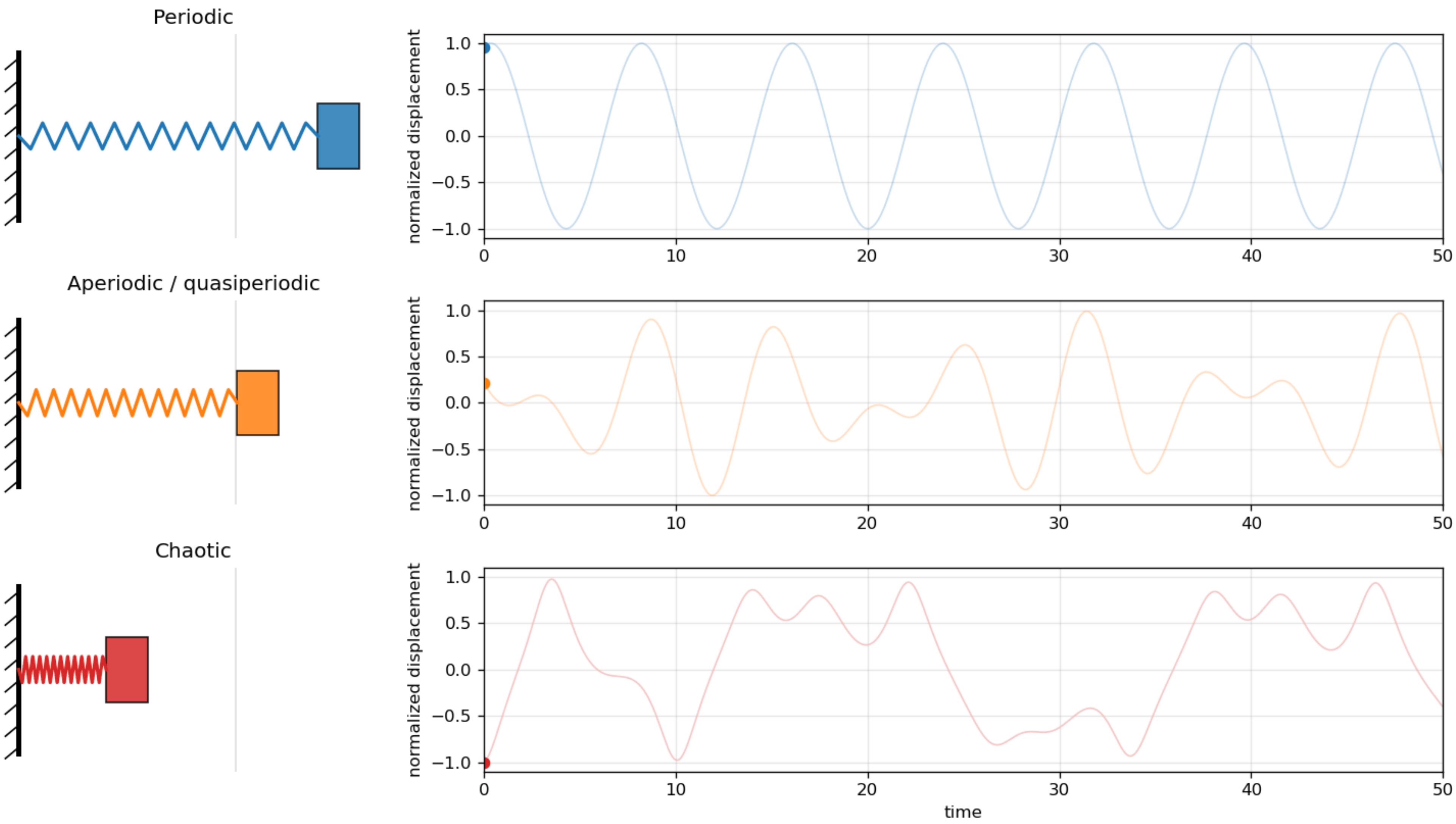
- Motivation
- Approach
- Modeling and Experimental Setup
- Model Calibration
- Results
- Future Work
- Conclusion

MOTIVATION | **IMPACT AND CHAOS**

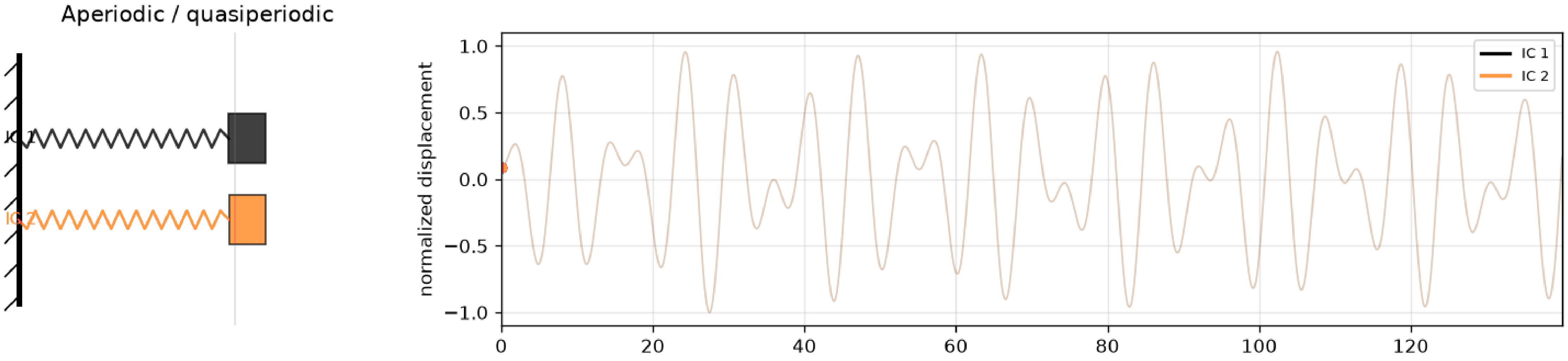
[2, Aussie50, YouTube 2013]

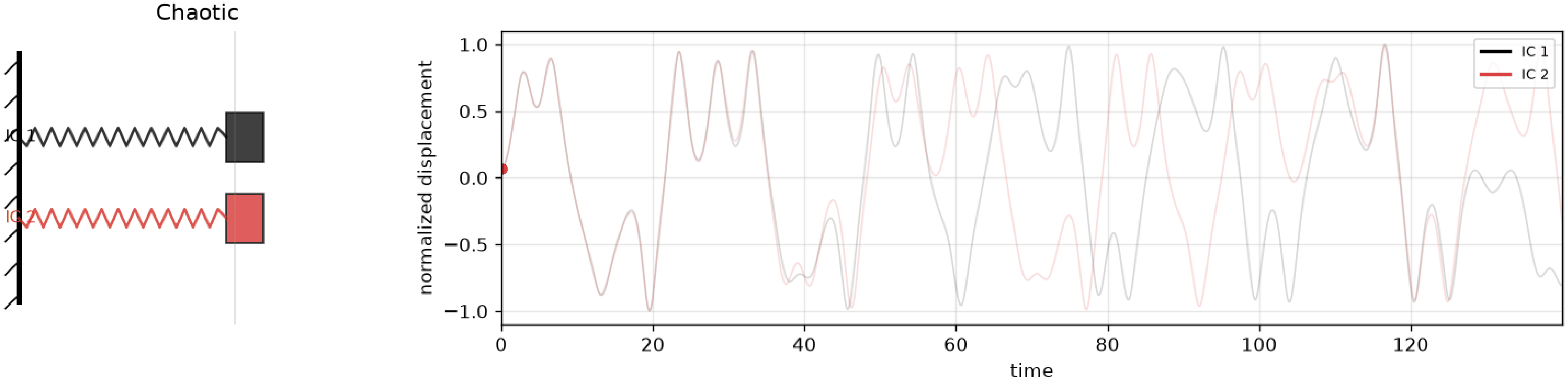
MOTIVATION | **TYPES OF VIBRATION RESPONSE**

Periodic vs. Aperiodic vs. Chaotic



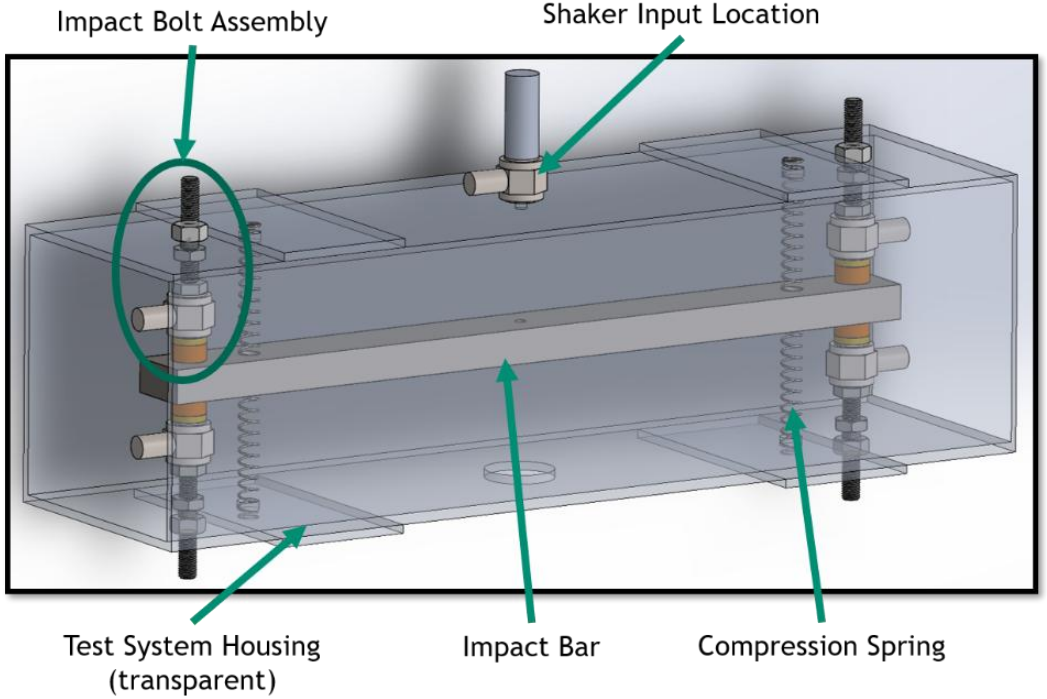
Perturbations to Initial Conditions



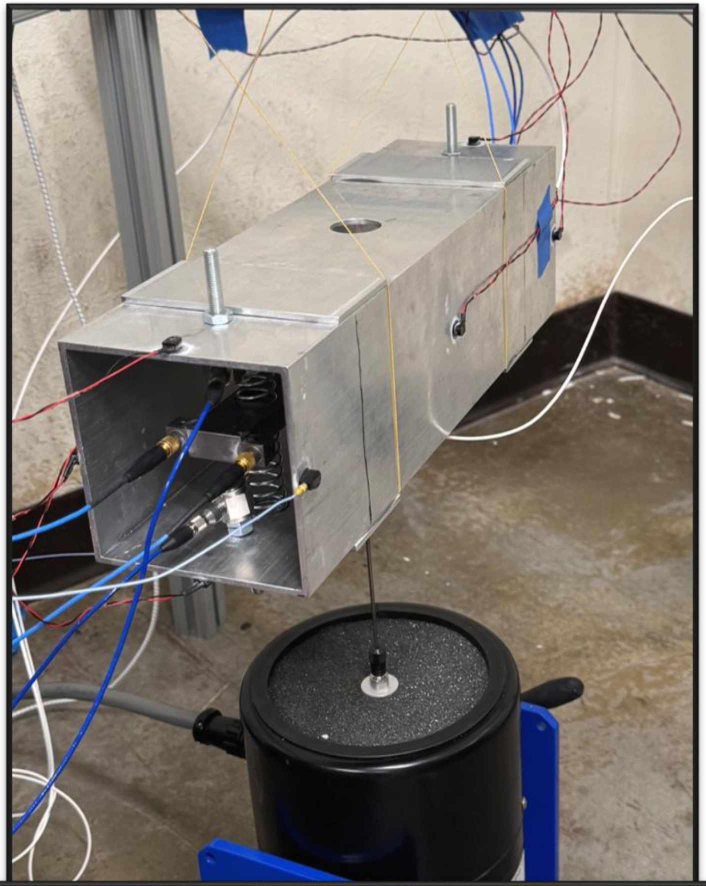


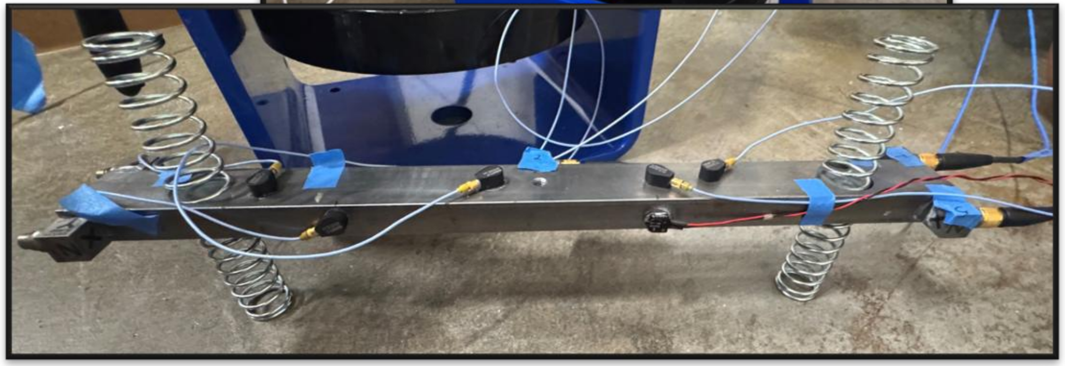
MOTIVATION | **SYSTEM**

- Benchmark vibro-impact system
- May produce chaotic response
- Pseudo-rigid body mode: 19.8 Hz



[1, Saunders et al. 2024]





MOTIVATION | **SCIENTIFIC QUESTION**

Can we build a model that can accurately predict if and when the vibration response of the vibro-impact bar is chaotic?

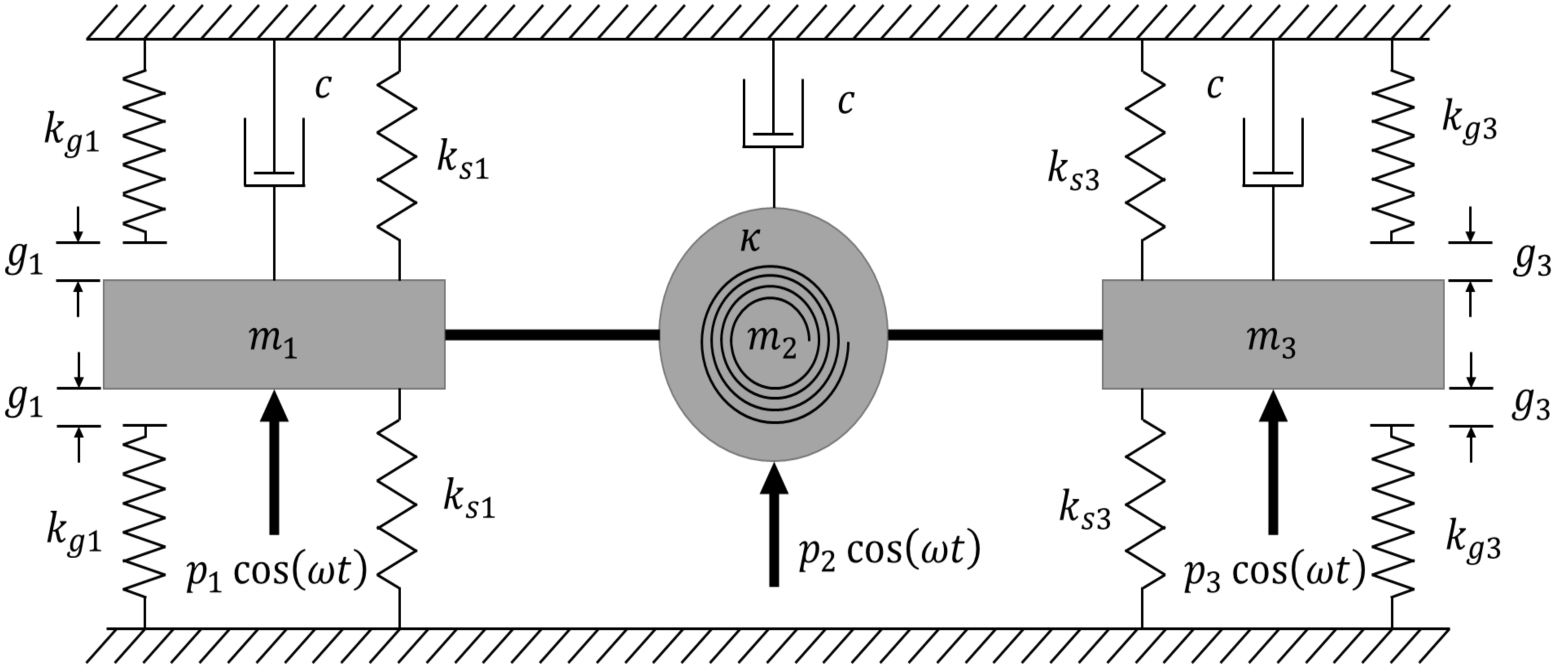
APPROACH | **CHAOS DETECTION FOR TIME SIGNALS**

- PS1 Test
  - Topological PS1 score via KDE sublevel-set persistence [5]
  - A **lower PS1 score indicates chaotic dynamics**
- 0-1 Correlation Test
  - Gottwald-Melbourne 0-1 test [4]
  - **K ≈ 1 is indicates chaotic dynamics**
- Lyapunov Exponents
  - Largest exponent (Rosenstein method) or full Lyapunov spectrum (Eckmann method) [3]
  - A largest **exponent greater than 0 indicates chaotic dynamics**

MODELING | **APPROACH**

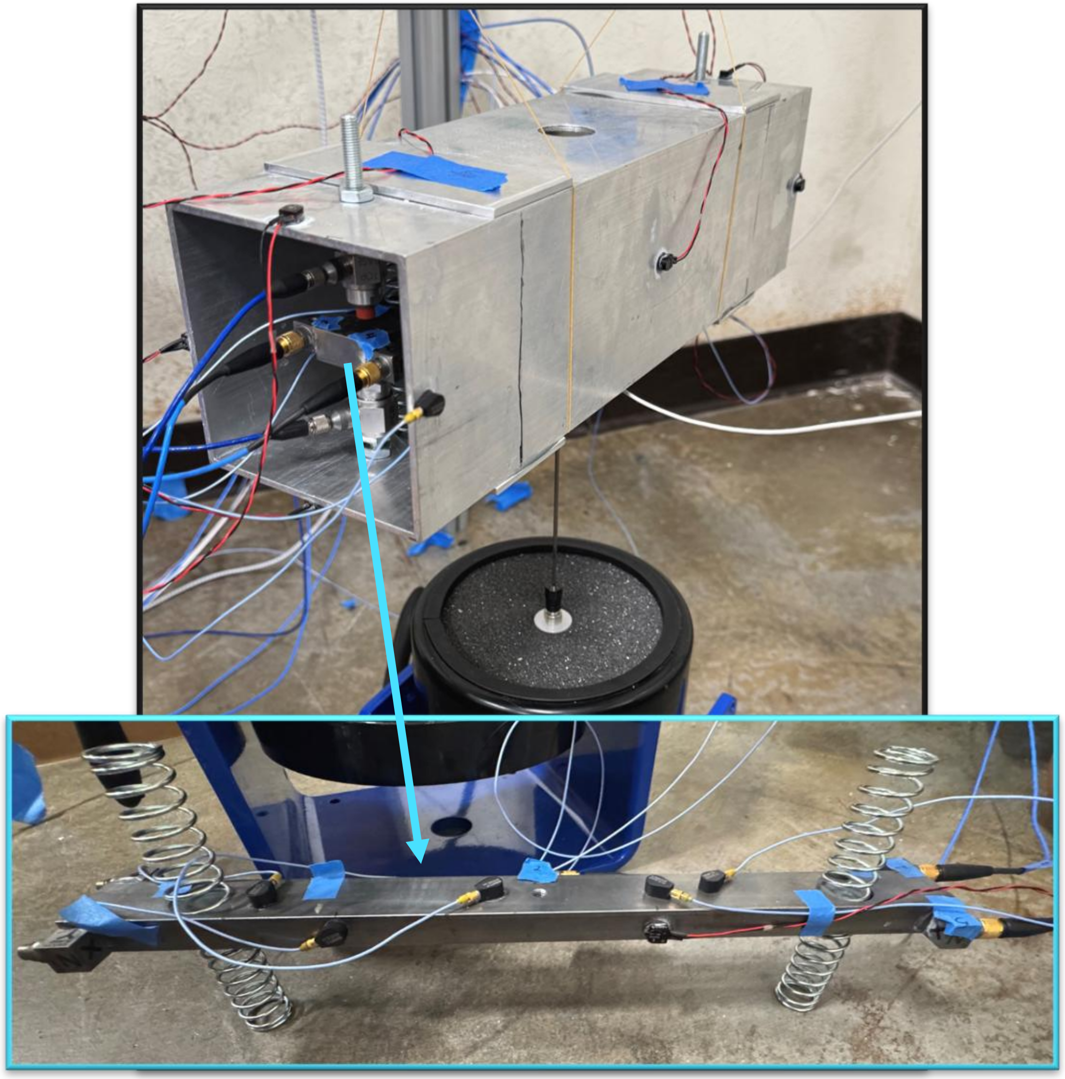
- Create a 3-DOF lumped-mass model

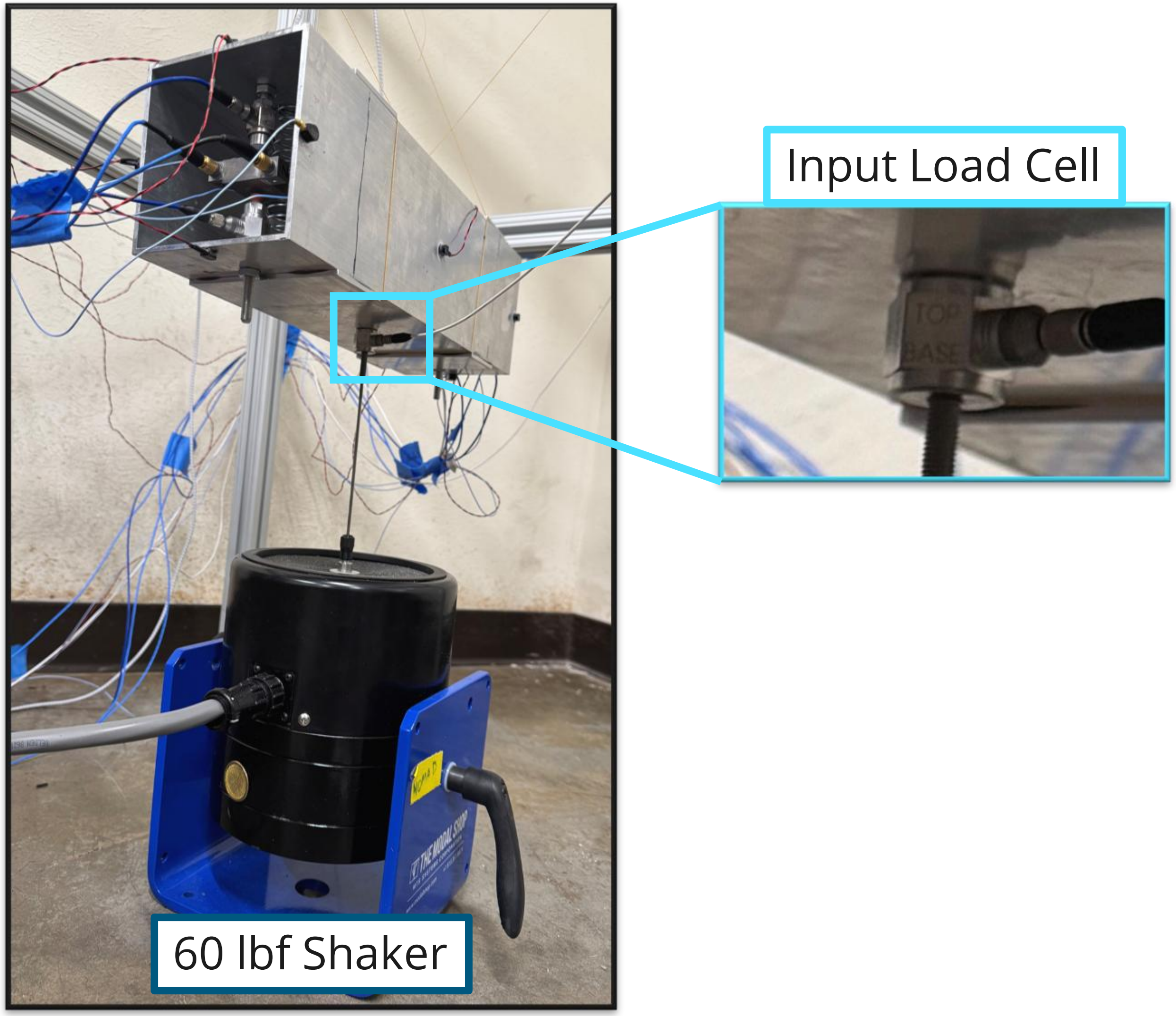
$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} 2k_s + \frac{\kappa}{L^2} & \frac{-2\kappa}{L^2} & \frac{\kappa}{L^2} \\ \frac{-2\kappa}{L^2} & \frac{4\kappa}{L^2} & \frac{-2\kappa}{L^2} \\ \frac{\kappa}{L^2} & \frac{-2\kappa}{L^2} & 2k_s + \frac{\kappa}{L^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} f_{g1}(x_1) \\ 0 \\ f_{g3}(x_3) \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \sin(\omega t)$$



[1]

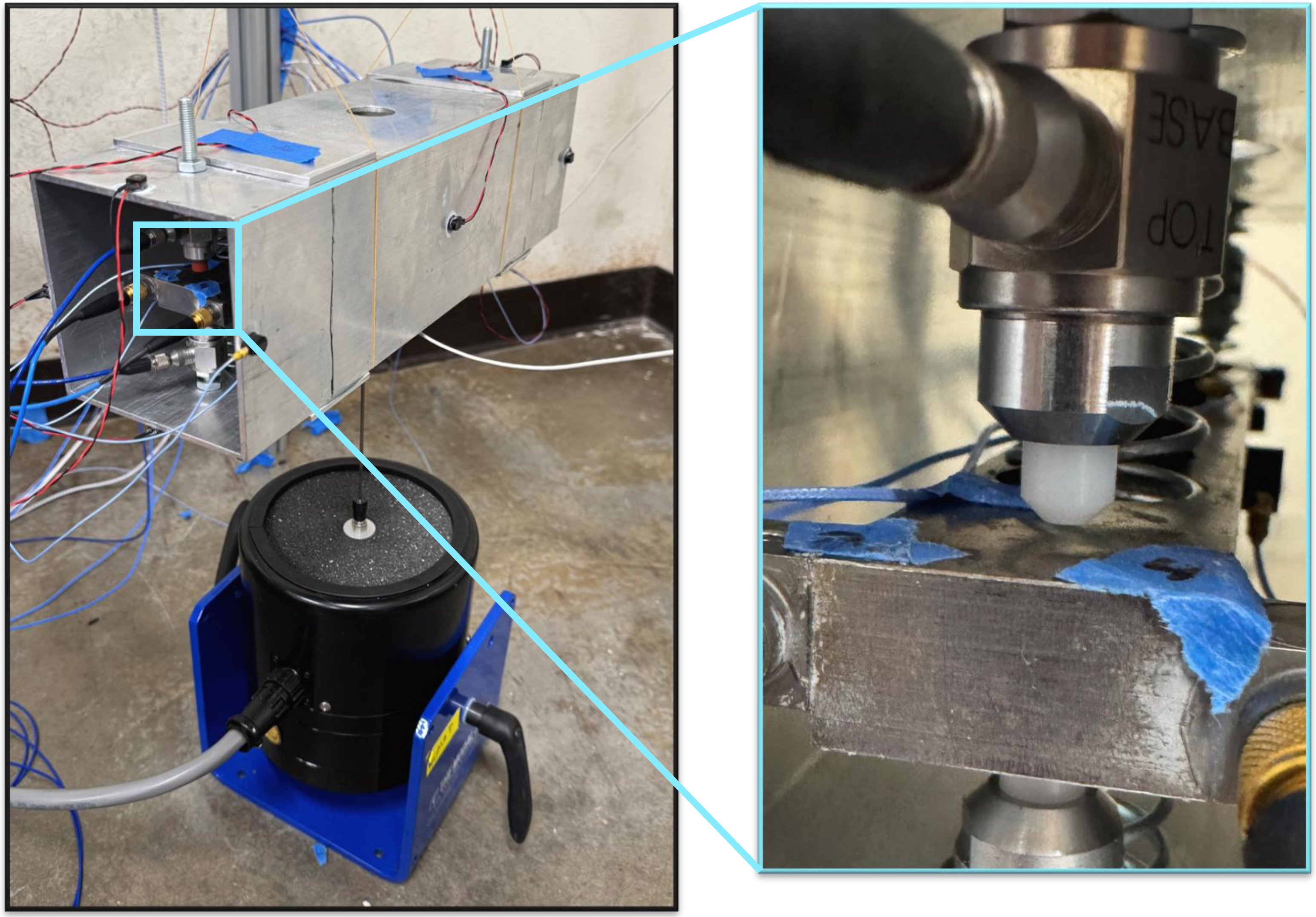
EXPERIMENTAL SETUP





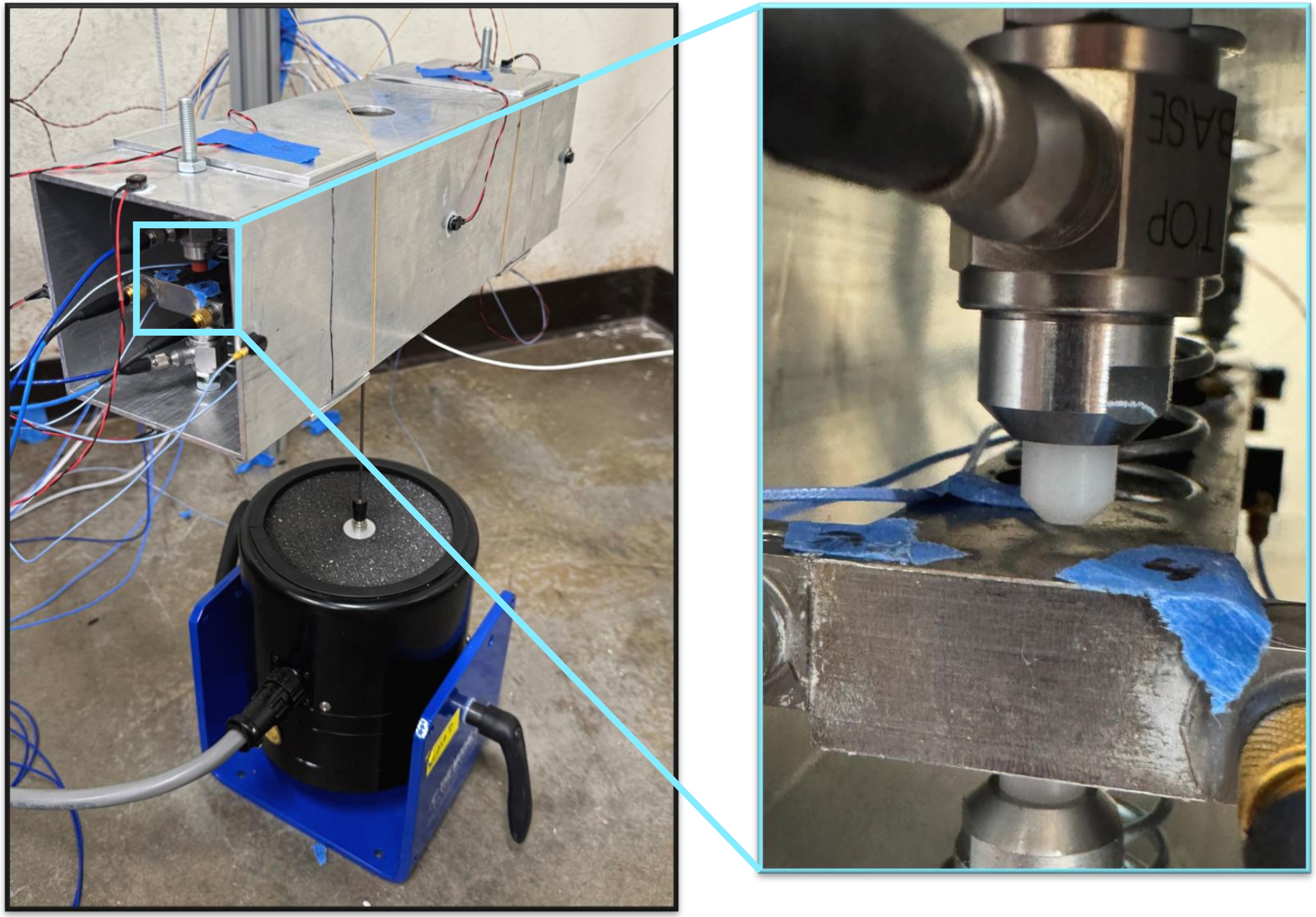
EXPERIMENTAL SETUP

- Shaker attached to box
- Suspended with bungee cords
- Impact rods
  - Hammer Tip
  - Load Cell
  - Adjustable

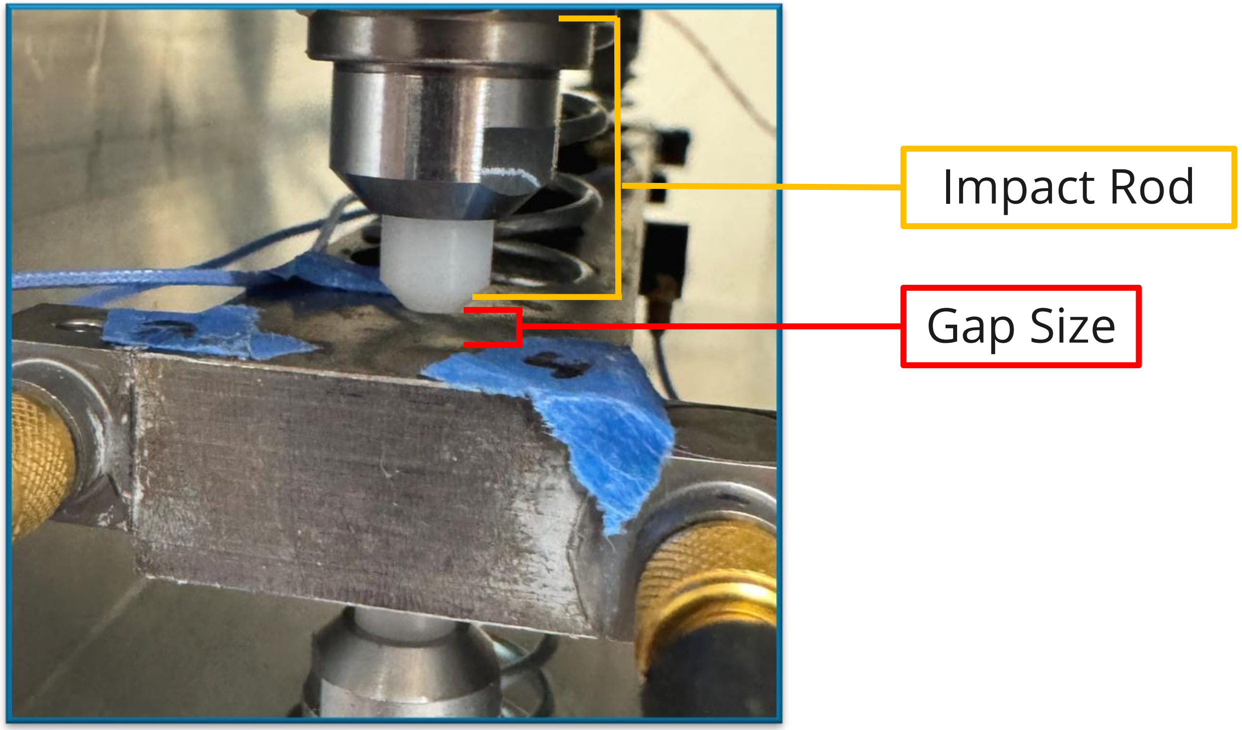


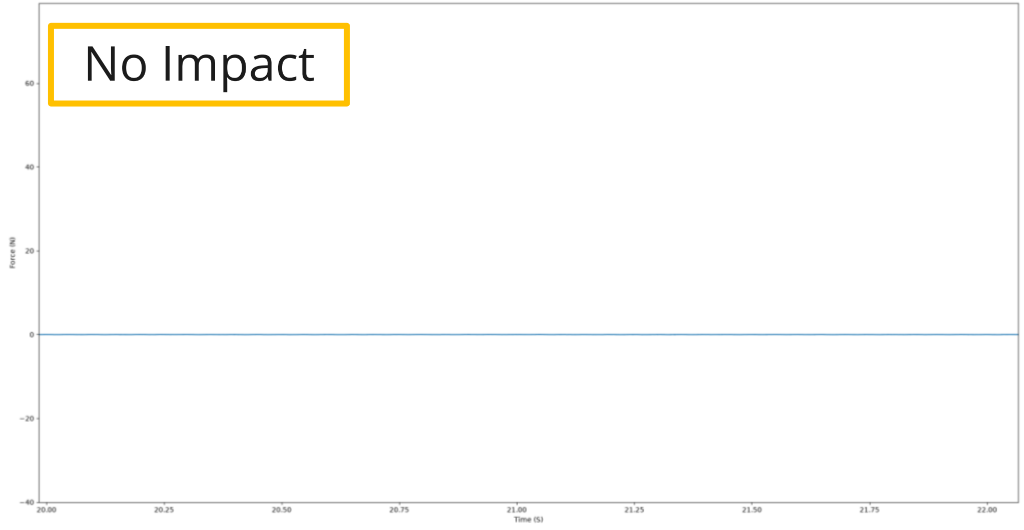
EXPERIMENTAL SETUP

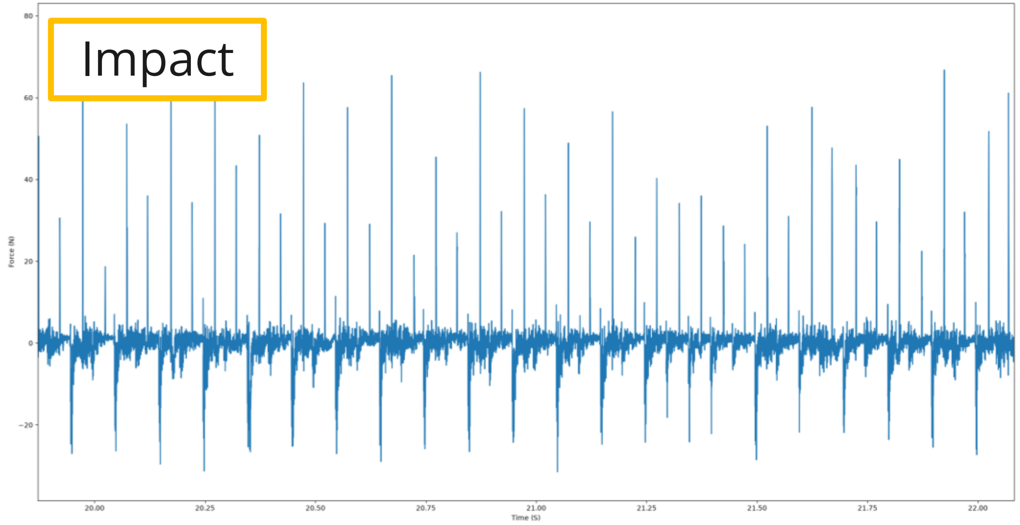
- Harmonic Force Input
- Parameters:
  - Frequency: 5 – 35 Hz
  - Amplitude: 0.01 – 0.3 V
  - Gap size: 0.01 – 0.03 in
  - Impact rod stiffness
- Build map of parameters and responses



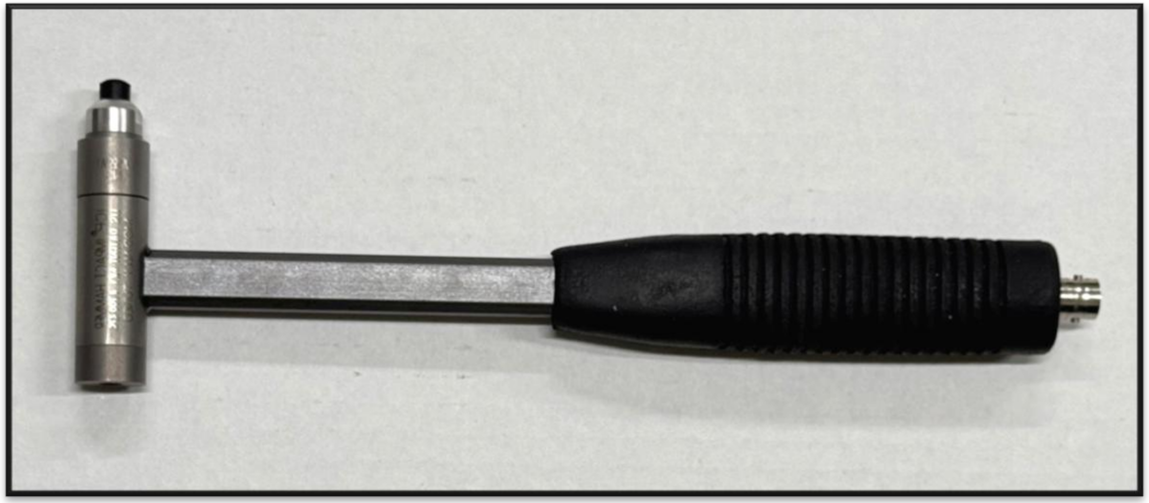
EXPERIMENTAL SETUP | **METHODS**

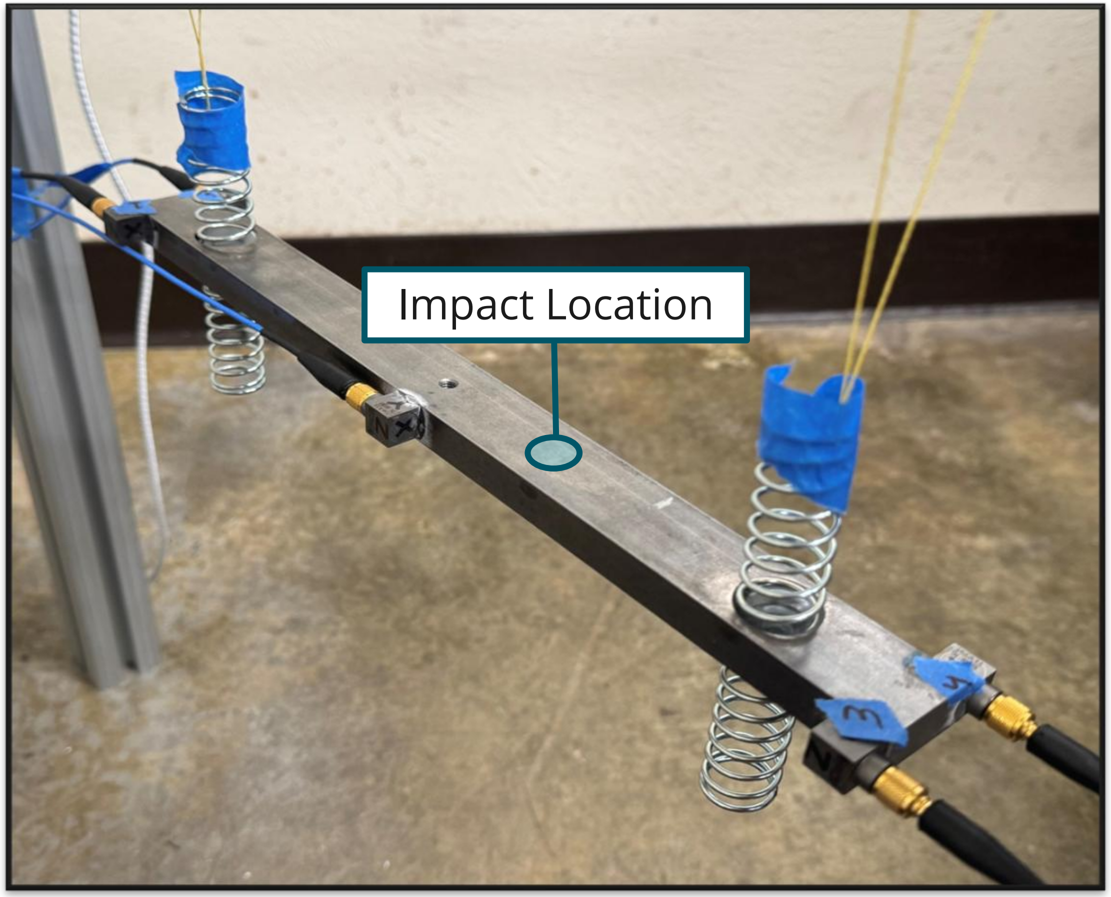




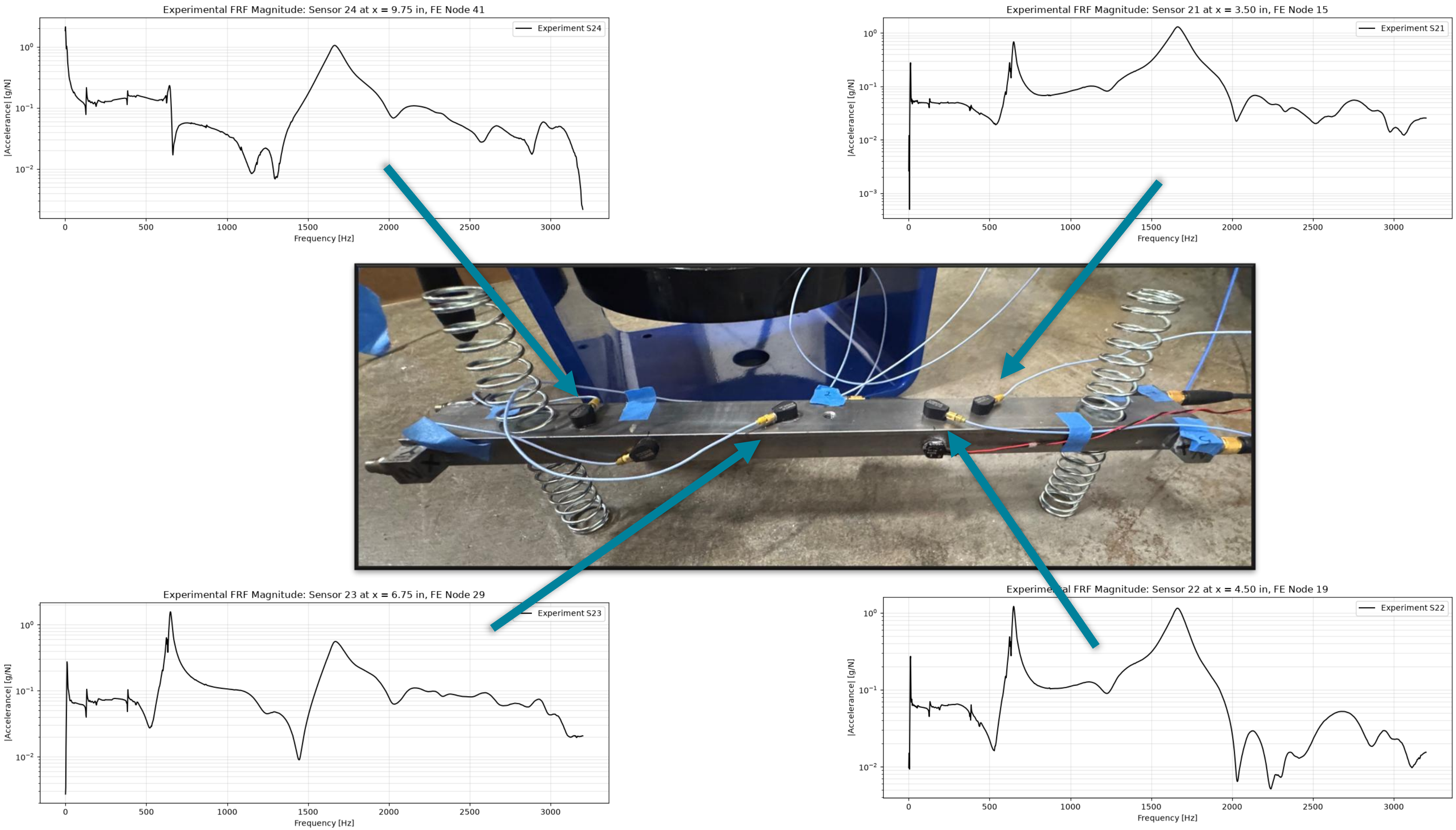


Beam modal characterization



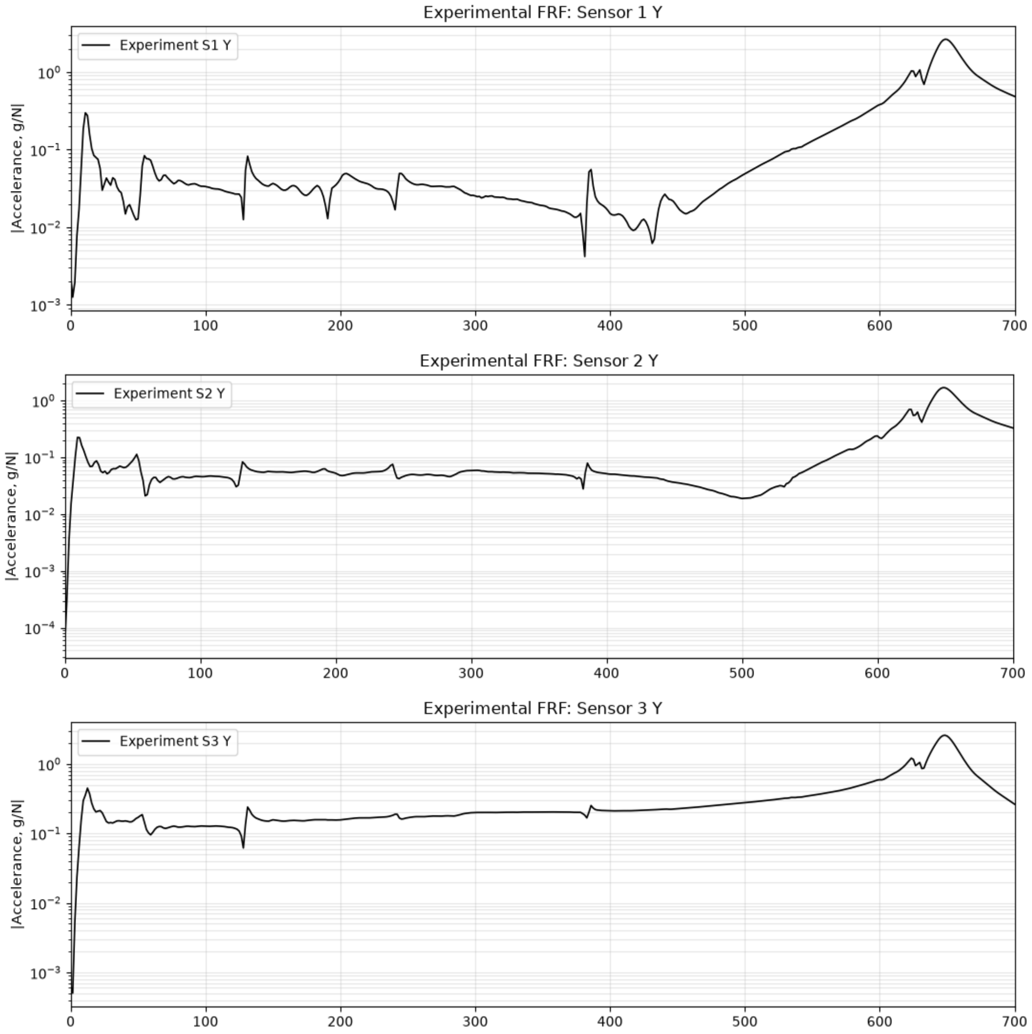


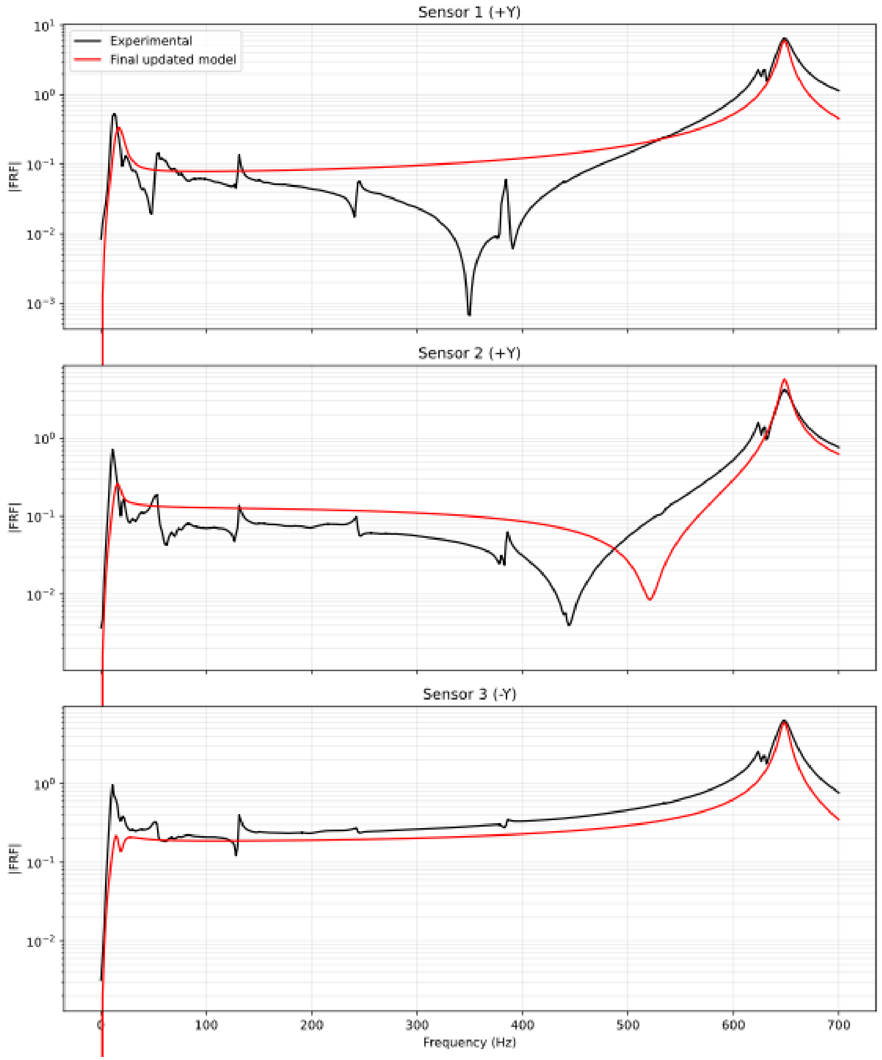
MODELING | **MODAL TESTING**



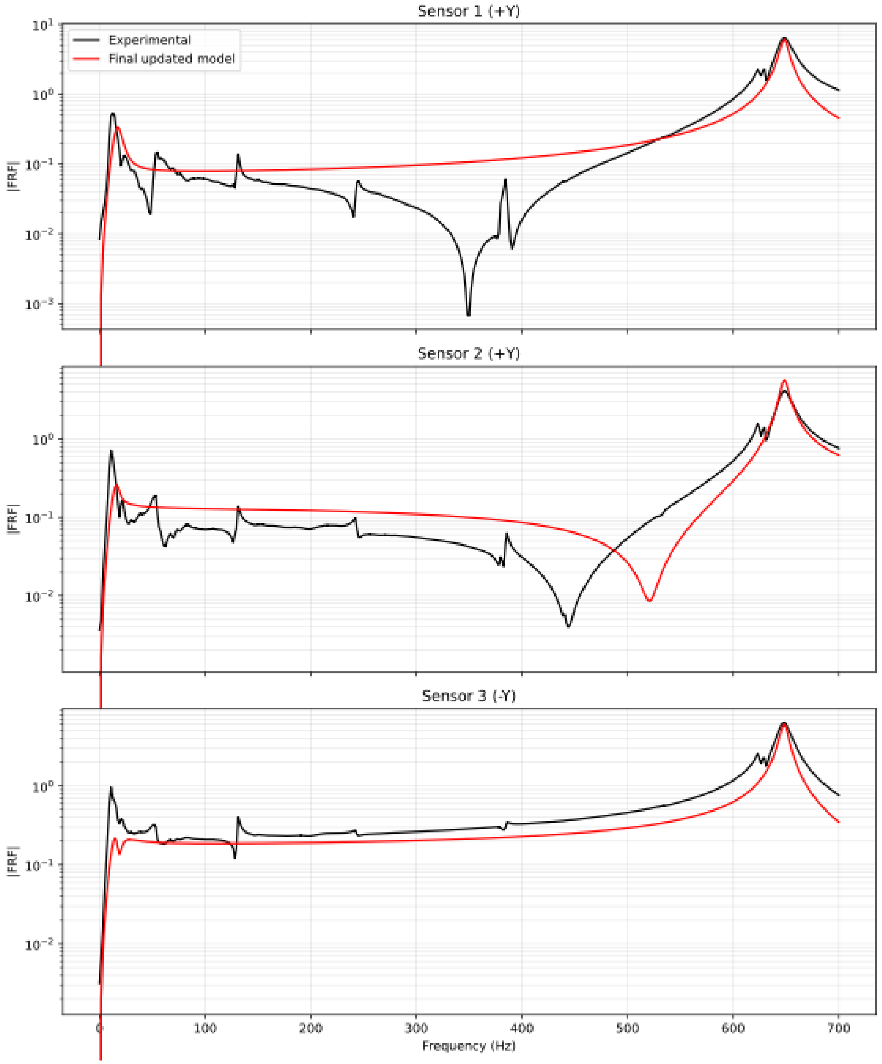
MODELING | **MODAL TESTING**

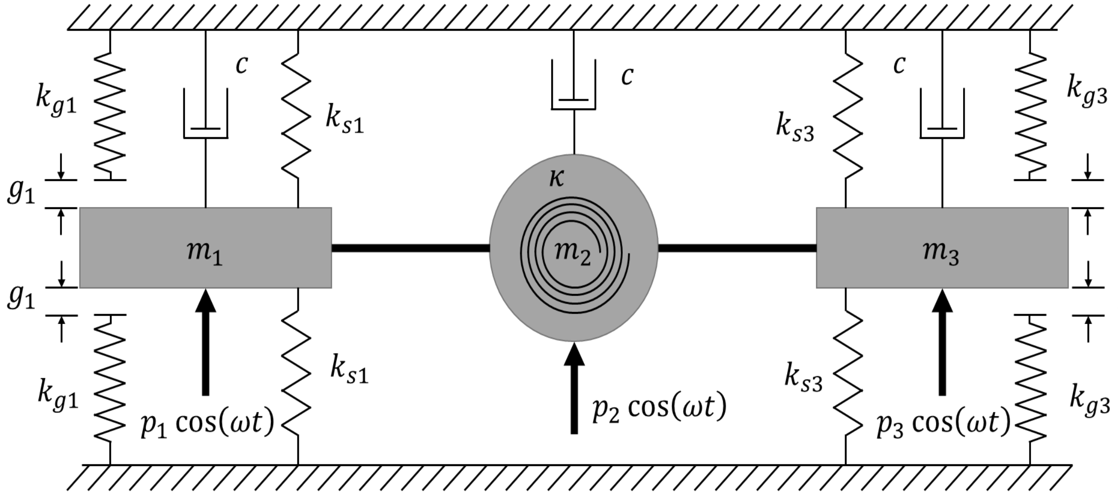
Adjust model to match experiments





MODELING | **MODAL TESTING**





[1]

Parameter	Optimized Value	Units
$m_1 = m_3$	0.19150	kg
m_2	0.40899	kg
k_s	1555.80816	N/m
κ	76204.98604	Nm/rad

MODELING | **IMPACT APPROXIMATION**

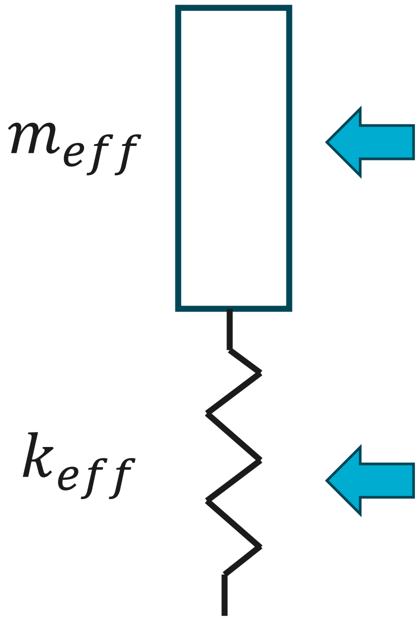
Effective Stiffness Estimation:

$$k_{eff} = \frac{\pi^2 m_{eff}}{\tau^2}$$

τ: contact time (from impact test)

Modeled tip as spring attached to mass

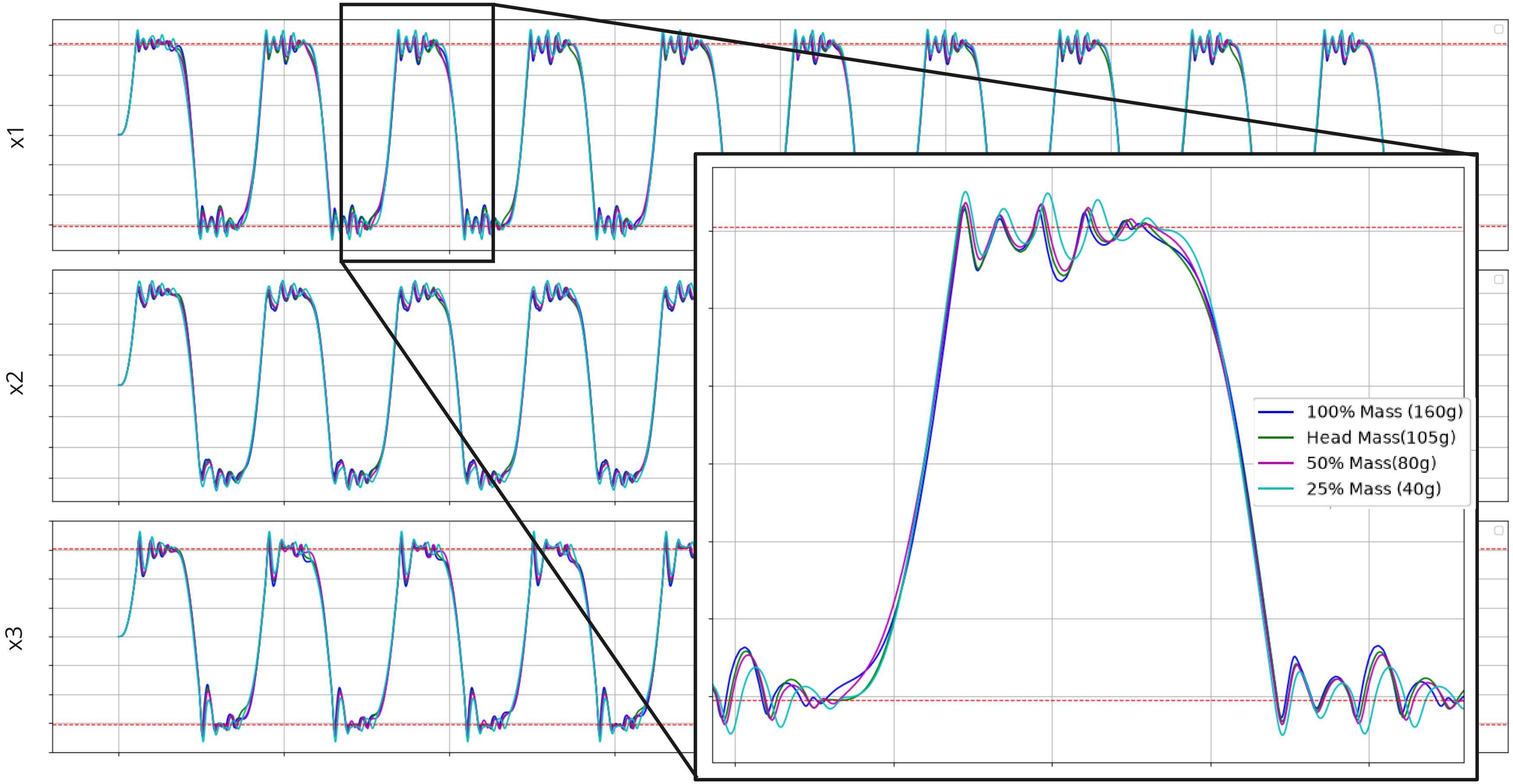
Stiffness applied to model



MODELING | **IMPACT APPROXIMATION**

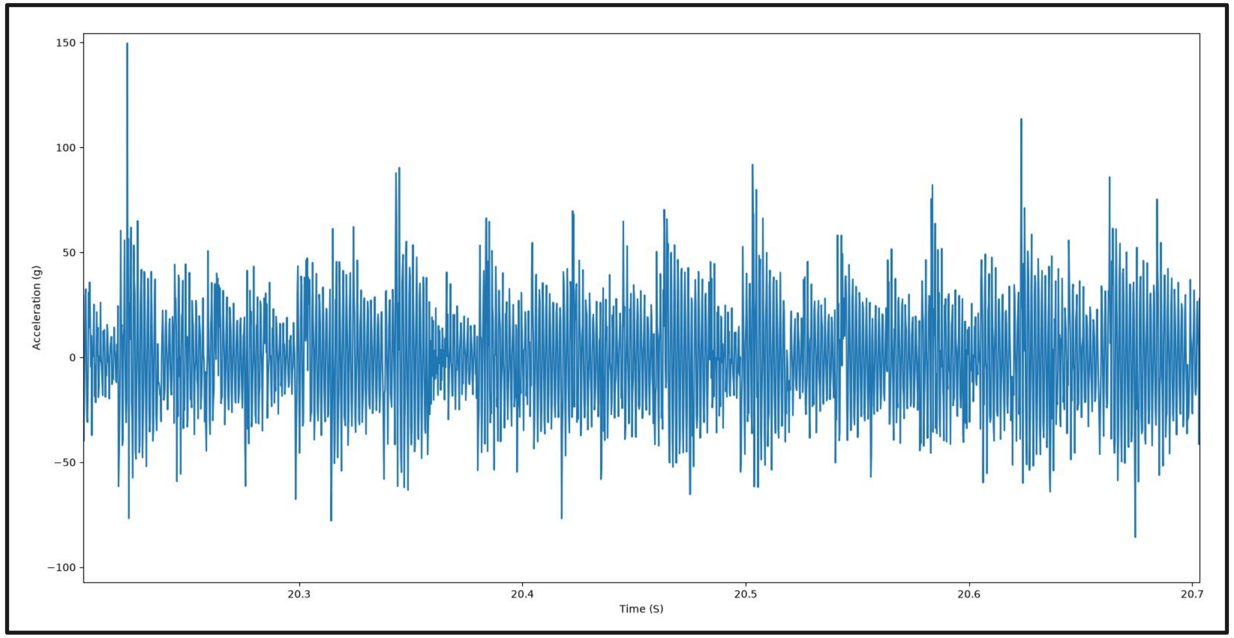
Stiffness Parameter Study

- Hammer Mass (160g)
- Head Mass (105g)
- 50% Total Mass (80g)
- 25% Total Mass (40g)

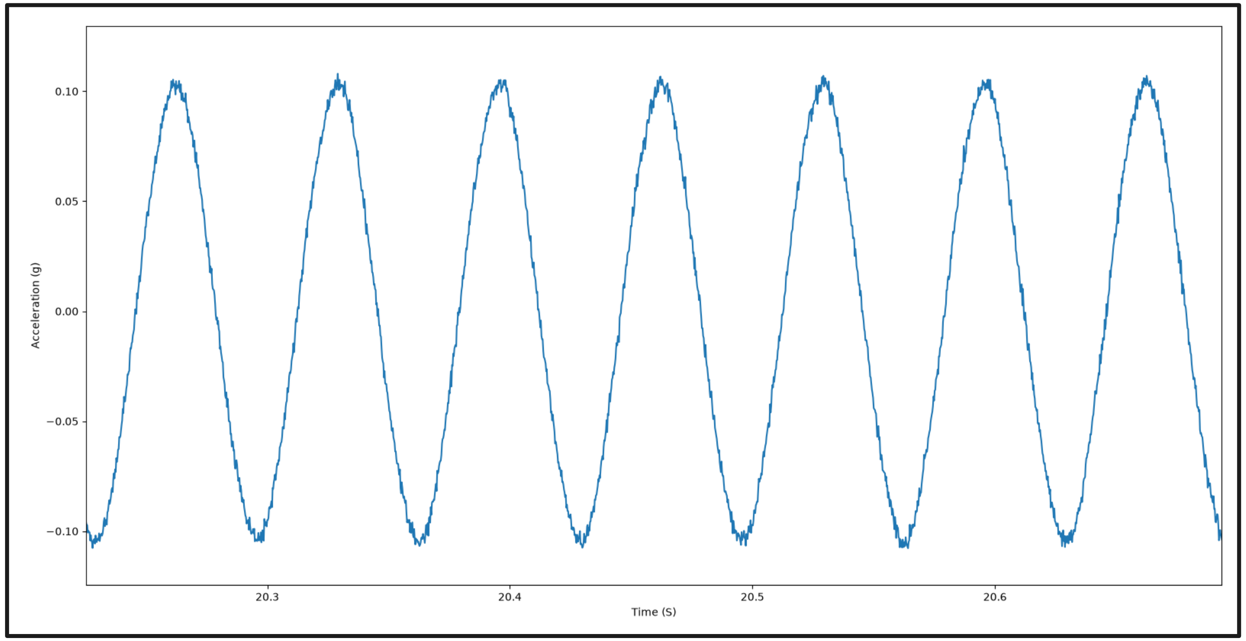


RESULTS | **TIME SERIES**

- Experimental results fed through low pass filter to mitigate effect of noise
- Parameter sweep of chaos detector options
- Time histories chaotic vs periodic

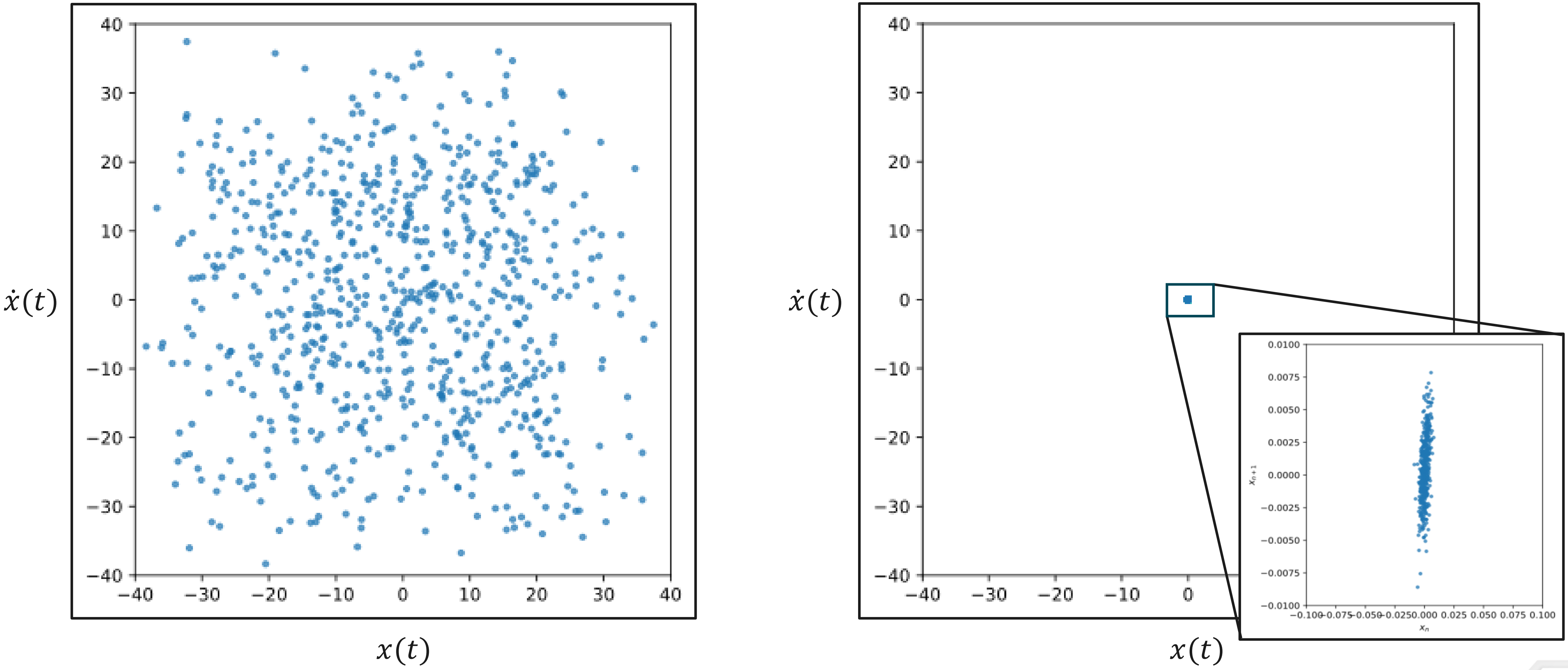


0.02 in Red Tip 25 Hz 0.3 V



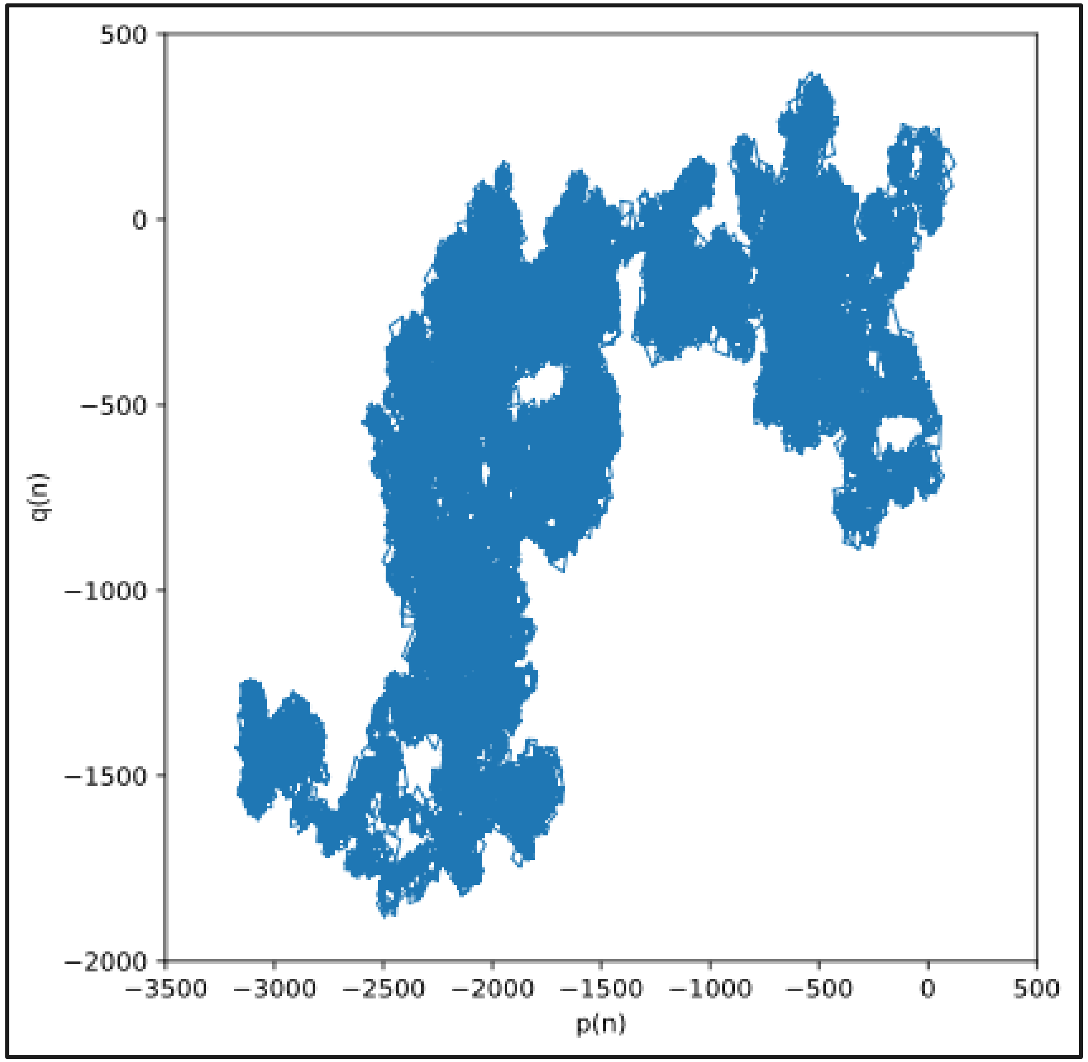
0.02 in Red Tip 15 Hz 0.01 V

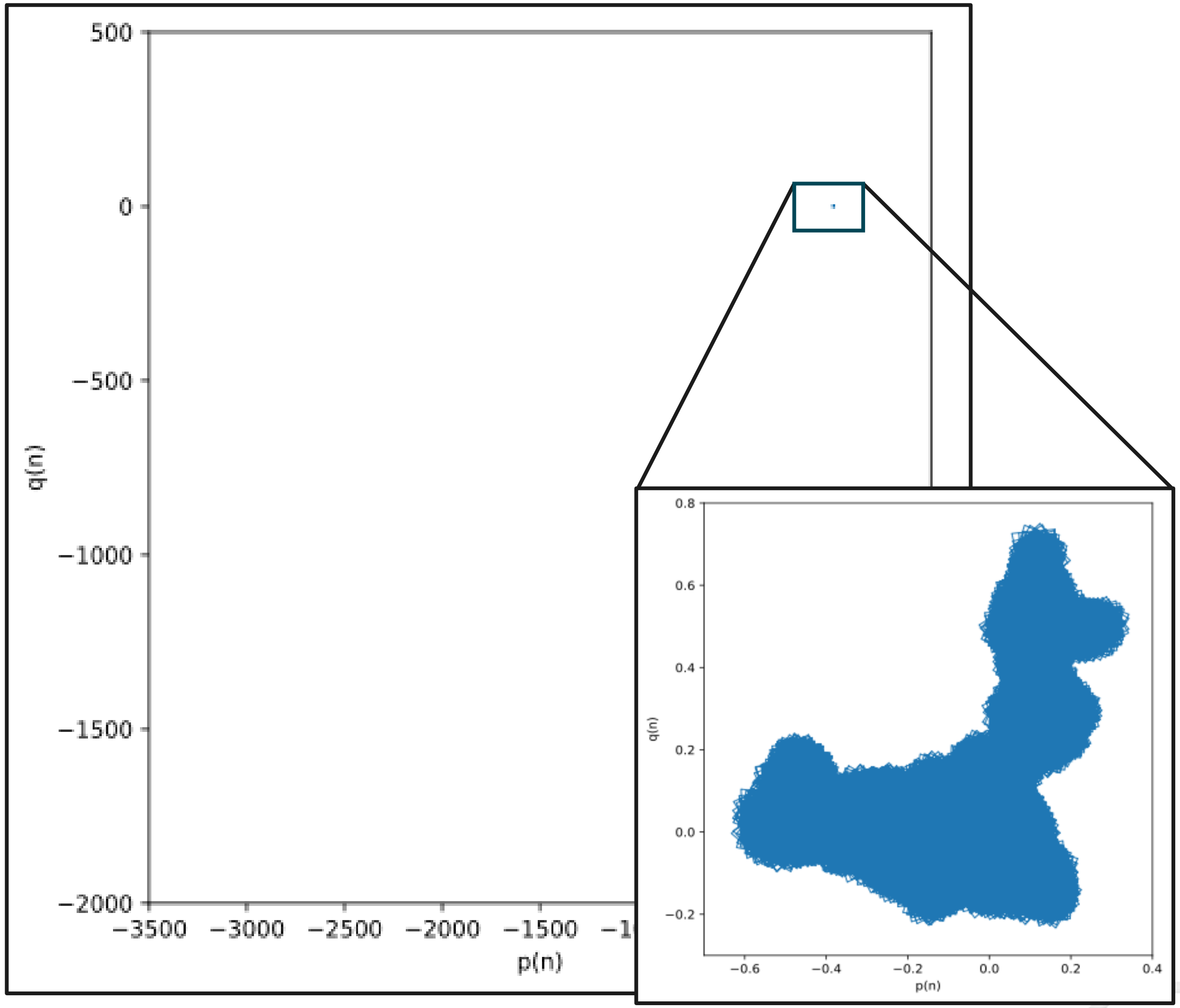
- Poincaré sections of chaotic vs. periodic response



RESULTS | P-Q PROJECTIONS

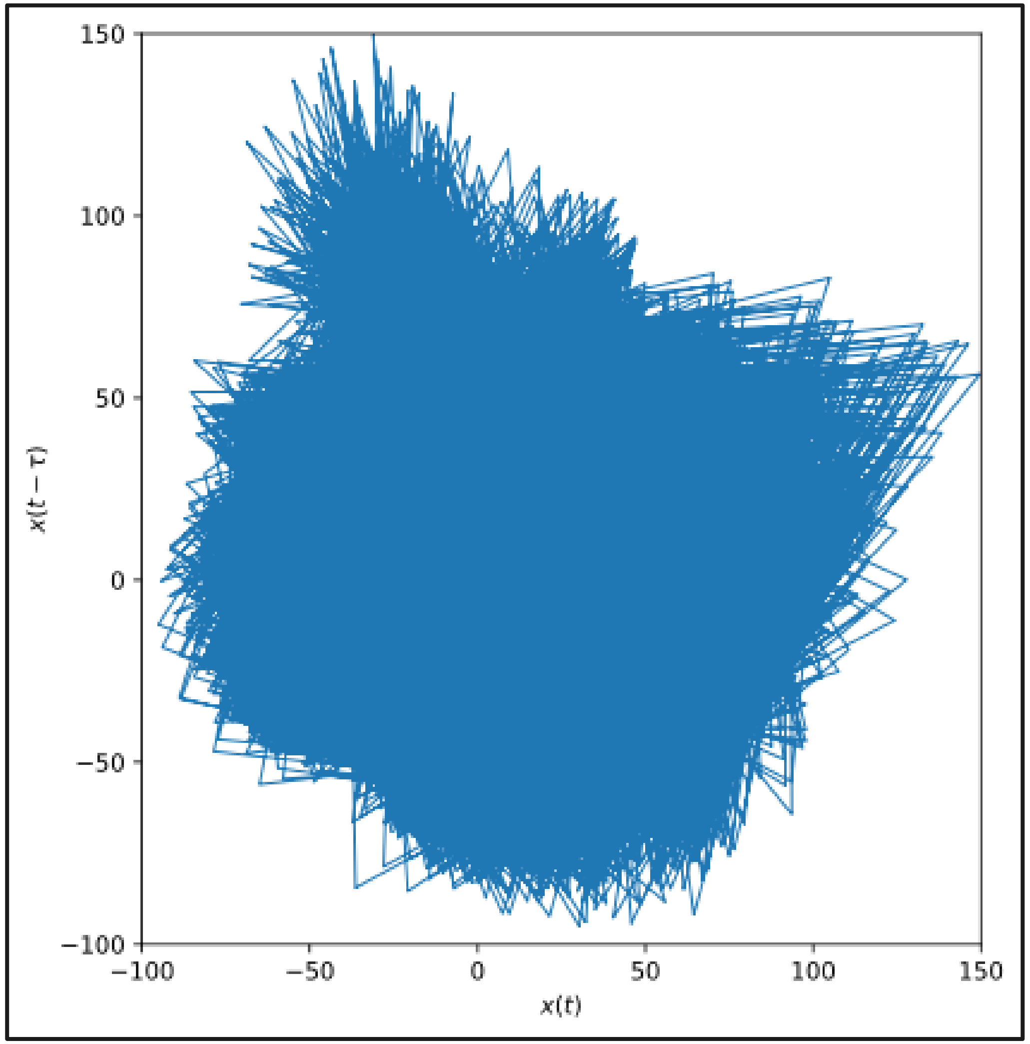
- p-q plots of chaotic vs. periodic response

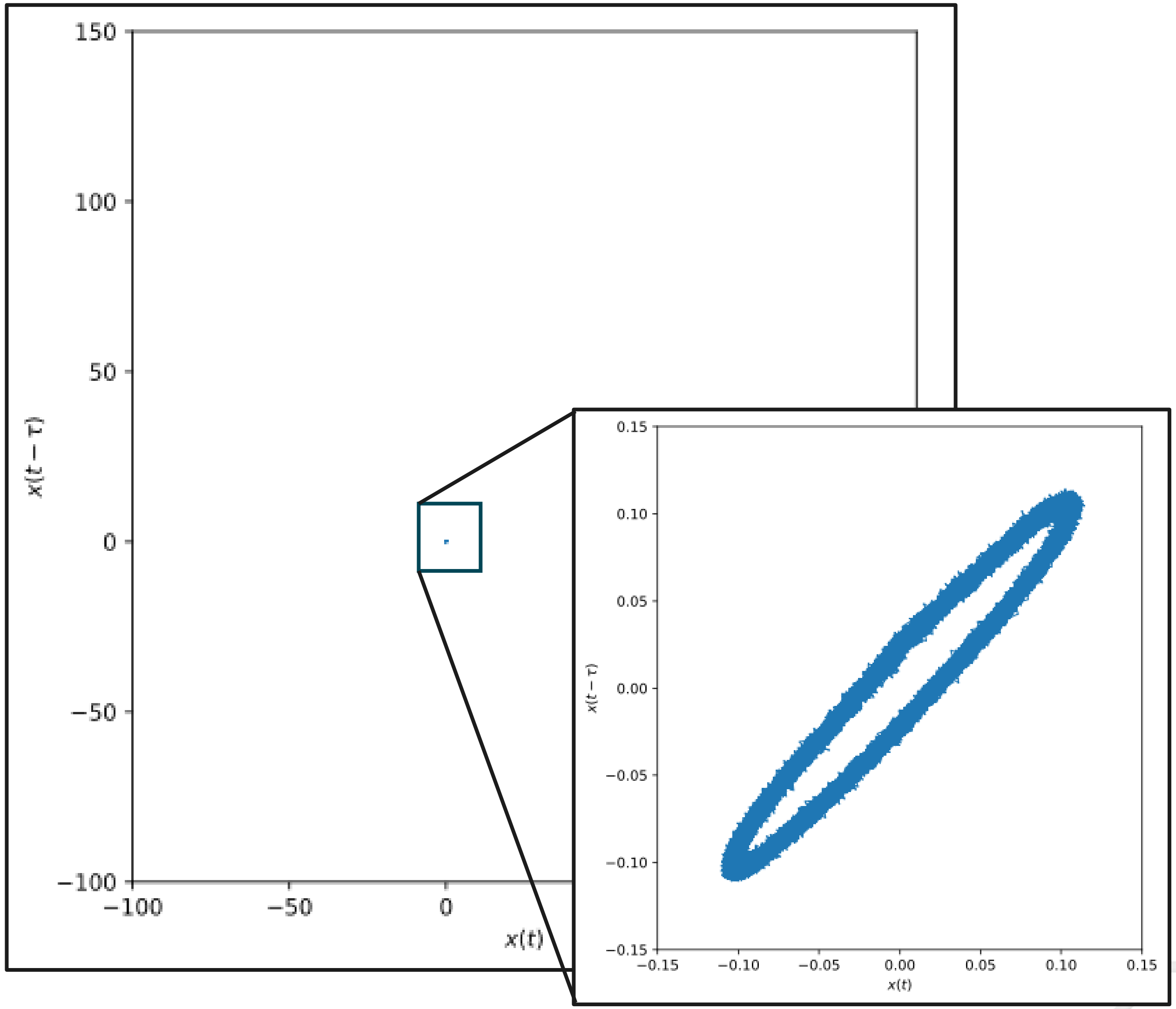




RESULTS | **TIME EMBEDDED PHASE SPACE**

- Time embedded phase space plots of chaotic vs. periodic response

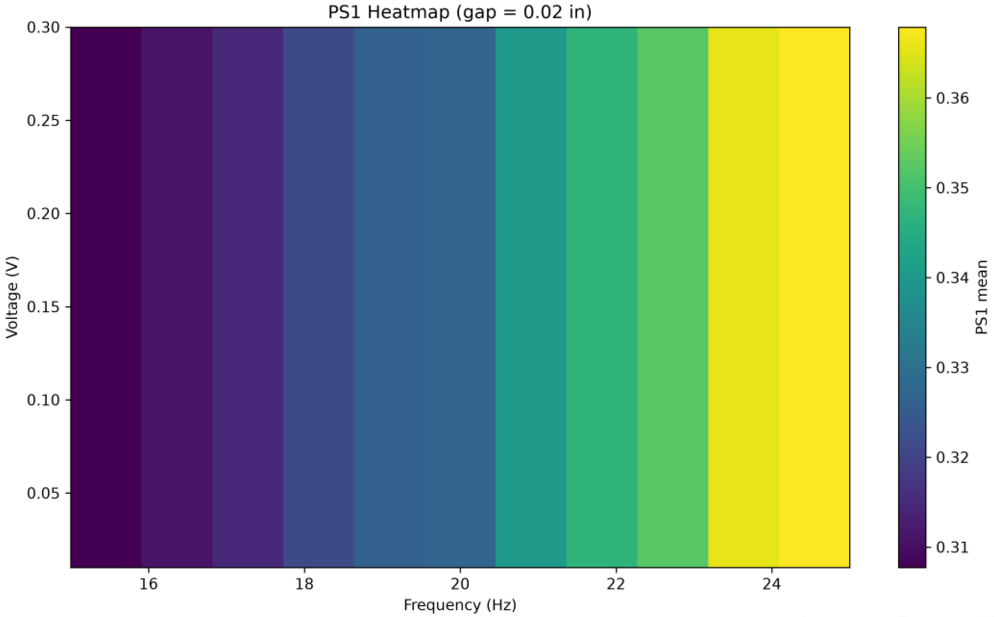


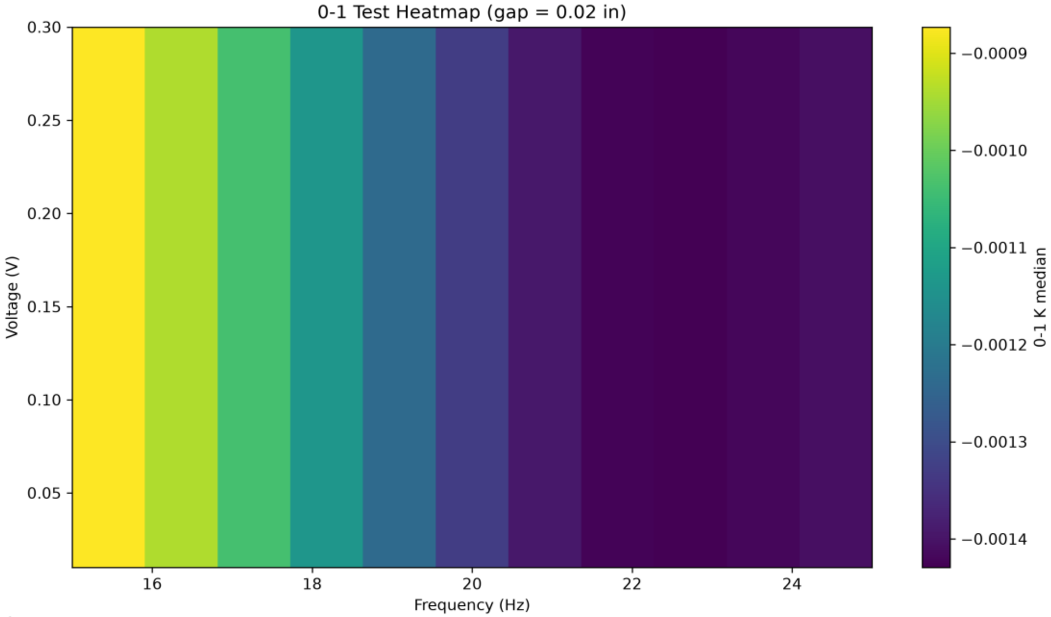


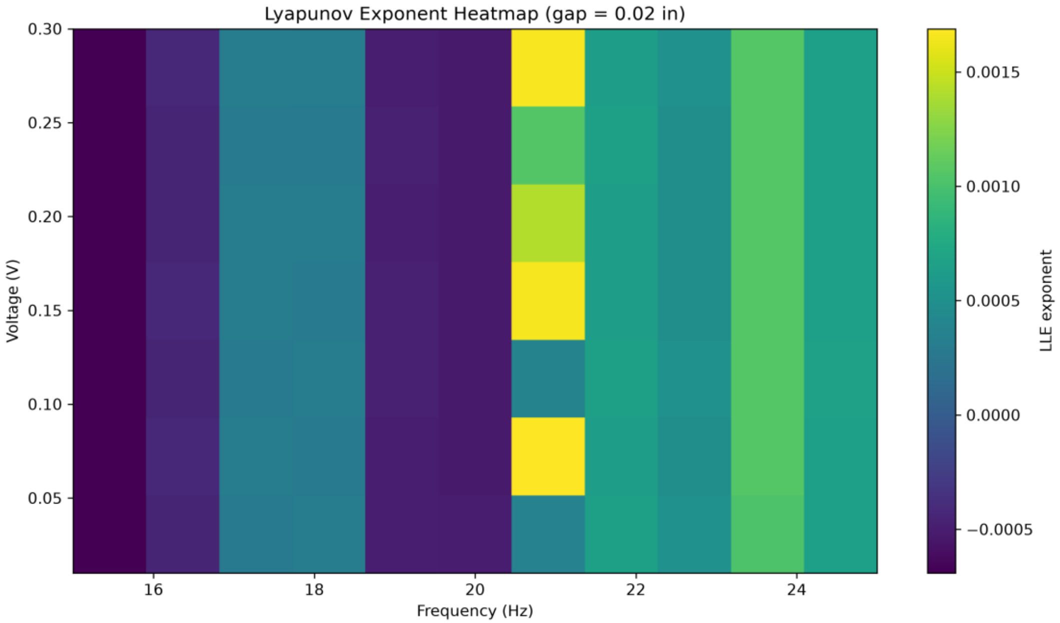
RESULTS

- Heat maps of chaos detector values were created for several system setups
  - 0.01- 0.02 in gaps, 15-25 Hz input frequencies, 0.01-0.3 V input voltages
- 3 detection algorithms used
  - PS1: Darker colors, closer to 0, indicate chaos
  - 0-1 Test: Brighter colors, closer to 1, indicate chaos
  - Lyapunov Exponent: : Brighter colors, above 0, indicate chaos
- Frequency vs. Voltage Heat maps for each gap size and tip stiffness

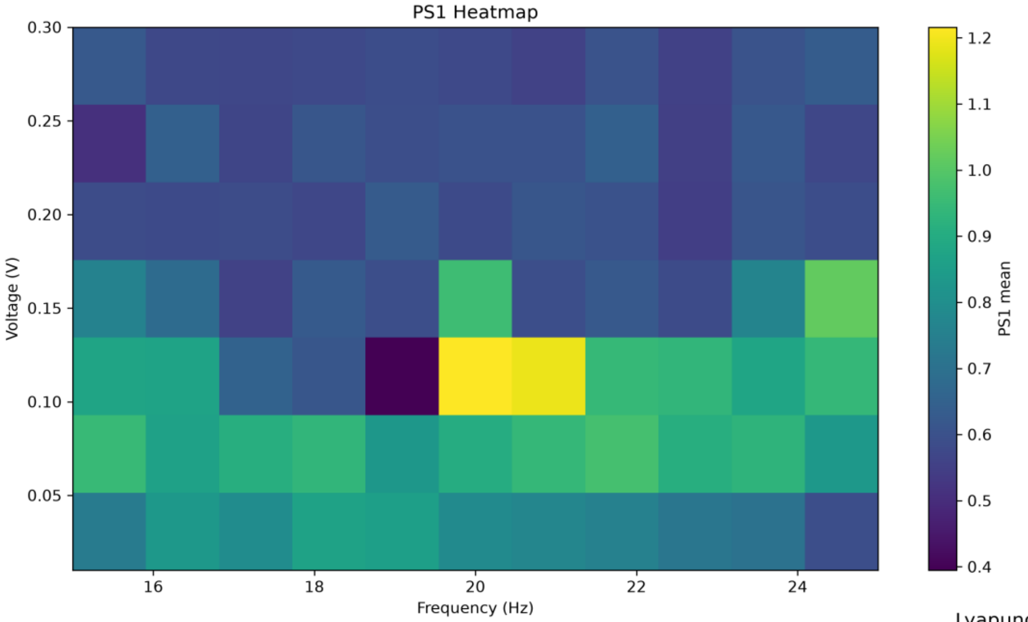
RESULTS | **MODEL**

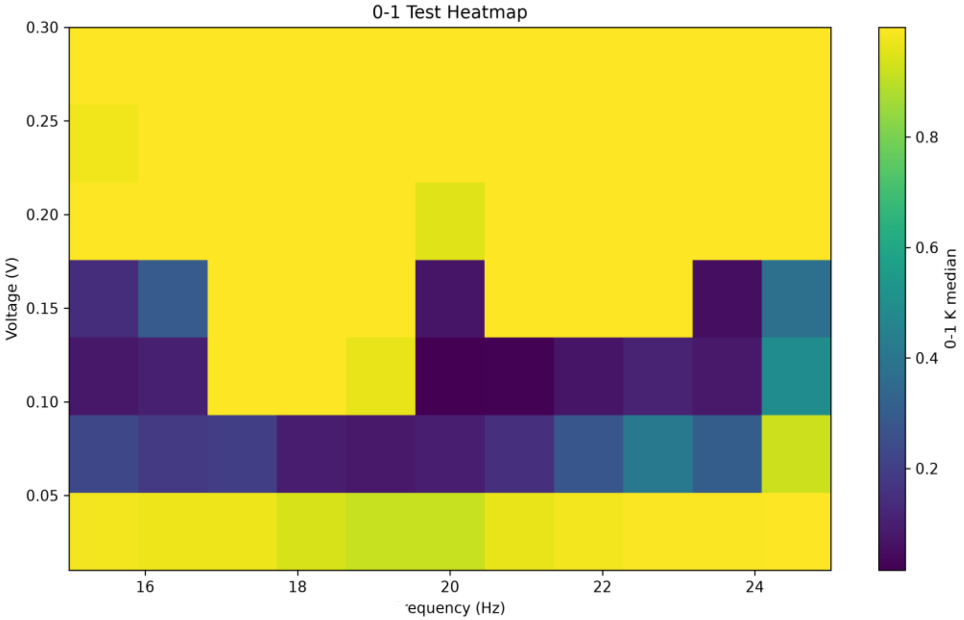


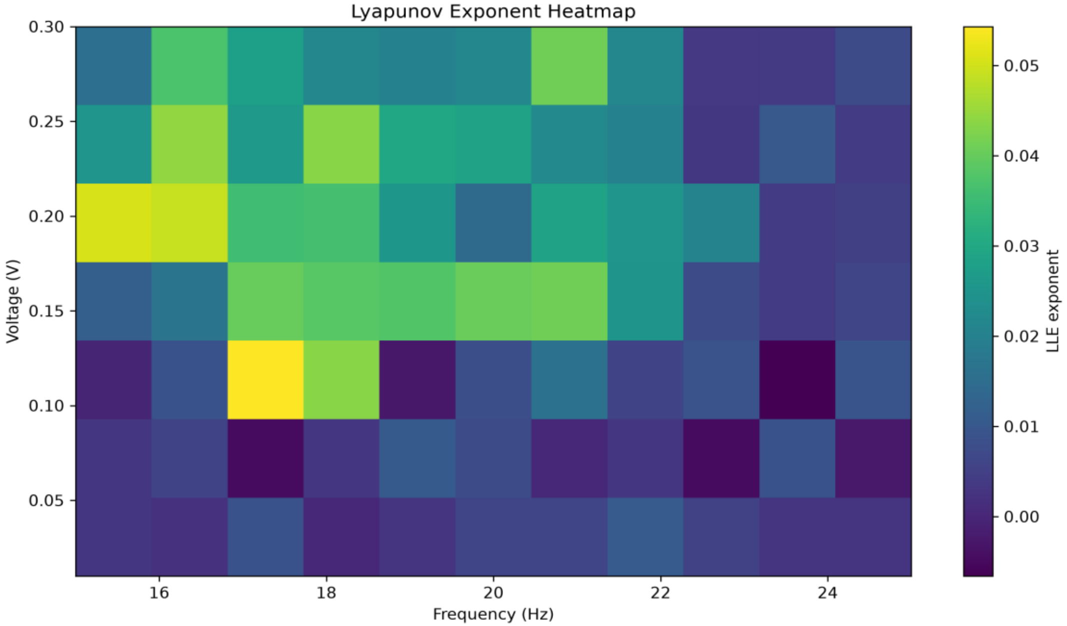




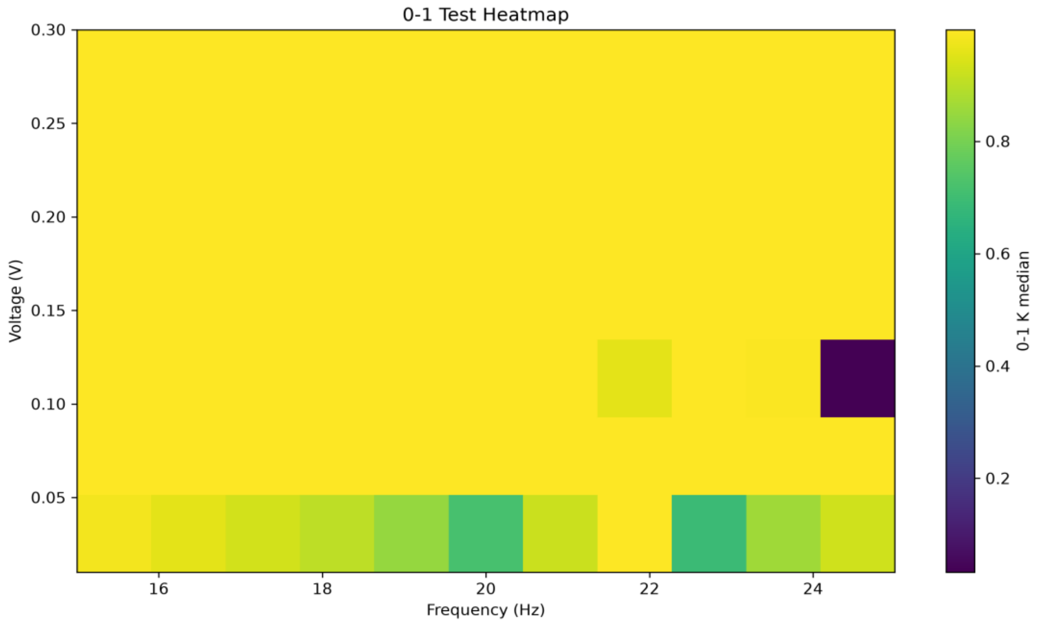
RESULTS | **0.05 CM GAP WHITE TIP**





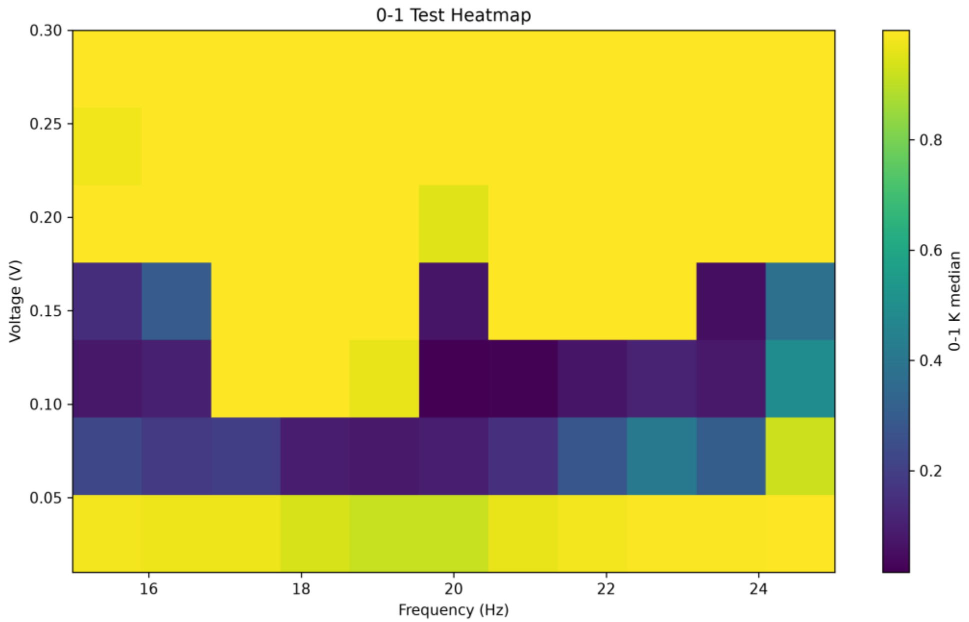


RESULTS | **0-1 CORRELATION TEST**



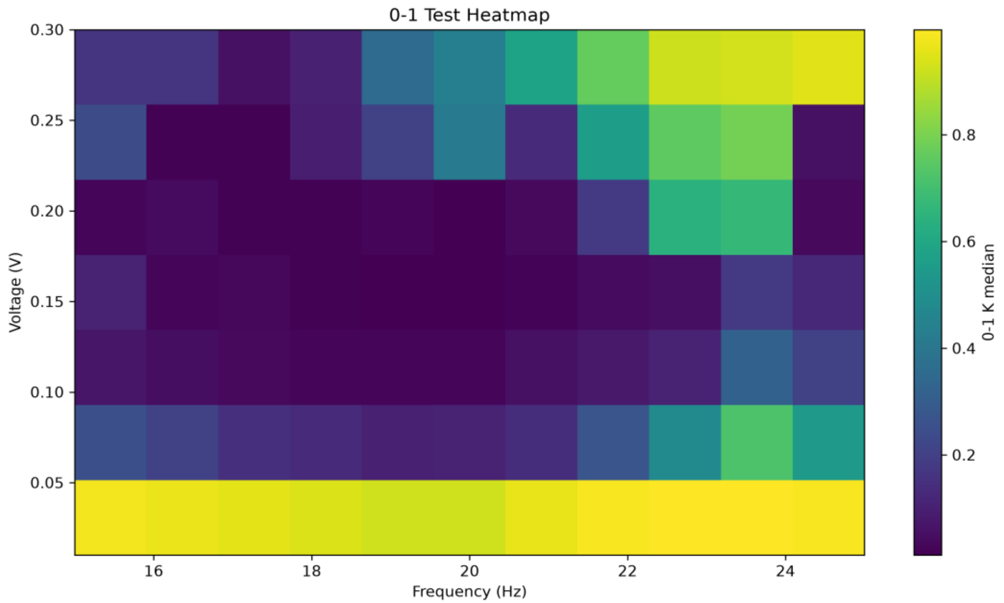
0.01 in

White Tip



0.02 in

White Tip



0.02 in

Red Tip

MAIN TAKEAWAYS

Experiment parameter sets run through 3DOF model simulations

- Time series passed into chaos detectors to test agreement
- Gap size delays onset of chaos in terms of voltage and reduces frequency range
- Softer impact rod results in smoother transition to chaos, with higher frequency and voltage threshold to observe chaos
- All three algorithms consistently detected chaos in the targeted frequency and voltage ranges
    - Chaos first appears near resonance

CONCLUSION

- Built an understanding of what conditions are required for chaos
  - Tested combinations of multiple parameters
  - Built map of chaos for different parameters
- Calibrated 3-DOF model against benchmark vibro-impact system
- Used multiple chaos detection algorithms to identify chaos
  - Compared reliability of detection algorithms
- Uncovered current limits of testing and modeling capabilities

EXISTING GAPS | EXPERIMENTAL

Gaps:

- Control algorithms are not designed for impact
- Beam is ~40% of the total mass of the system
- Software could not control to right force near resonance
  - Switch to open loop voltage input

Future Changes:

- Increase mass of box and/or decrease mass of beam
- Standardize gap size setting → Even small variance in gap sizes has enormous effects
- Stay with open loop voltage input → Include electromechanical model of shaker in model
- Expand parameter range

EXISTING GAPS | MODELING AND DETECTION

Gaps:

- Detection algorithms did not always agree
- Heuristic metrics to determine chaos
- Algorithm settings have large effect on the outcome
- 3DOF model too coarse

Future Changes:

- Finer mesh FEA model better covered beam behavior
- Further exploration of chaos detection methods
  - Best options, parameters for set of data

ACKNOWLEDGEMENTS

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BIBLIOGRAPHY

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[2] Aussie50, “Front Load Washer Total Carnage – Washer goes Chain Chomp on me”, April 20, 2013, https://www.youtube.com/watch?v=dq6T5BojXc8

[3] A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastano, "Determining Lyapunov exponents from a time series," Physica D: nonlinear phenomena, vol. 16, no. 3, pp. 285-317, 1985.

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[5] J. R. Tempelman and F. A. Khasawneh, "A look into chaos detection through topological data analysis," Physica D: Nonlinear Phenomena, vol. 406, p. 132446, 2020.

THANKS FOR LISTENING!

QUESTIONS?