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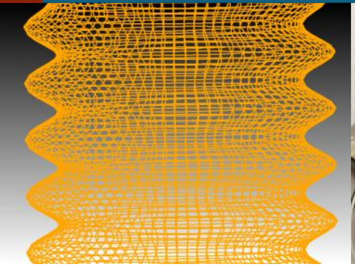
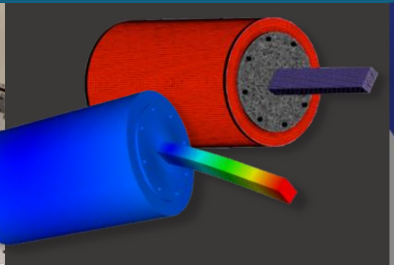
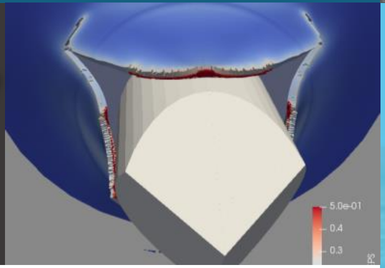
Analytical Modeling of Piezoelectric Stack Actuators for Vibrafuge Applications







Alen Golpashin (UIUC), Mark Jackson (UGA), Joshua Siktar (UTK)

Moheimin Khan (SNL), Rich Jepsen (SNL), David Siler (SNL), Gregory Bunting (SNL), Abdessattar Abdelkefi (NMSU)

August 8, 2023

SAND2023-07599PE

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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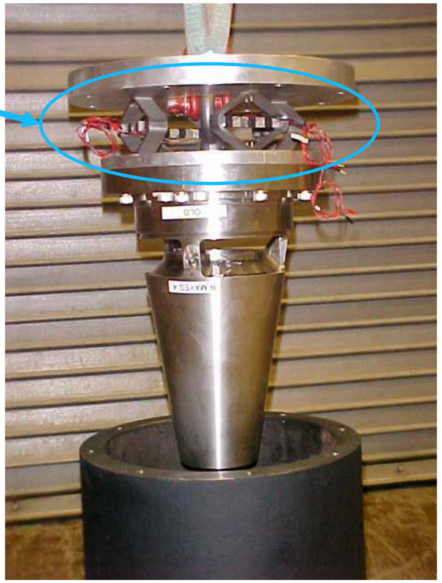
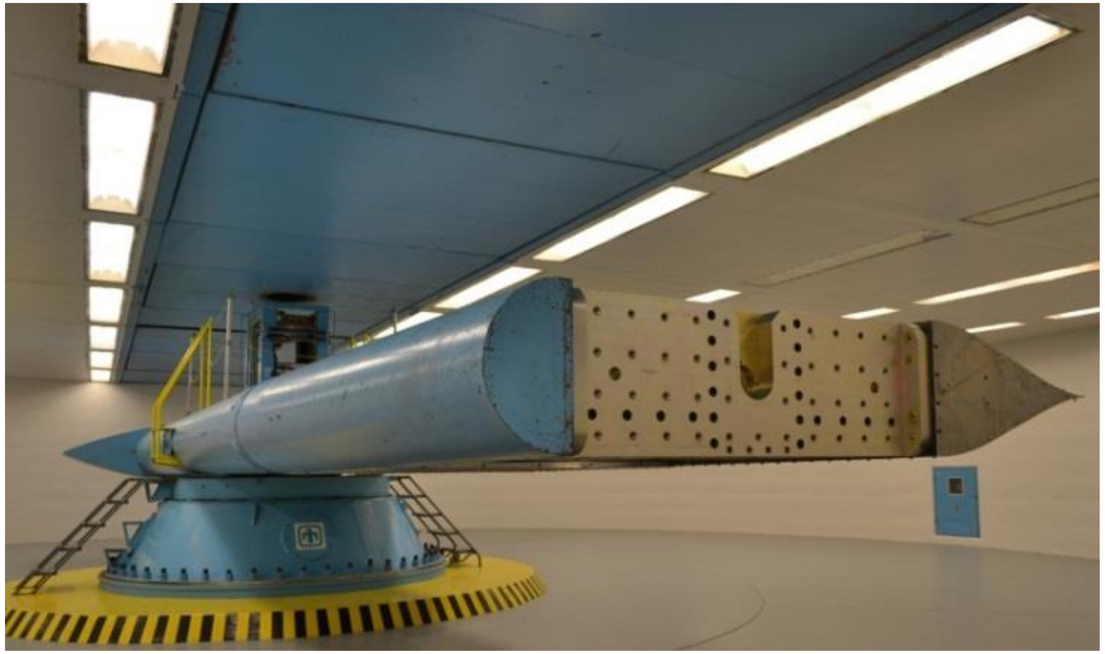
Overview



- Background
  - Application of amplified piezoelectric actuators (APAs)
- Motivation
  - What behavior are we modeling?
- Experimental setup
- Model selection
- Parameter estimation/System ID
- Results



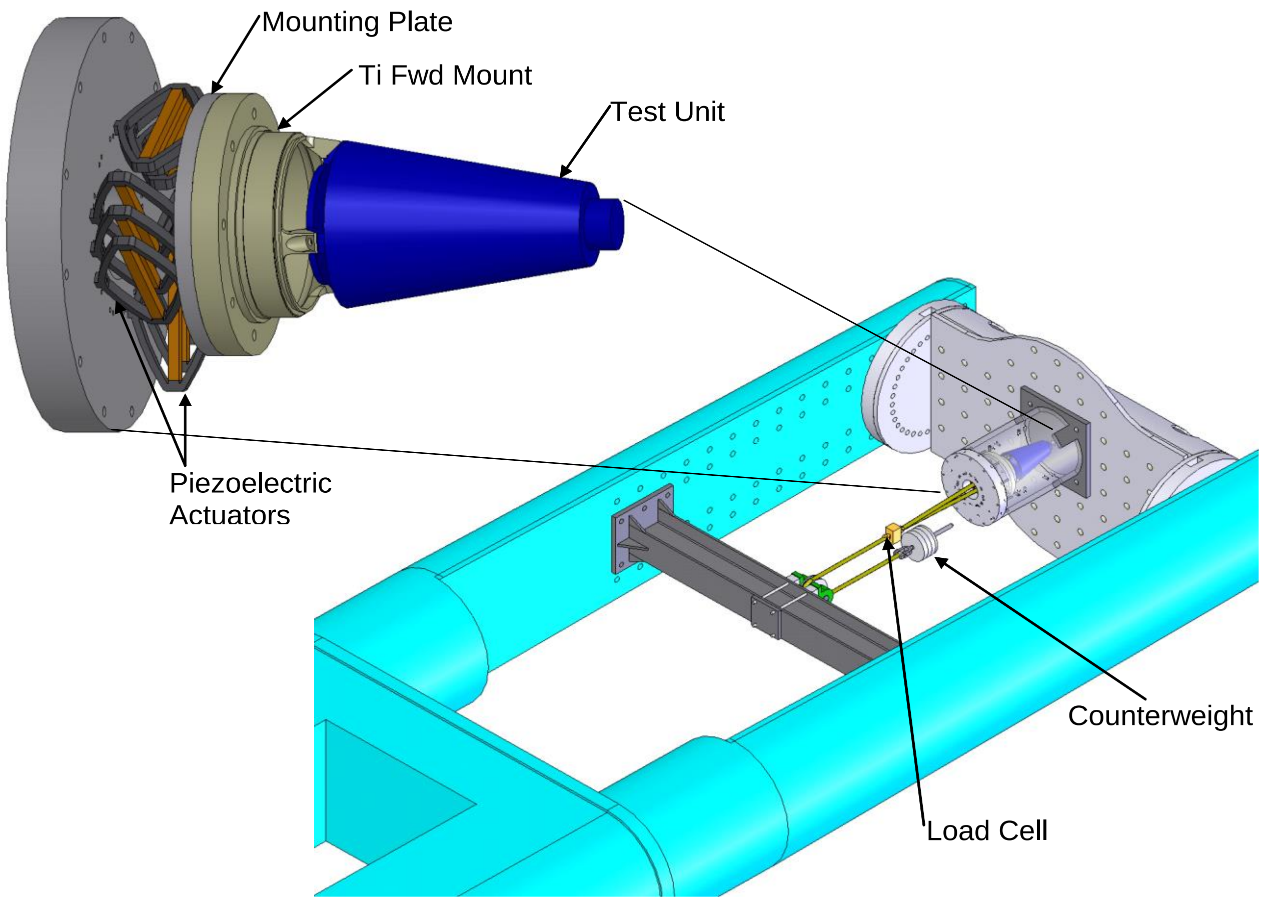
Background

- Often times, device under test is exposed to inertial and dynamic loading simultaneously

- Combined loading environment required for qualification testing
  - This is achieved through Sandia's vibrafuge setup

- Vibrafuge exposes parts to inertial and dynamic loading
  - Electromechanical shakers unable to perform under inertial environment

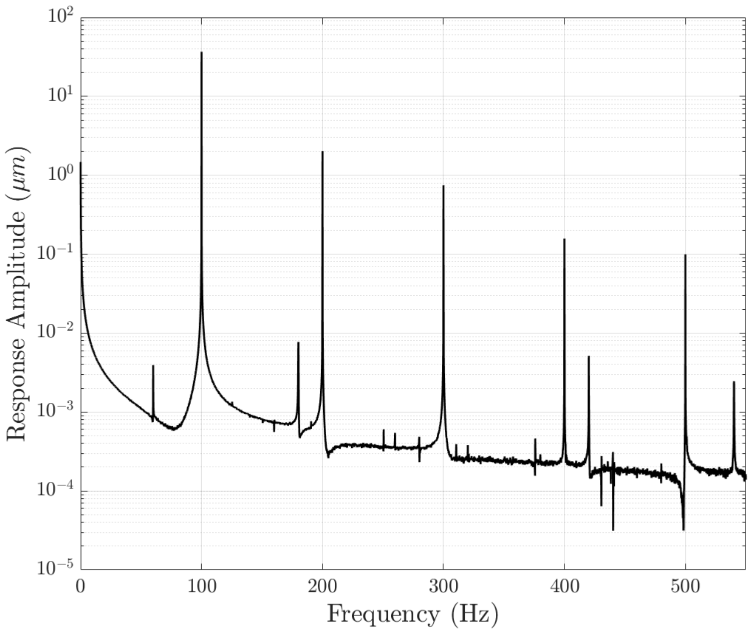
- APA converts an electrical field into mechanical strain through the inverse piezoelectric effect

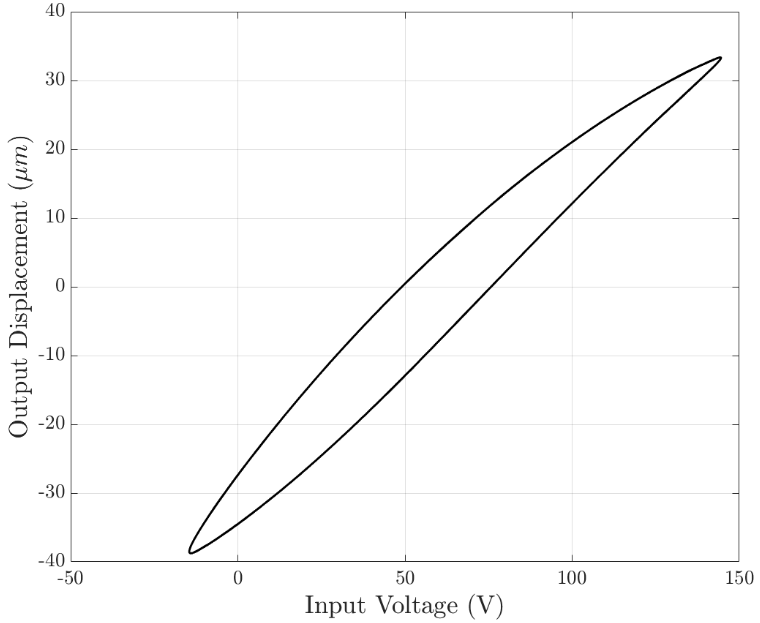


Motivation

- Input signal to actuator needs to be delivered to test unit without distortion
- Nonlinear motion observed in actuator as a result of hysteresis
- Hysteresis described as multiple stable equilibria of a given state
  - Equilibrium point dependent on history of state
- Must model the nonlinear behavior to account for how motion in vibrafuge testing can be affected

Input Signal: 100 Hz

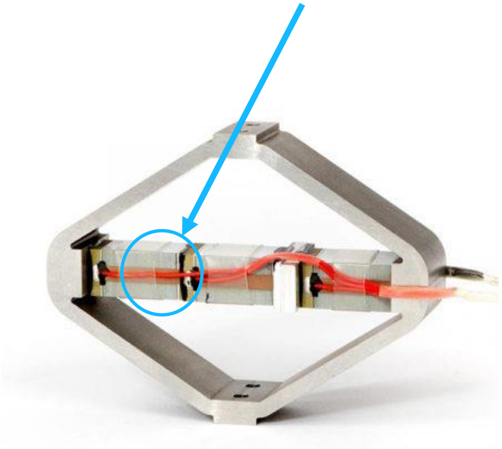




Experimental Setup

- Actuator considered: APA95ML from Cedrat technologies
  - Piezo stack extends/contracts within frame resulting in dynamic motion of the actuator head
- Input signal generated with waveform generator
  - Signal passed through Cedrat RK42F4U-LC75C linear amplifier
- Response of piezo stack measured with strain gauge affixed to stack
  - Cedrat SG-75 signal conditioner
- 10 Hz, 100 Hz, 1000 Hz and band limited (1-4000Hz) stationary random signal used during testing

Strain Gauge Location



APA95ML Actuator

Hysteresis Model Selection

Classical Bouc-Wen model:

$$\dot{h} = \alpha d_{33}\dot{u} - \beta|\dot{u}|h - \gamma\dot{u}|h|$$

- h denotes hysteresis, u denotes input voltage
- d_{33} is the piezoelectric constant, $\alpha,\ \beta,\ \gamma$ are the model parameters

Model drawbacks

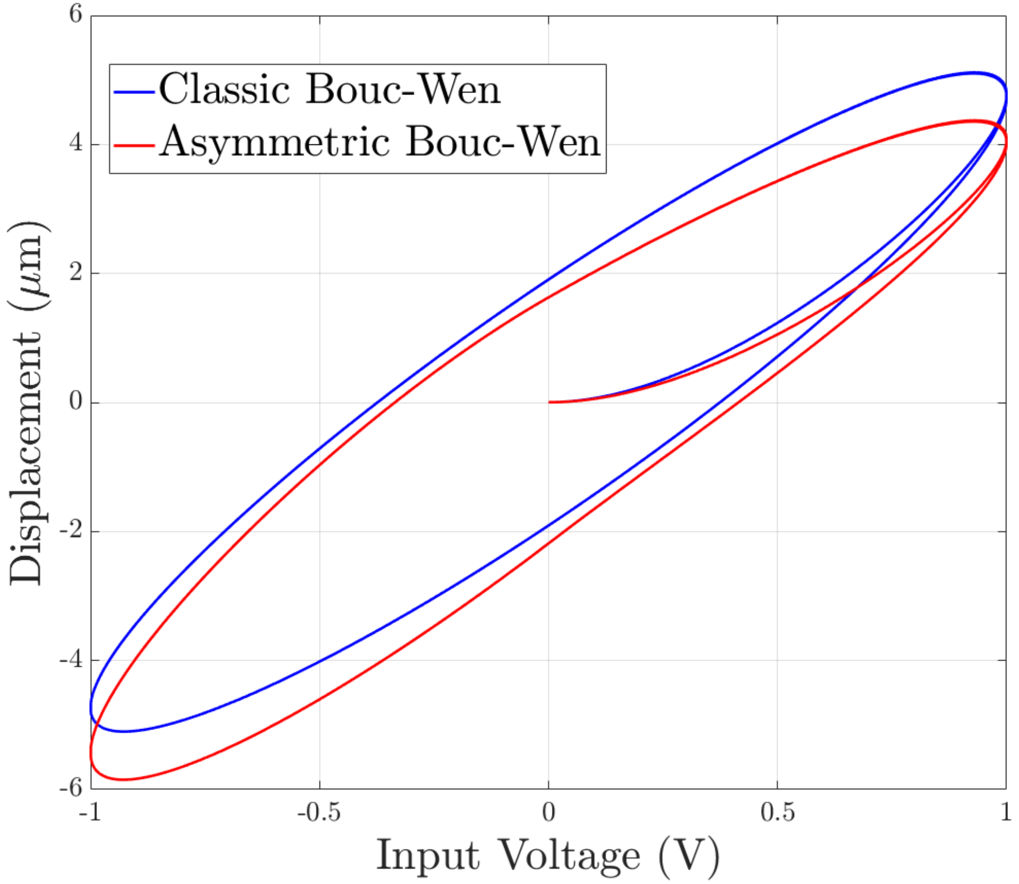
- Model unable to produce asymmetric hysteresis curve for constant parameters
- Numerical instability, especially for large input values
- Parameters cannot be uniquely determined

Hysteresis Model Selection

Classical Bouc-Wen model with asymmetry

$$\dot{h} = \alpha d_{33}\dot{u} - \beta|\dot{u}|h - \gamma\dot{u}|h| - \delta\,\dot{u}\mathrm{sgn}(u)$$

- Additional parameter δ accounts for asymmetry



Model Drawbacks

- Model is unstable unless δ is very small
- After nondimensionalizing, the small magnitude of δ restricts the optimizer's search-space
- When coupled with equation of motion, most non-dimensional models are still numerically unstable

Hysteresis Model Selection

Generalized Bouc-Wen Model

$$\dot{h} = \alpha d_{33}\dot{u} - \beta_1 h|\dot{u}| - \beta_2 |h\dot{u}| \operatorname{sgn}(u) - \beta_3 h\dot{u} \operatorname{sgn}(u) - \beta_4 |h\dot{u}| - \beta_5 \dot{u}h - \beta_6 \dot{u}|h| \operatorname{sgn}(u)$$

- h is hysteresis, u is applied actuation voltage
- $\beta_1, \beta_2, \ldots, \beta_6$ are shape parameters for the hysteresis curve
- $\operatorname{sgn}(w) = \begin{cases} 1, & w > 0 \\ -1, & w < 0 \\ 0, & w = 0 \end{cases}$ is the *signum*, or sign function

Model Drawbacks

- Larger dimensional search-space with more parameters

Hysteresis Model Selection

Normalized Bouc-Wen

$$\dot{w}(t) = \rho(\dot{u}(t) - \sigma|\dot{u}(t)|w(t) - (\sigma - 1)\dot{u}(t)|w(t)|)$$

- In this model, w denotes normalized hysteresis (h)
- ρ and σ are model parameters of the hysteresis curve
- This normalization removes numerical instability issues

Model Drawbacks

- Unable to produce asymmetric hysteresis curves with constant parameters

Actuator Model

Equation of motion

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = k_u u(t) + k_h h(t)$$

Neglecting the dynamic effects of the actuator: $(\ddot{x} = \dot{x} = 0)$

$$x(t) = k_u u(t) + k_h h(t)$$

- With this assumption, the displacement of the actuator is expressed as a linear combination of the input voltage $u(t)$ and hysteresis h(t)

Coupled Model Selection

	Hysteresis model	Actuator model
Classical Bouc-Wen	$\dot{h} = \alpha d_{33}\dot{u} - \beta\lvert\dot{u}\rvert h - \gamma\dot{u}\lvert h\rvert$	Complete equation of motion
Asymmetric Bouc-Wen	$\dot{h} = \alpha d_{33}\dot{u} - \beta\lvert\dot{u}\rvert h - \gamma\dot{u}\lvert h\rvert - \delta\,\dot{u}\,\mathrm{sgn}(u)$	$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = k_u u(t) + k_h h(t)$
Generalized Bouc-Wen	$\dot{h} = \alpha d_{33}\dot{u} - \beta_1 h\lvert\dot{u}\rvert - \beta_2\lvert h\dot{u}\rvert\,\mathrm{sgn}(u) - \beta_3 h\dot{u}\,\mathrm{sgn}(u) - \beta_4\lvert h\dot{u}\rvert - \beta_5\dot{u}h - \beta_6\dot{u}\lvert h\rvert\,\mathrm{sgn}(u)$	Quasi-static equation of motion
Normalized Bouc-Wen	$\dot{w}(t) = \rho(\dot{u}(t) - \sigma\lvert\dot{u}(t)\rvert w(t) - (\sigma - 1)\dot{u}(t)\lvert w(t)\rvert)$	$x(t) = k_u u(t) + k_h h(t)$
Selected model: $\dot{w}(t) = \rho(t)(\dot{u}(t) - \sigma(t)\lvert\dot{u}(t)\rvert w(t) - (\sigma(t) - 1)\dot{u}(t)\lvert w(t)\rvert)$		Selected model: $x(t) = k_u(t)u(t) + k_w(t)w(t)$

Kalman Filter Parameter Estimation

- Consider the Bouc-Wen nonlinear dynamical system:

$$\dot{x}(t) = f\big(x(t), \dot{u}(t)\big)$$

where,

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5] = [w \quad k_u \quad k_w \quad \rho \quad \sigma].$$

- **Goal**: estimate $x(t)$ based on data $\mathcal{Y}(t) = \{\Phi\big(u(s)\big) : 0 \leq s \leq t \}$

- **Nonlinear sensor model**: $y(t) = h\big(x(t), u(t)\big)$

where,

$$h\big(x(t), u(t)\big) = x_2(t)u(t) + x_3(t)x_1(t) = k_u(t)u(t) + k_w(t)w(t).$$

Discrete Equations

Model Equation: $x_k = F(x_{k-1}, \dot{u}_{k-1}) + W_k, \quad x_o \in \mathbb{R}^5$

Sensor Equation: $y_k = h(x_k, u_k) + V_k, \quad y_o \in \mathbb{R}$

$$W_k \sim \mathcal{N}(0, \Sigma_k) \qquad V_k \sim \mathcal{N}(0, \mathrm{R}_k)$$

- **Assumption:** the data is normally distributed around the true value of the estimate.
- **Define:**

Prior: $\hat{x}_{k|k-1} = \mathbb{E}[x_k | y_{0:k-1}]$ $\qquad P_{k|k-1} = \mathrm{Cov}(x_k | y_{0:k-1})$

Posterior: $\hat{x}_{k|k} = \mathbb{E}[x_k | y_{0:k}]$ $\qquad P_{k|k} = \mathrm{Cov}(x_k | y_{0:k})$

Extended Kalman Filter (EKF)

$$A_k = \frac{\partial F}{\partial x}\Big|_{\hat{x}_{k-1|k-1}, \dot{u}_k}$$

$$H_k = \frac{\partial h}{\partial x}\Big|_{\hat{x}_{k|k-1}, u_k}$$

Prior:
$$\begin{cases} \hat{x}_{k|k-1} = F\left(\hat{x}_{k-1|k-1}, \dot{u}_k\right) \\ P_{k|k-1} = A_k P_{k-1|k-1} A_k^T + \Sigma_k \end{cases}$$

Posterior:
$$\begin{cases} \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left(y_k - h\left(\hat{x}_{k|k-1}, u_k\right)\right) \\ P_{k|k} = (I - K_k H_k) P_{k|k-1} \\ K_k = P_{k|k-1} H_k^T \left(H_k P_{k|k-1} H_k^T + R_k\right)^{-1} \end{cases}$$

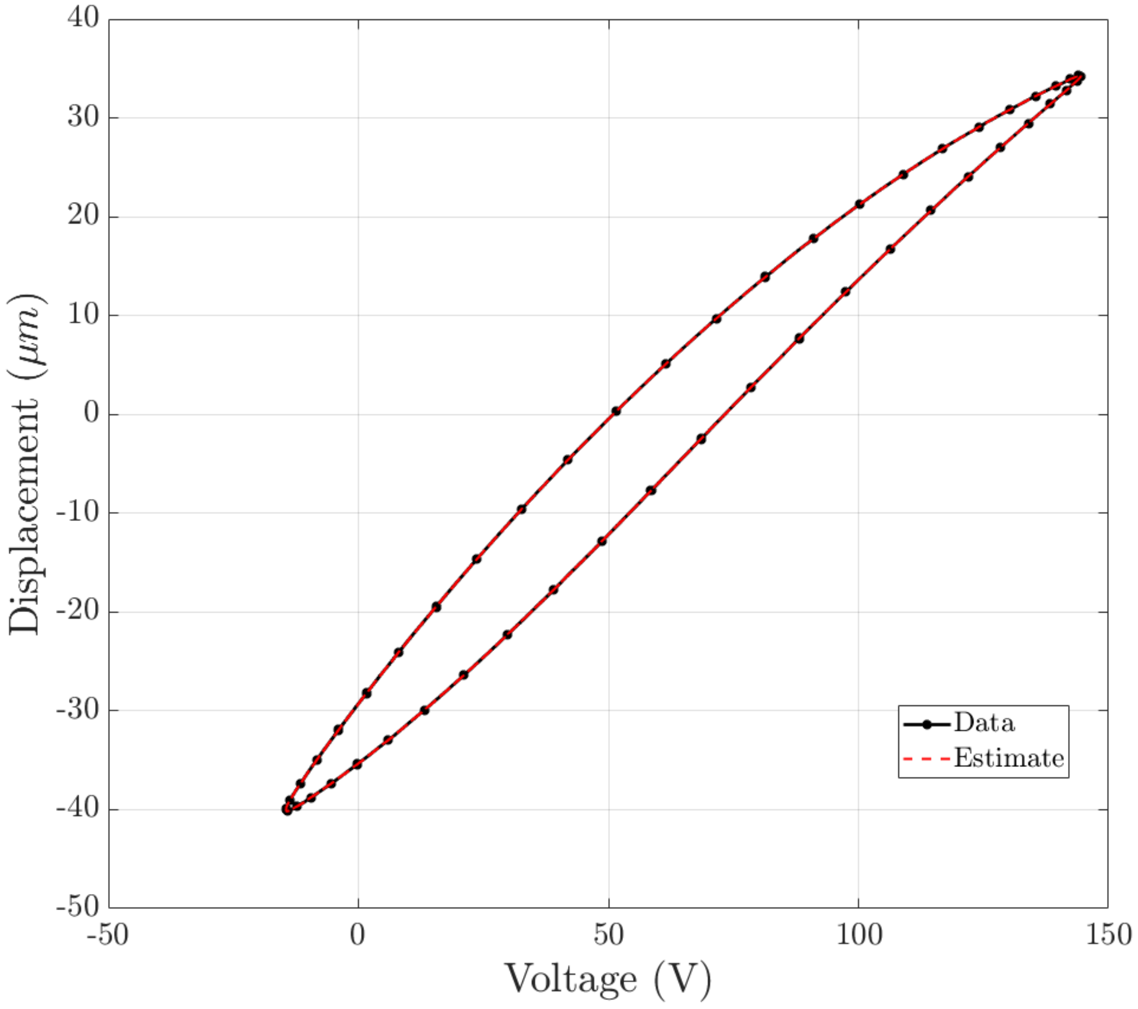
Constrained Extended Kalman Filter (CEKF)

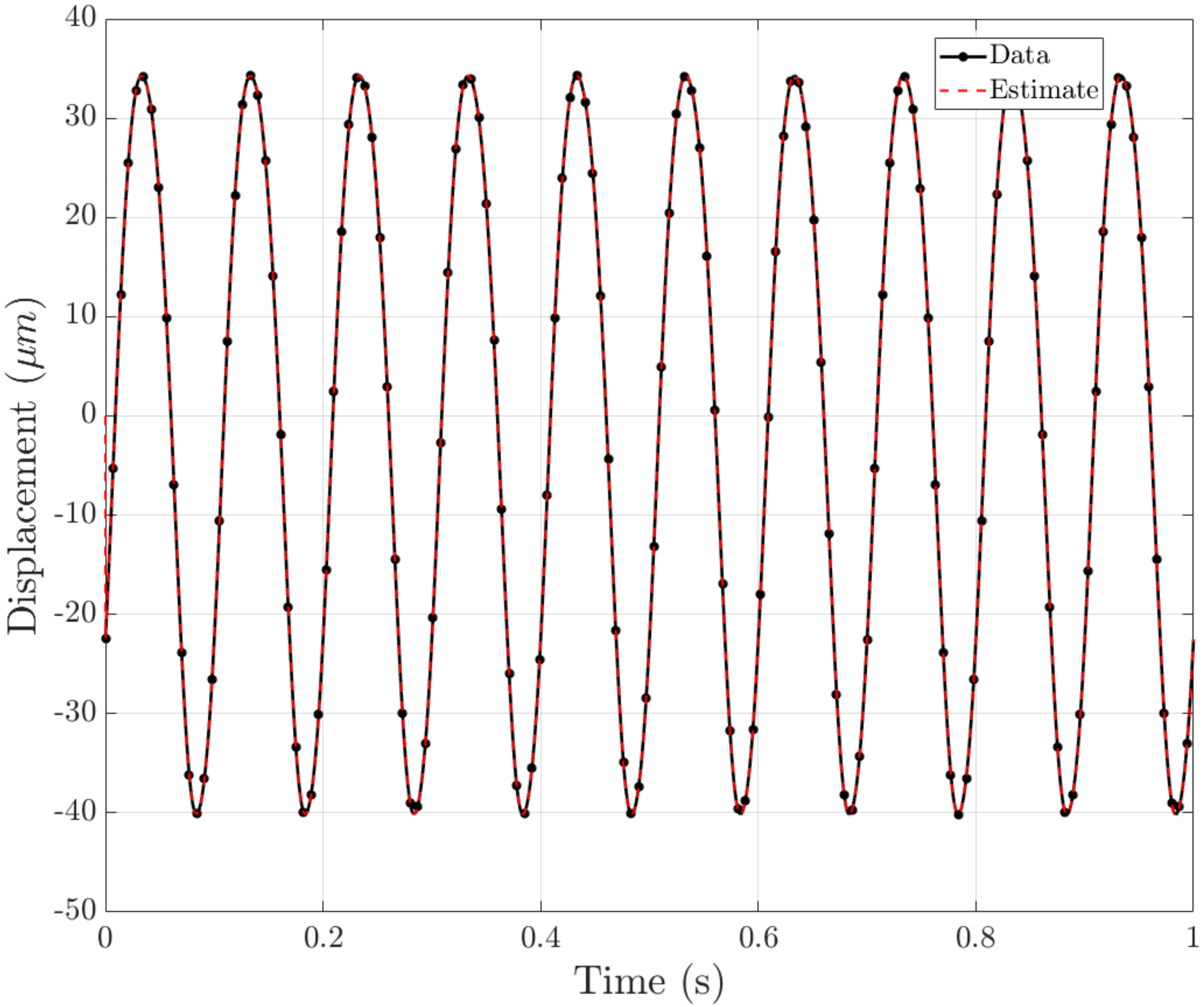
- Kalman filter goal: **minimize** the error variance
- Formulate the Kalman gain equation as a constrained optimization problem
- Posterior error covariance: $P_{k|k} = (I - K_k H_k) P_{k|k-1} (I - K_k H_k)^T + K_k R_k K_k^T$

$$\min_{K} \ \mathrm{trace}(P_{k|k})$$

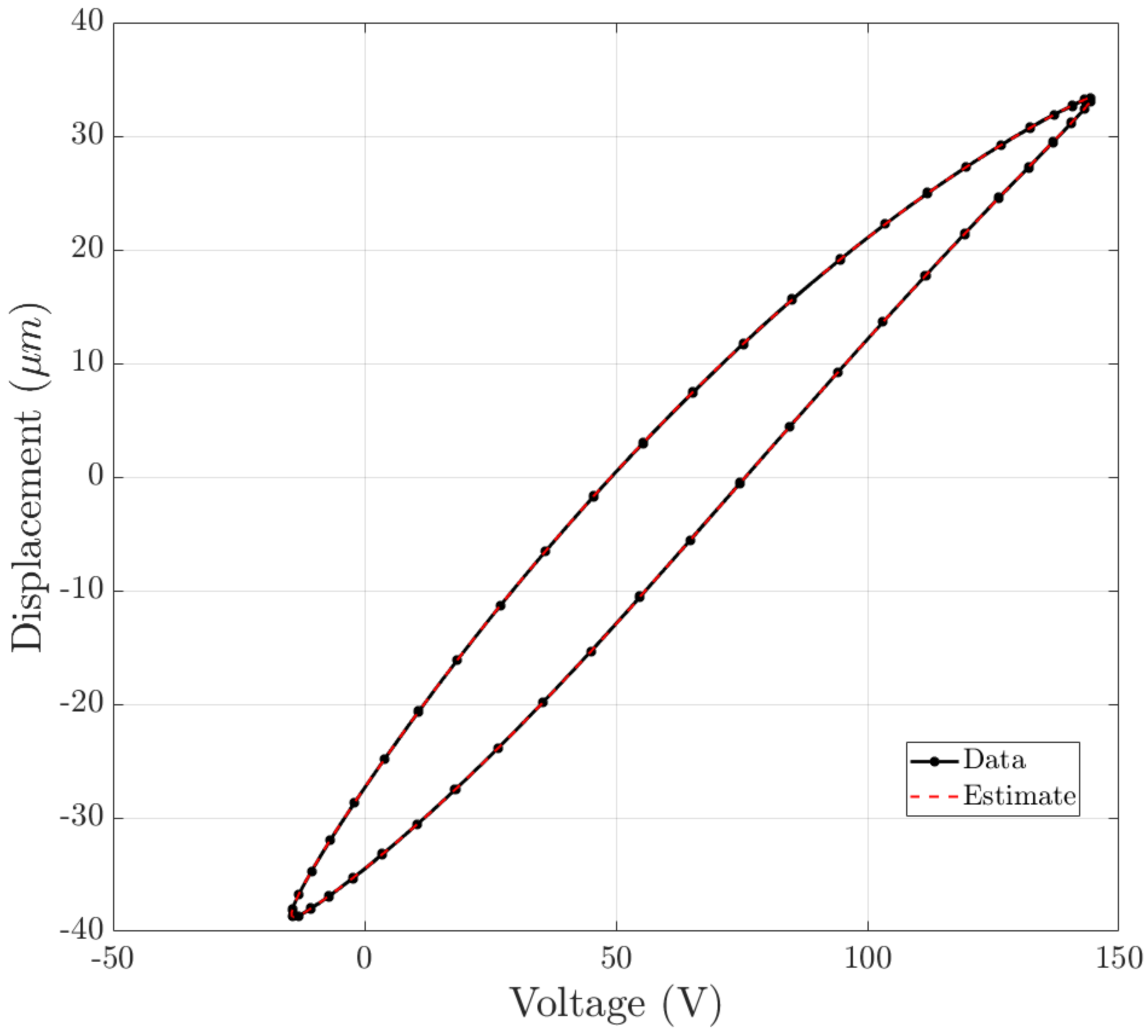
$$\text{subject to} \begin{cases} -1 \le \hat{x}_{1_{k|k}} \le 1 \\ 0 < \hat{x}_{2_{k|k}} \\ 0 < \hat{x}_{3_{k|k}} \\ 0 < \hat{x}_{4_{k|k}} \\ \frac{1}{2} \le \hat{x}_{5_{k|k}} \end{cases}$$

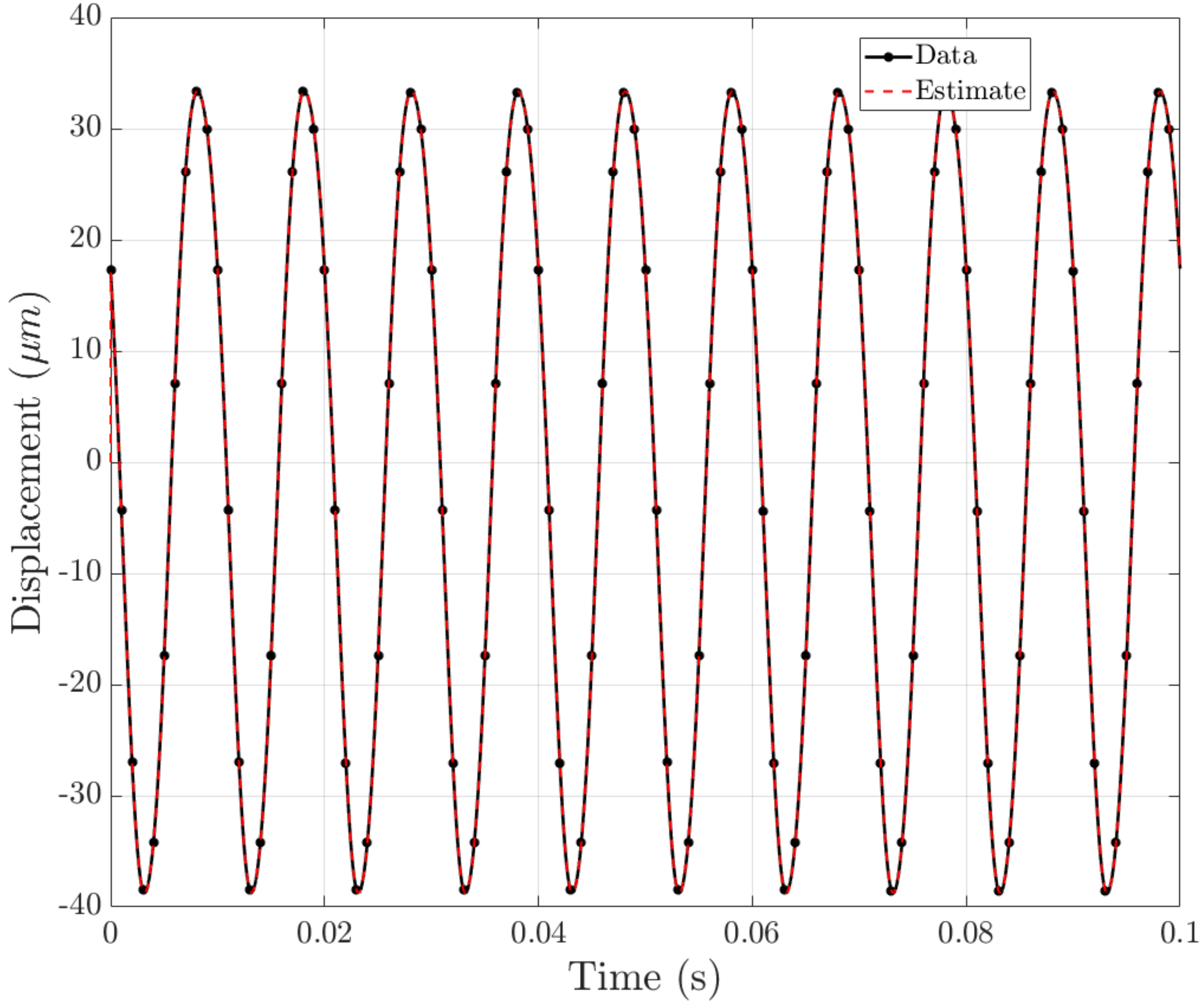
Parameter Estimation Results: 10 Hz



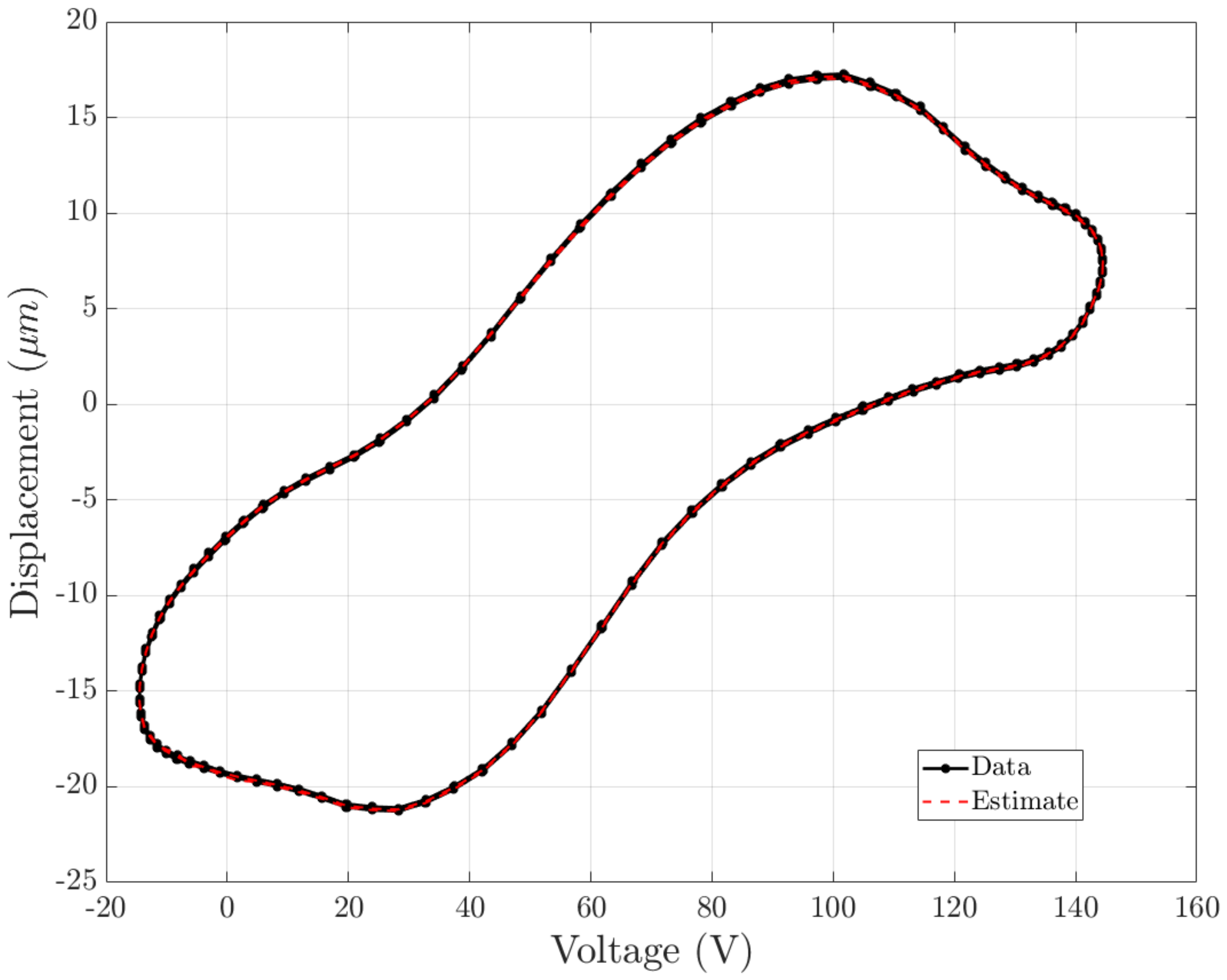


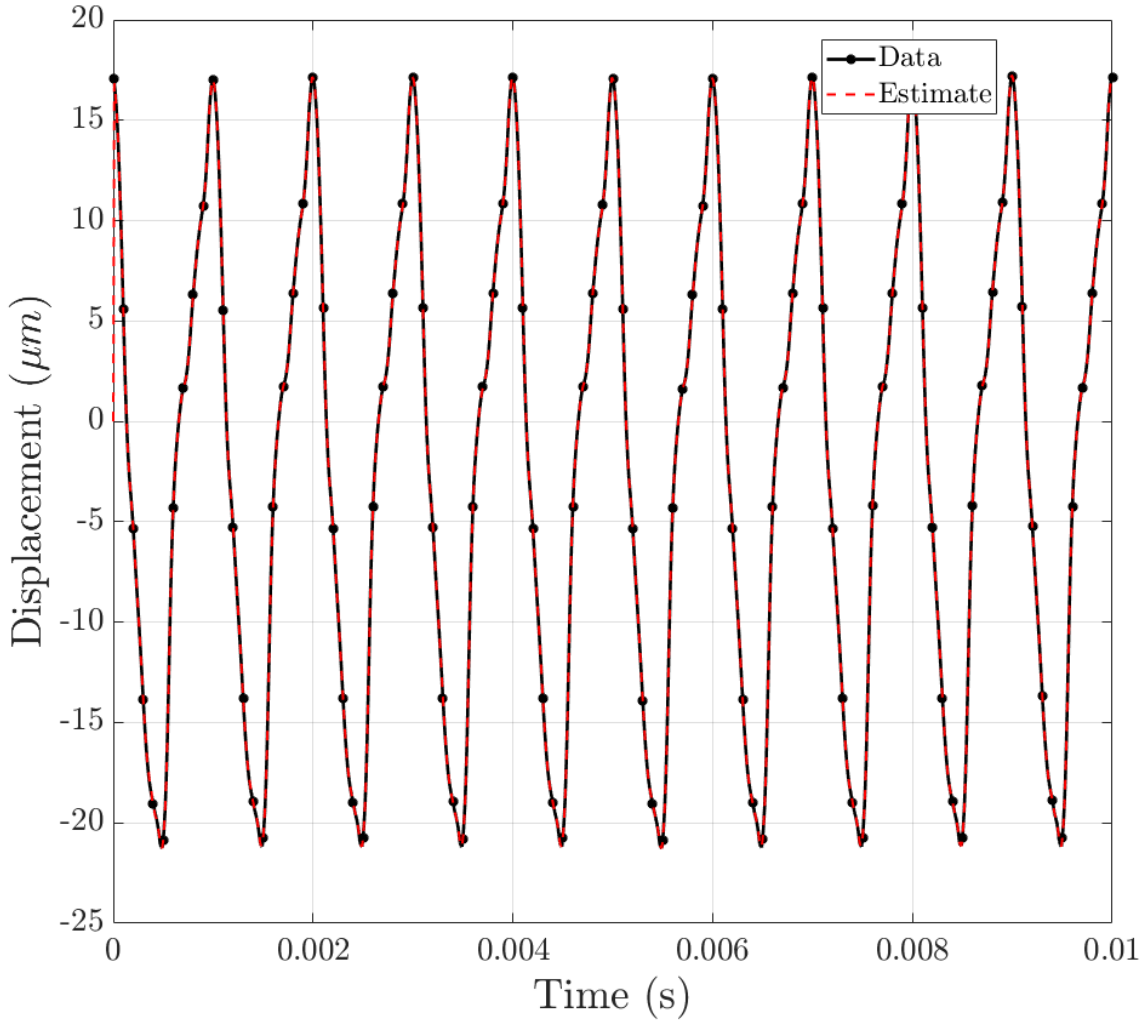
Parameter Estimation Results: 100 Hz



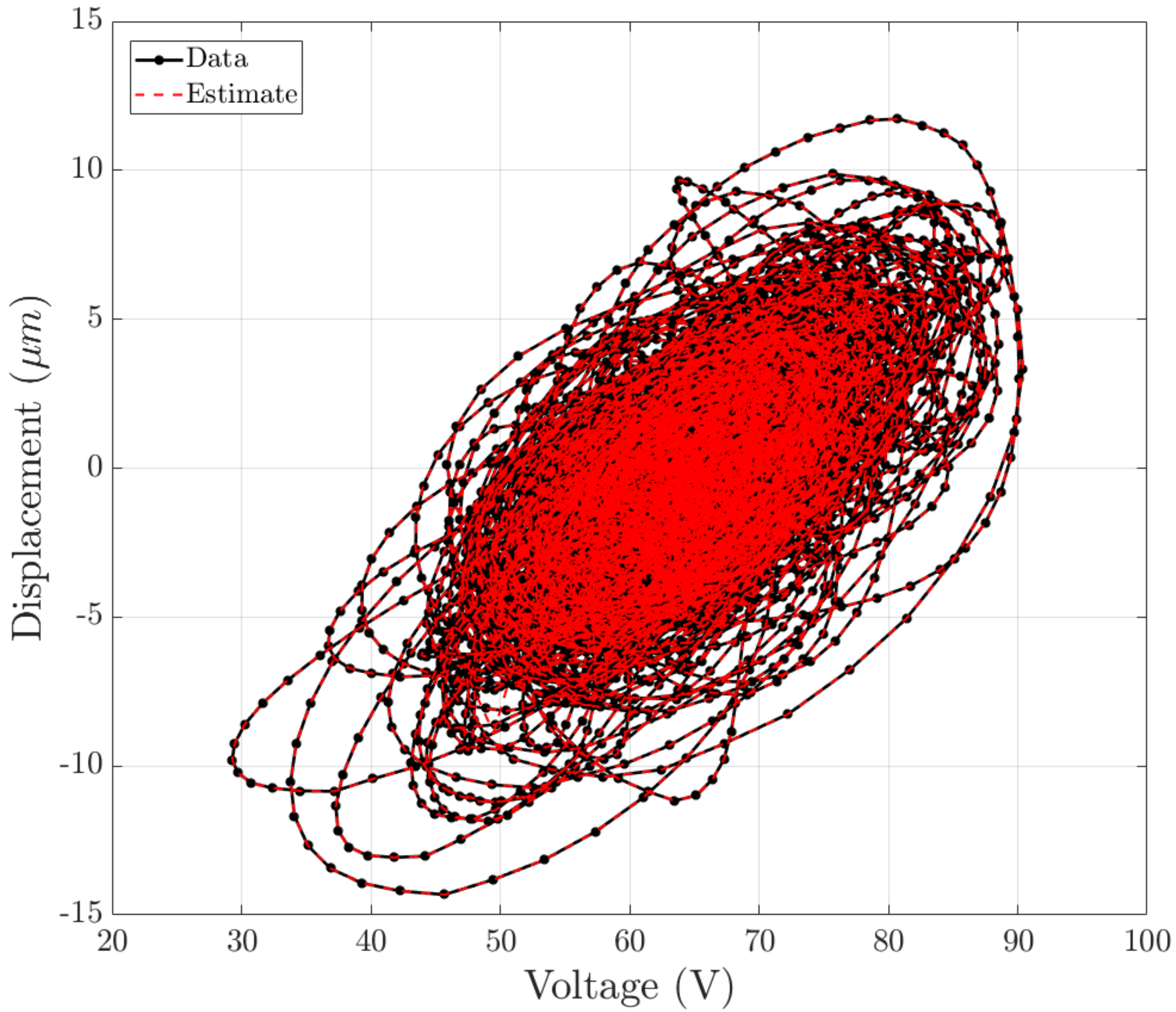


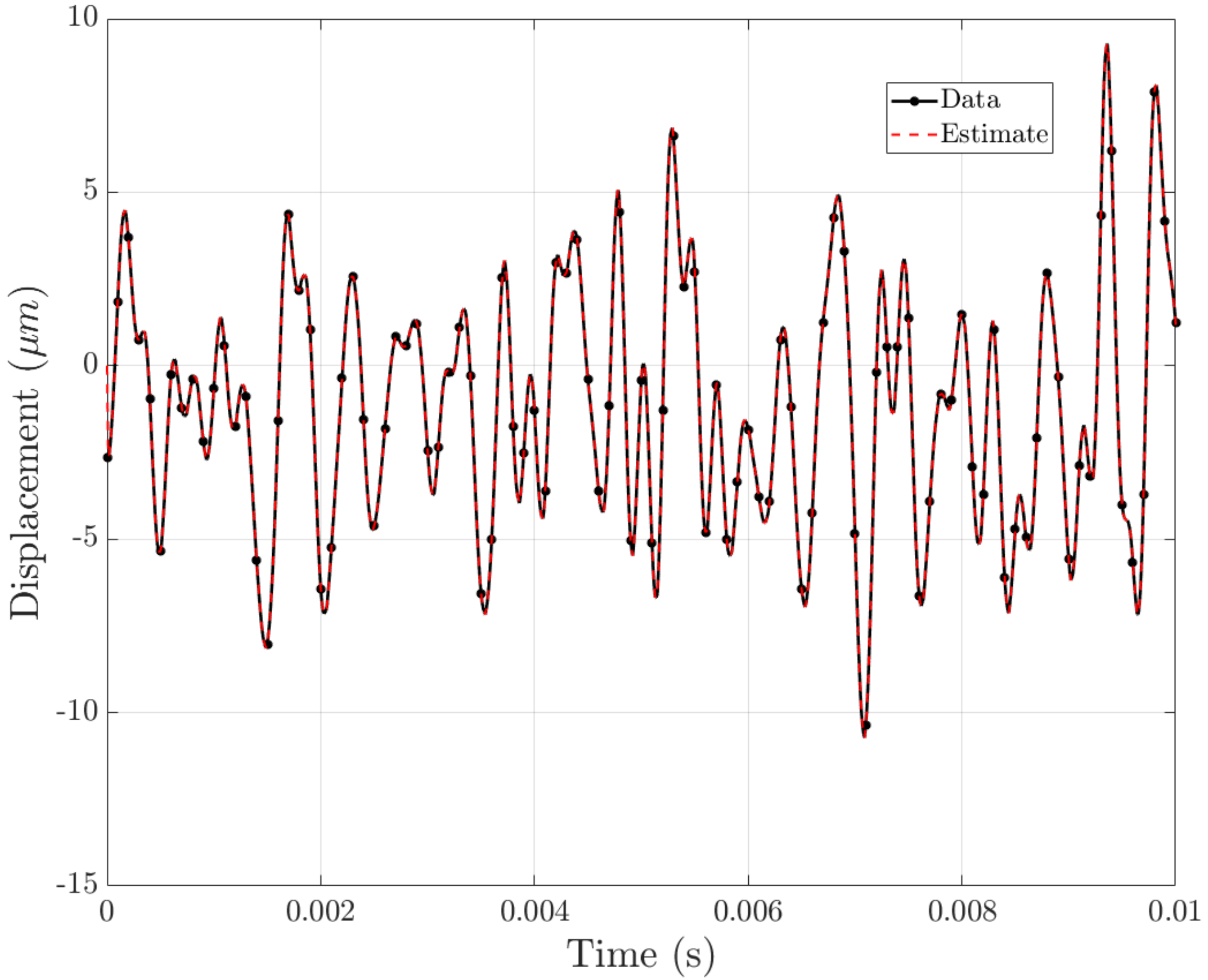
Parameter Estimation Results: 1000 Hz



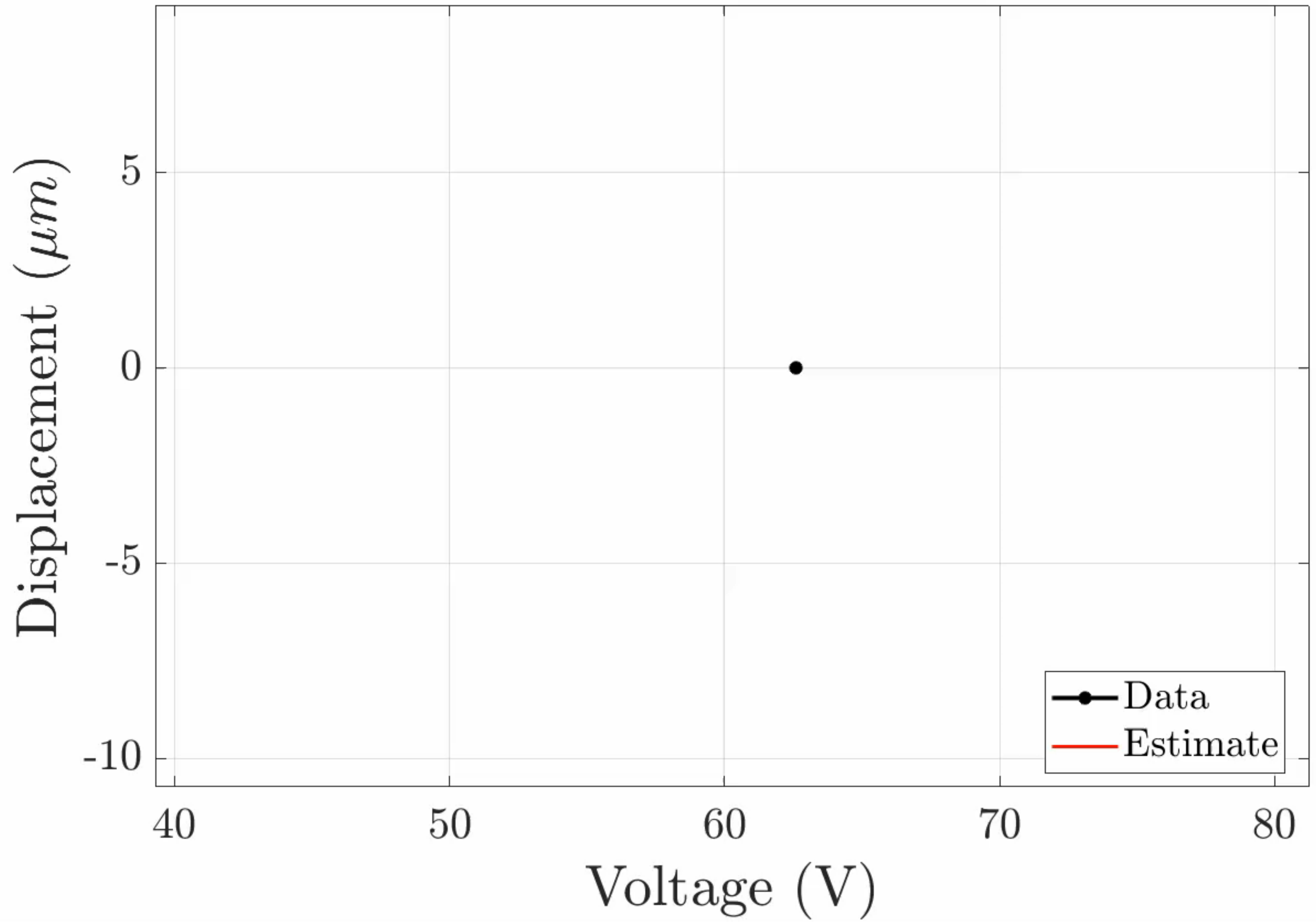


Parameter Estimation Results: Random

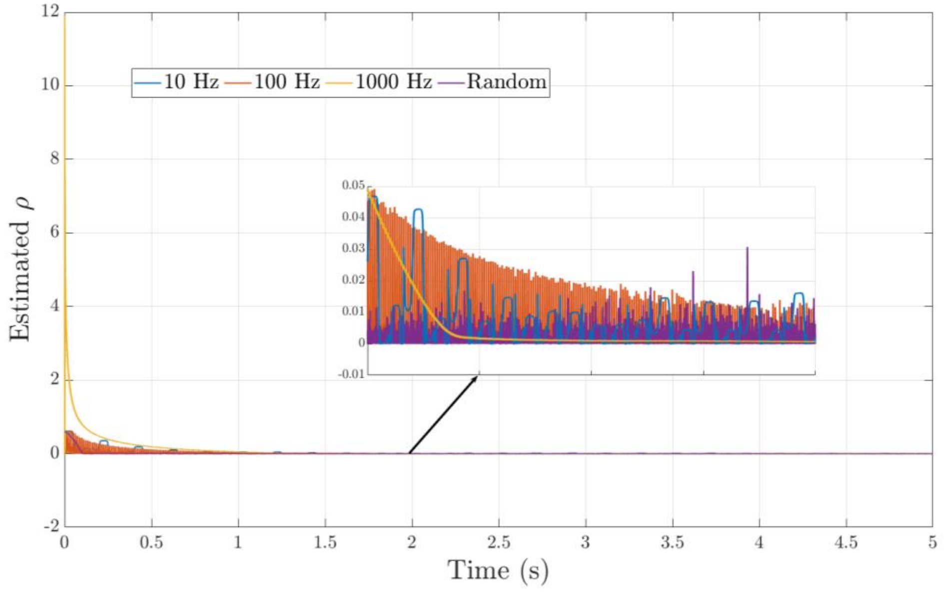


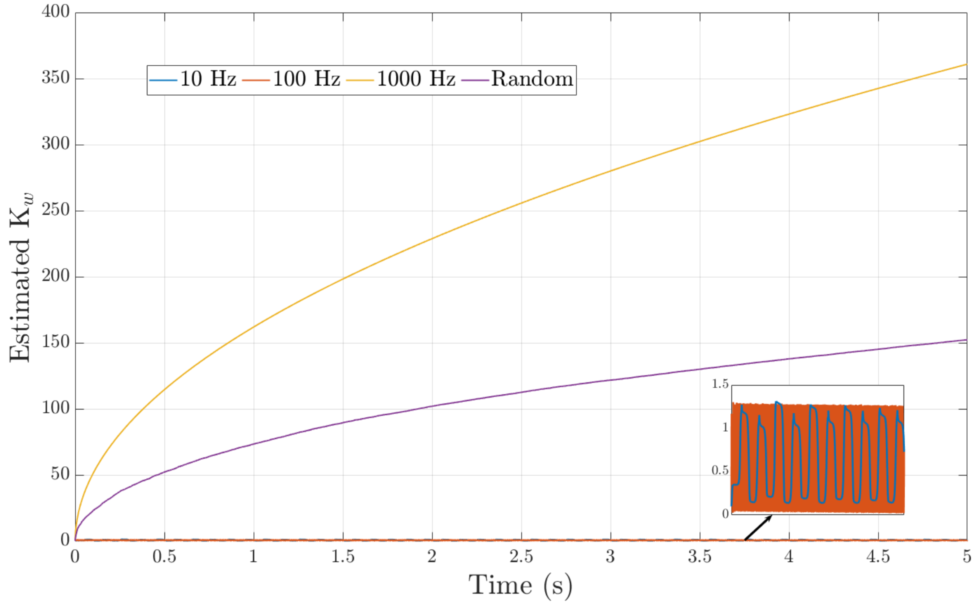


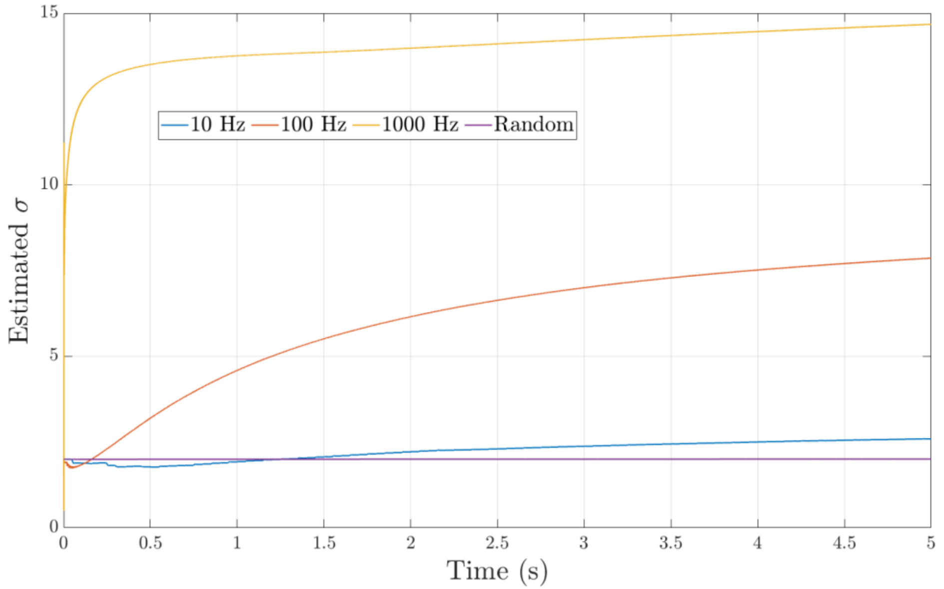
Kalman Filter Visualization

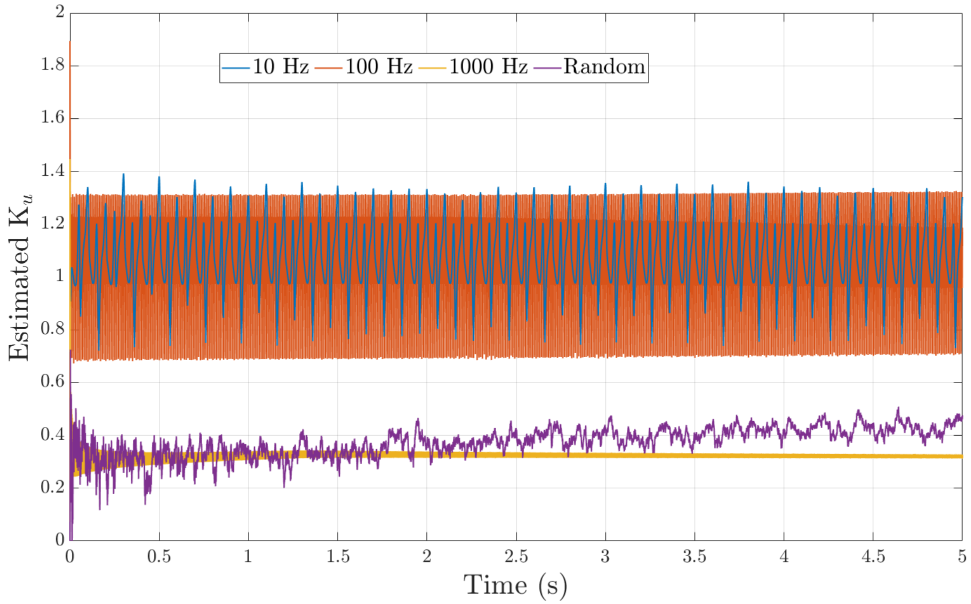


Estimated Parameters









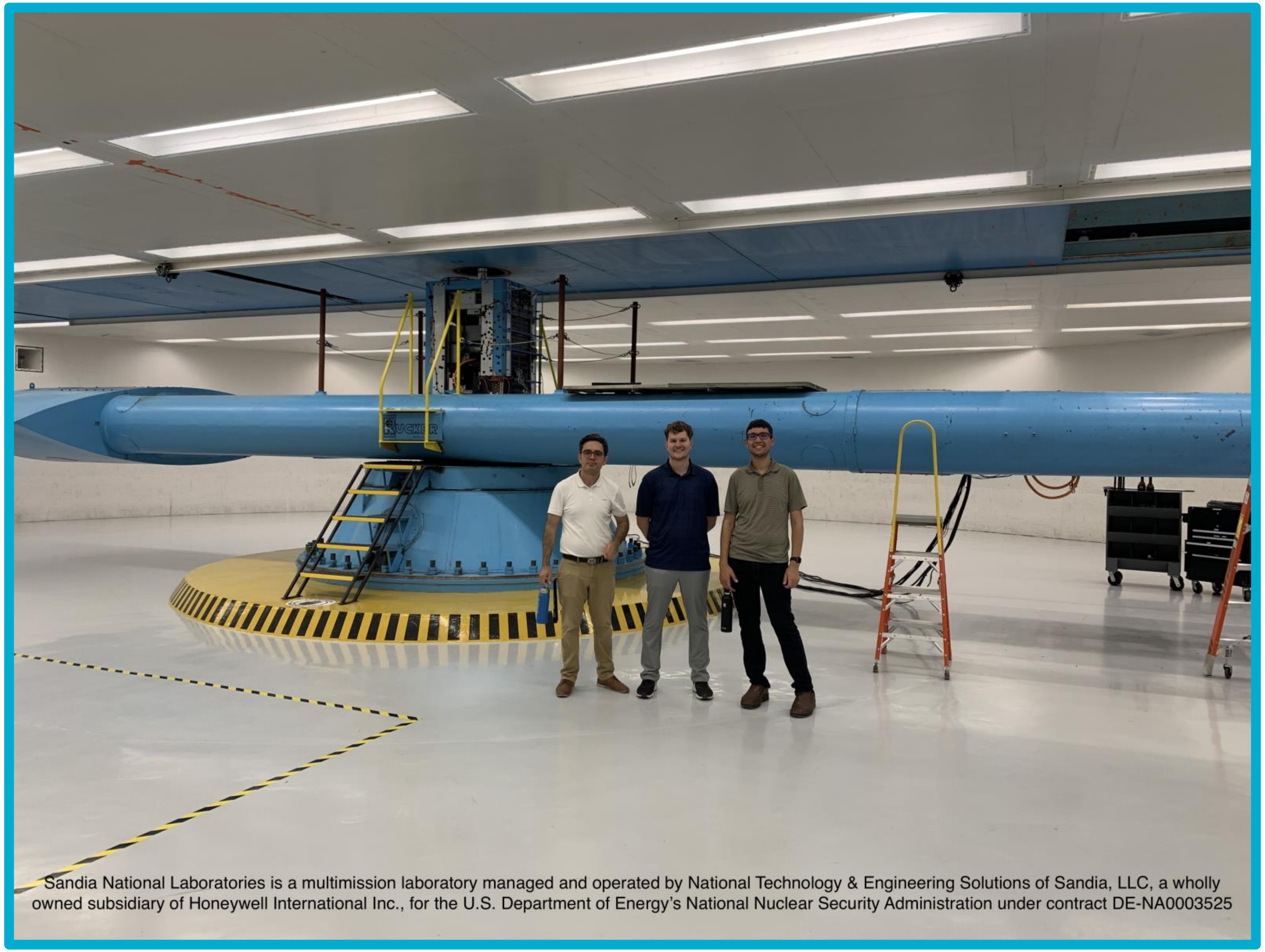
Conclusions/Future Work

- Static parameter estimation:
  - Implement a heuristic optimization algorithm to minimize objective functions of static parameter estimation, fitting the data to the generalized Bouc-Wen model.
- Hysteresis compensation:
  - Use the output of the filter to design a feedback controller that stabilizes the hysteresis.
- Further Experimental Work:
  - Expand this work to consider different actuators. Additional data is to be collected at various frequencies.

Acknowledgments

- This research was conducted at the 2023 Nonlinear Mechanics and Dynamics Research Institute supported by Sandia National Laboratories and hosted by the University of New Mexico. Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.
- The students would like to thank their mentors Moheimin Khan, Richard Jepsen, David Siler, Gregory Bunting, and Abdessattar Abdelkefi for their technical support
- The students would also like to thank Deborah Fowler, Ben Moldenhauer, and Rob Kuether for their technical and organizational support

Questions?



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