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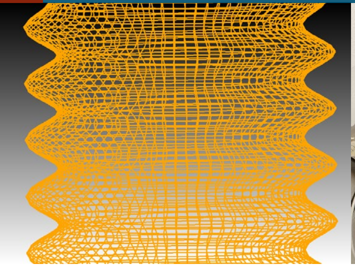
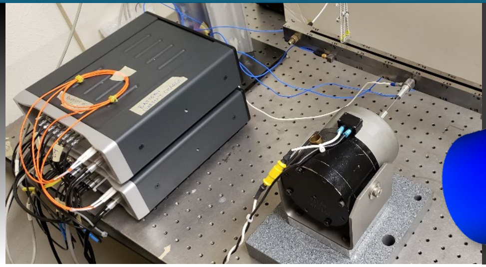
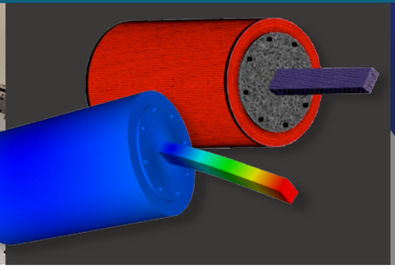
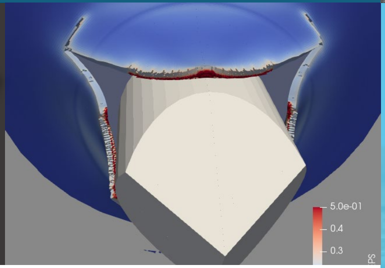
Optimizing Test Setup Parameters for Force Appropriation Testing









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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Table of Contents

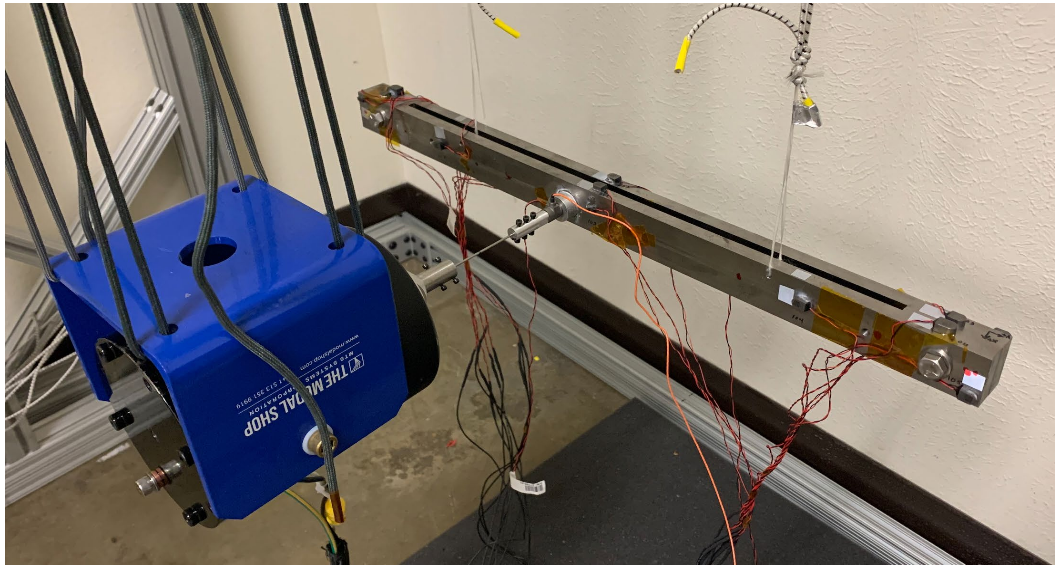
Current Problem to Address

- Dynamic Nonlinearities Widespread in Mechanical Systems
- Better methods needed to test nonlinear structures
    - Material nonlinearities (stress and strain relationship is nonlinear)
    - Contact nonlinearities such as Bolted Joints (stick, slip, open)

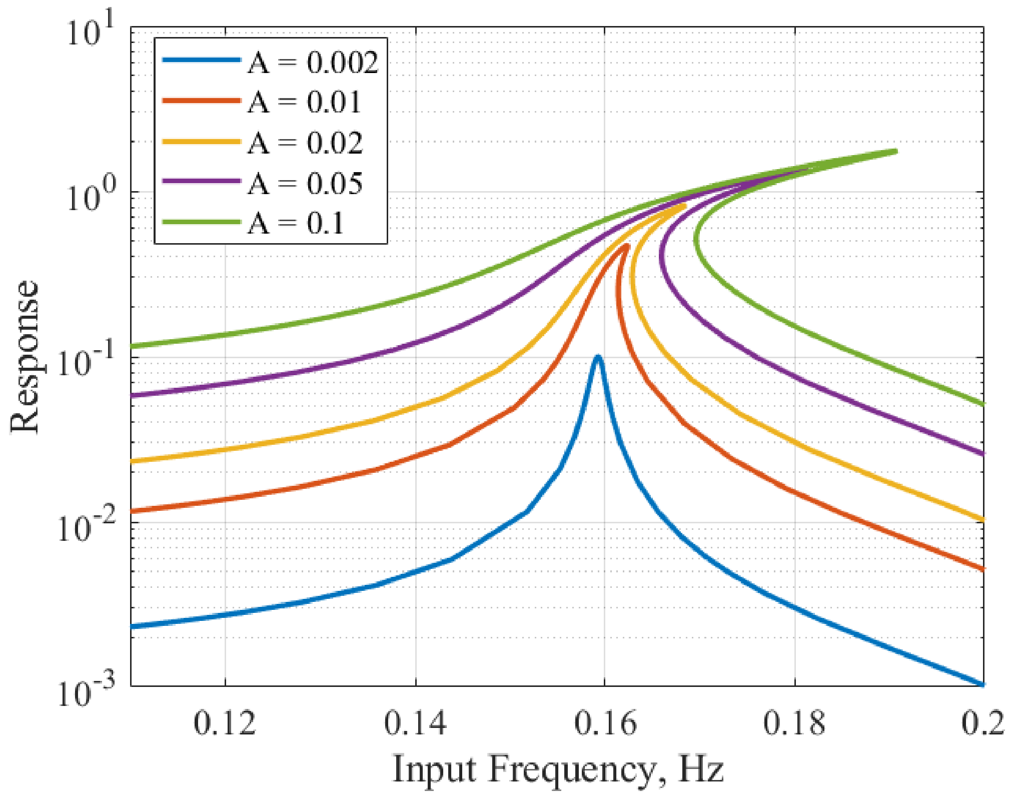
Current Experimental Methods to Test Nonlinear Parameters

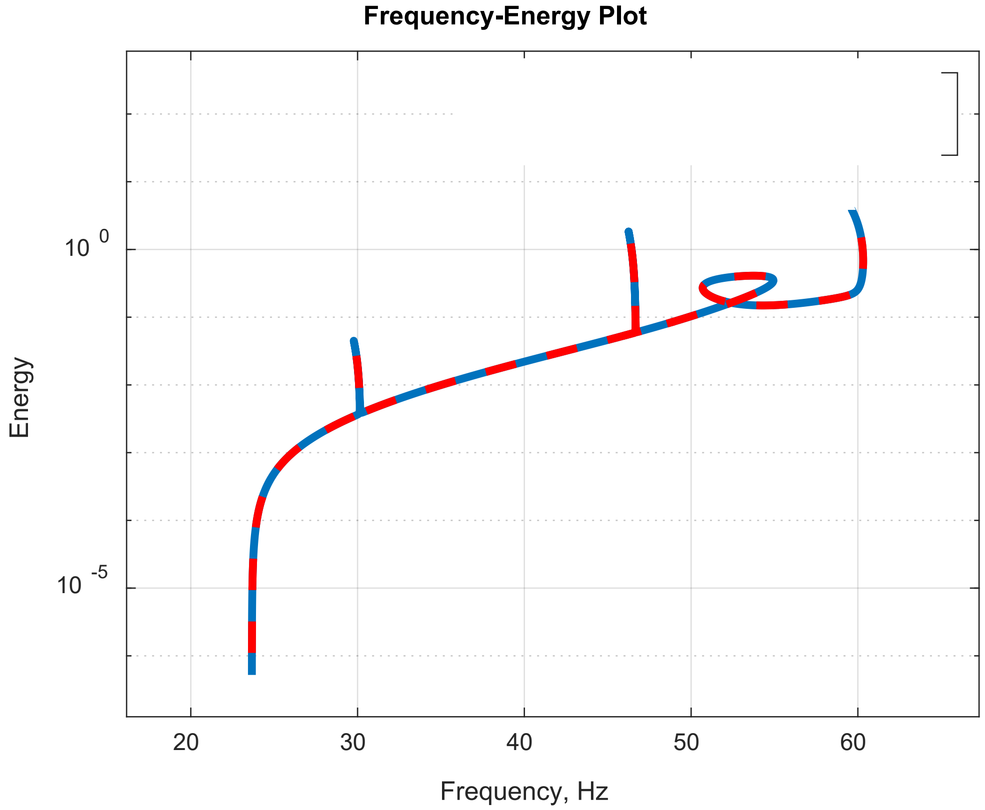
- Nonlinear swept sine testing
- Nonlinear Resonant Decay
- Force appropriation testing

OUR FOCUS



Nonlinear Normal Modes





Resonant frequency changes based on energy in the system

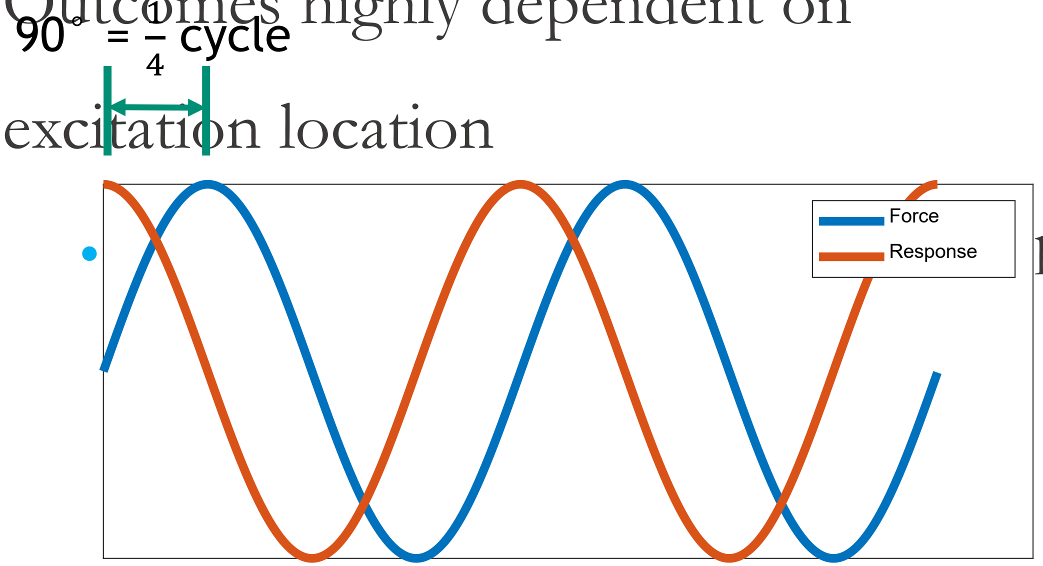
The resonant frequencies of a nonlinear system depend on energy, so experimentally the resonant frequency must be found at each energy step

Force Appropriation Procedure

- Constraint 90 degree phase difference between excitation force and response to identify resonance – allows measurement of nonlinear mode
- Control system used to
  - Excite at constant voltage
  - Sweep frequency looking for resonance
  - Once resonance is found, step up the signal voltage and repeat

Force Appropriation Challenges

- Outcomes highly dependent on excitation location



  - h
  - Excite at high-amplitude point results in large shaker-structure interaction which reaches maximum force of shaker

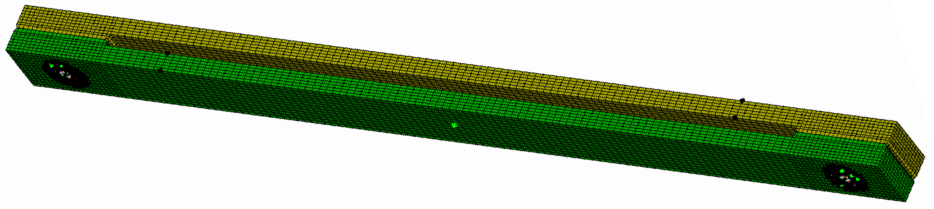
Where is the Best Excitation Location?

Project Description

- Compare two simulation methods to predict the optimal force input location
  - Optimal = Maximize modal response for input voltage and force our equipment can support
- Confirm simulated predictions with experimental data and determine best method

Method 1: Linearized Frequency Response Functions (FRF)

Linearize nonlinear contact elements and compute frequency response functions between response and voltage



Method 2: Harmonic Balance

Utilize harmonic balance to simulate a force appropriation test by enforcing a phase constraint and using a nonlinear model. Compare outcomes at several input locations

Simulation Methods-Linearized System about Preloaded State

- Shaker-structure combined system EOM linearized about preloaded equilibrium

$$\boldsymbol{M}\ddot{\boldsymbol{y}} + \boldsymbol{C}\dot{\boldsymbol{y}} + \overline{\boldsymbol{K}}\boldsymbol{y} = \boldsymbol{f}_{\boldsymbol{ext}}(t),$$

Given:

$\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{x}_{\boldsymbol{0}}$, ($\boldsymbol{x}_{\boldsymbol{o}}$ is a preloaded equilibrium point)

$\overline{\boldsymbol{K}} = \boldsymbol{K} + \frac{\partial \boldsymbol{f}_{\boldsymbol{nl}}}{\partial \boldsymbol{x}}\Big|_{\boldsymbol{x}=\boldsymbol{x}_{\boldsymbol{0}}}$ (linearized stiffness)

$\boldsymbol{f}_{\boldsymbol{nl}}$ corresponds to the nonlinear contact forces

$\boldsymbol{f}_{\boldsymbol{ext}}(t)$ corresponds to the external excitation (harmonic voltage input to amplifier)

- From the EOM, we can further derive the linearized FRF Matrix as

$$\boldsymbol{H} = [\overline{\boldsymbol{K}} - \omega^2 \boldsymbol{M} + j\omega \boldsymbol{C}]^{-\boldsymbol{1}}$$

Multi-Harmonic Balance (MHB) - Theory

Multi-harmonic balance is a method to solve nonlinear equations of motion of the form

$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{C}\,\dot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} + \mathbf{f}_{nl}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{f}_{pre} + \mathbf{f}_{ext}(t)$$

The response vectors ($\ddot{\mathbf{x}}, \dot{\mathbf{x}}, \mathbf{x}$) as well as the forcing ($\mathbf{f}_{ext}(t)$) are unknown

Assuming a periodic steady state response, the Fourier series are written as,

$$\mathbf{x}(t) = \frac{\mathbf{c}_0^x}{\sqrt{2}} + \sum_{k=1}^{N_h} [\mathbf{s}_k^x \sin(k\omega t) + \mathbf{c}_k^x \cos(k\omega t)]$$

$$\mathbf{f}_{nl}(\mathbf{x}, \dot{\mathbf{x}}) = \frac{\mathbf{c}_0^{nl}}{\sqrt{2}} + \sum_{k=1}^{N_h} [\mathbf{s}_k^{nl} \sin(k\omega t) + \mathbf{c}_k^{nl} \cos(k\omega t)]$$

$$\mathbf{f}_{ext}(t) = \frac{\mathbf{c}_0^f}{\sqrt{2}} + \sum_{k=1}^{N_h} [\mathbf{s}_k^f \sin(k\omega t) + \mathbf{c}_k^f \cos(k\omega t)]$$

Sort Fourier coefficients into vectors

$$\mathbf{z} = \left[(\mathbf{c}_0^x)^{\mathrm{T}} \quad (\mathbf{s}_1^x)^{\mathrm{T}} \quad (\mathbf{c}_1^x)^{\mathrm{T}} \quad \dots \quad (\mathbf{s}_{N_h}^x)^{\mathrm{T}} \quad (\mathbf{c}_{N_h}^x)^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\mathbf{b} = \left[(\mathbf{c}_0^{nl})^{\mathrm{T}} \quad (\mathbf{s}_1^{nl})^{\mathrm{T}} \quad (\mathbf{c}_1^{nl})^{\mathrm{T}} \quad \dots \quad (\mathbf{s}_{N_h}^{nl})^{\mathrm{T}} \quad (\mathbf{c}_{N_h}^{nl})^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\mathbf{b}_{ext} = \left[(\mathbf{c}_0^f)^{\mathrm{T}} \quad (\mathbf{s}_1^f)^{\mathrm{T}} \quad (\mathbf{c}_1^f)^{\mathrm{T}} \quad \dots \quad (\mathbf{s}_{N_h}^f)^{\mathrm{T}} \quad (\mathbf{c}_{N_h}^f)^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\mathbf{b}_{pre} = \left[\left(\frac{\mathbf{f}_{pre}}{\sqrt{2}}\right)^{\mathrm{T}} \quad 0^{\mathrm{T}} \quad \dots \quad 0^{\mathrm{T}}\right]^{\mathrm{T}}$$

The Fourier coefficients are combined to form the harmonic balance equations of motion

$$\mathbf{r}(\mathbf{z}, \mathbf{b}_{ext}, \omega) = \mathbf{A}(\omega)\mathbf{z} + \mathbf{b}(\mathbf{z}) - \mathbf{b}_{pre} - \mathbf{b}_{ext}$$

With solutions of the form

$$\mathbf{y} = [\mathbf{z} \quad \mathbf{b}_{ext} \quad \omega]^{\mathrm{T}}$$

Multi-Harmonic Balance – Force Resonant Constraint

At a given *i*th excitation location, the force and response are described using the Fourier coefficients of the first harmonic

$$f_{ext,1,i}(t) = s_{1,i}^{f}\sin(\omega t) + c_{1,i}^{f}\cos(\omega t)$$

$$x_{1,i}(t) = s_{1,i}^{x}\sin(\omega t) + c_{1,i}^{x}\cos(\omega t)$$

The phase resonance constraint requires the phase lag to be 90°, resulting in

$$\Delta\varphi_i = -\tan^{-1}\left(\frac{s_{1,i}^{f}}{c_{1,i}^{f}}\right) + \tan^{-1}\left(\frac{s_{1,i}^{x}}{c_{1,i}^{x}}\right) - \frac{\pi}{2} = 0$$

To form a solvable system, monophase excitation is assumed

$$f_{ext,k,i}(t) = s_{1,i}^{f}\sin(k\omega t) + \cancelto{0}{c_{1,i}^{f}\cos(k\omega t)}$$

$$x_{k,i}(t) = \cancelto{0}{s_{1,i}^{x}\sin(k\omega t)} + c_{1,i}^{x}\cos(k\omega t)$$

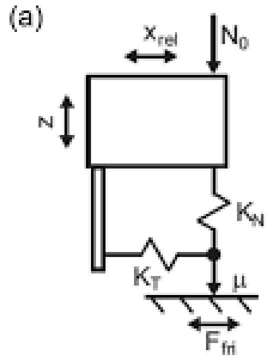
$$\longrightarrow$$

$$f_{ext,k,i}(t) = s_{1,i}^{f}\sin(k\omega t)$$

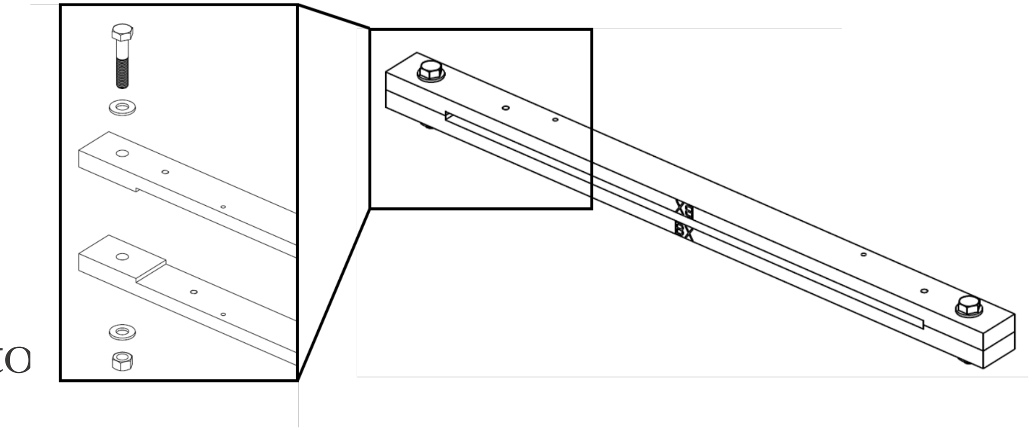
$$x_{k,i}(t) = c_{1,i}^{x}\cos(k\omega t)$$

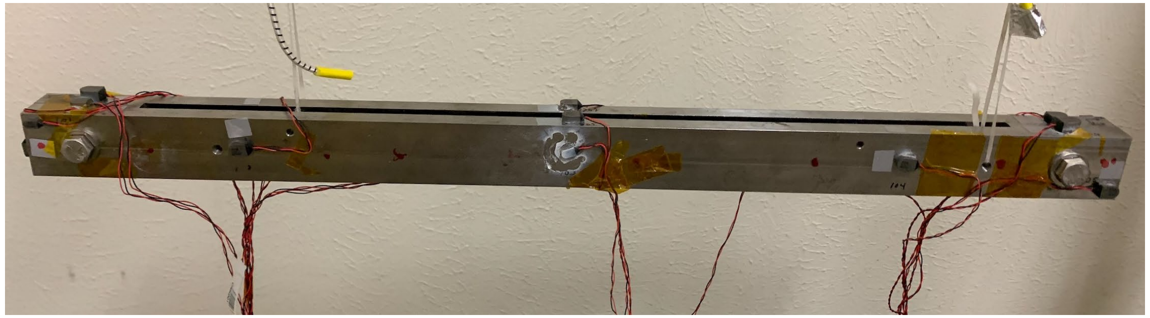
Test Structure: "C-Beam"

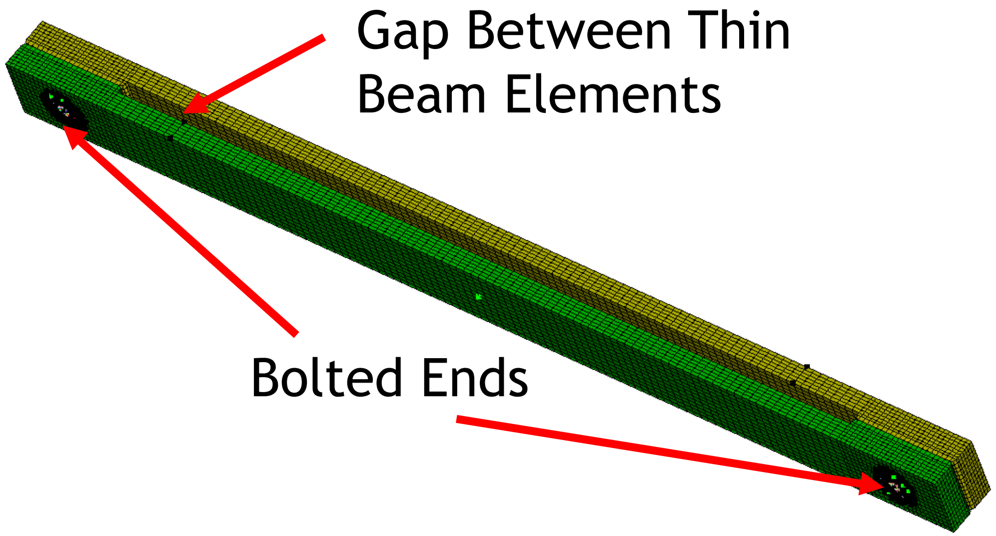
States
Stick
Slip
Open



- Two beams held together with bolts at both ends
- When excited at high enough energy levels the bolted joint area exhibits nonlinearity
- A model of this system has been developed with nonlinear Jenkins elements connecting the joints to simulate the nonlinear contact







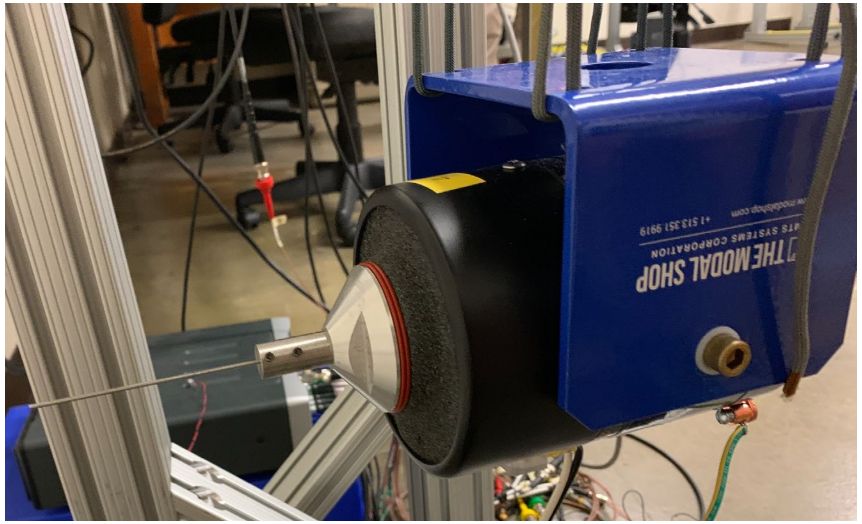
[1] Gross, J., et al. "A numerical round robin for the prediction of the dynamics of jointed structures." *Dynamics of Coupled Structures, Volume 4*. Springer, Cham, 2016. 195-211.

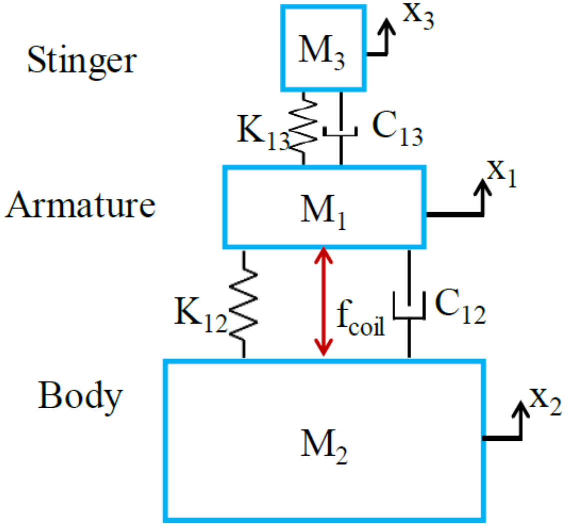
Shaker Characterization

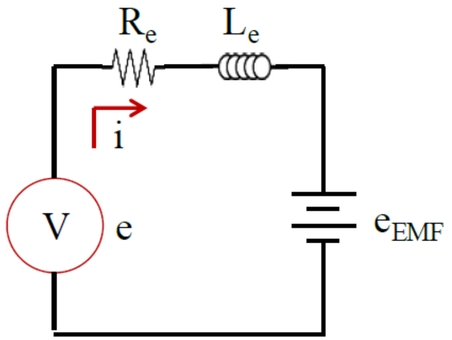
We model the shaker as a series of masses connected by springs and dampers

Important for Representing Shaker-Model Interaction

Why is this important?







$$\frac{1}{k_a}\dot{e}_{amp} + \frac{\omega_b}{k_a}e_{amp} = e_{DAQ}$$

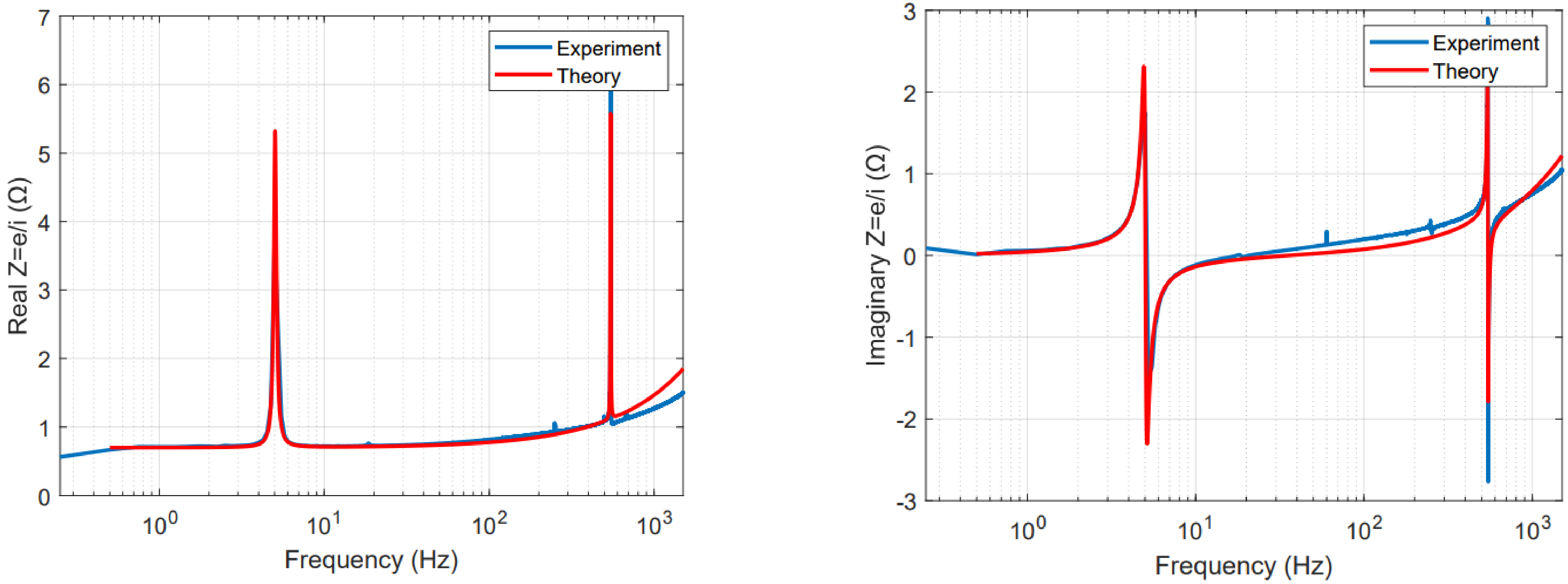
$$M_{skr} = \begin{bmatrix} M_1 & 0 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 \\ 0 & 0 & M_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} C_{skr} = \begin{bmatrix} (c_{12}+c_{13}) & -c_{12} & -c_{13} & 0 & 0 \\ -c_{12} & c_{12} & 0 & 0 & 0 \\ -c_{13} & 0 & c_{13} & 0 & 0 \\ BL & -BL & 0 & L_e & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{k_a} \end{bmatrix} K_{shk} = \begin{bmatrix} (k_{12}+k_{13}) & -k_{12} & -k_{13} & -BL & 0 \\ -k_{12} & k_{12} & 0 & BL & 0 \\ -k_{13} & 0 & k_{13} & 0 & 0 \\ 0 & 0 & 0 & R_e & -1 \\ 0 & 0 & 0 & 0 & \frac{\omega_b}{k_a} \end{bmatrix}$$

[2] Pacini, B.R., Roettgen, D.R., Rohe, D.P. (2020). Investigating Nonlinearity in a Bolted Structure Using Force Appropriation Techniques. In: Kerschen, G., Brake, M., Renson, L. (eds) Nonlinear Structures and Systems, Volume 1. Conference Proceedings of the Society for Experimental Mechanics Series. Springer, Cham. https://doi.org/10.1007/978-3-030-12391-8_23

[3] Schultz, R. (2021). Calibration of Shaker Electro-mechanical Models. In: Epp, D.S. (eds) Special Topics in Structural Dynamics & Experimental Techniques, Volume 5. Conference Proceedings of the Society for Experimental Mechanics Series. Springer, Cham. https://doi.org/10.1007/978-3-030-47709-7_12

Shaker Characterization

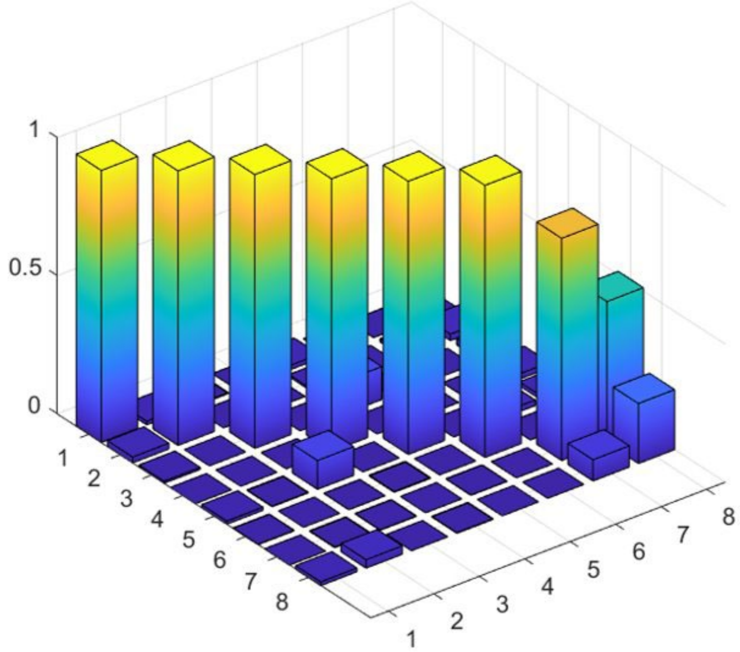
- The resistance and three masses of the system are known
- The spring constants, damping coefficients, and inductance can be curve fit using the known impedance FRFs



Linear Modal Analysis

- To confirm the linear model was representative of our test system, modes from the linear modal test were compared to a finite element model using a Modal Assurance Criterion (MAC)
- All MAC values were above 98% which demonstrates a high correlation between the finite element model and the test structure
- This indicates that simulations using this model are representative of the test at the linear level

Sim. Mode	Freq. (Hz)	Exp. Mode	Freq. (Hz)	Diff. (%)	MAC (%)
1	282.35	1	282.81	-0.16	98.47
2	346.95	2	352.27	-1.51	99.69
3	491.41	3	505.75	-2.83	99.04
4	581.80	4	593.28	-1.94	98.99
5	772.97	5	775.88	-0.37	99.31
6	945.23	6	925.78	2.10	98.53

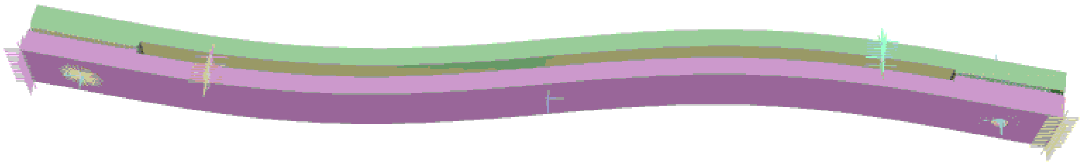


Model Mode Shapes of "C-Beam" Structure

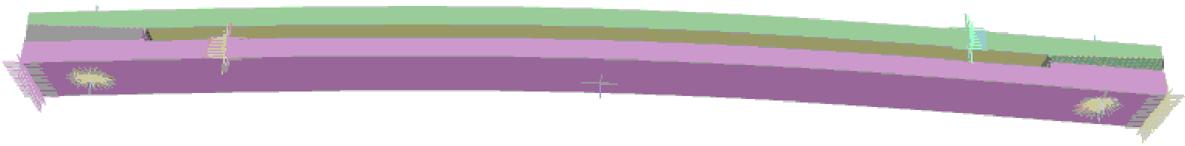
Mode 1: 282 Hz

Mode 2: 352 Hz

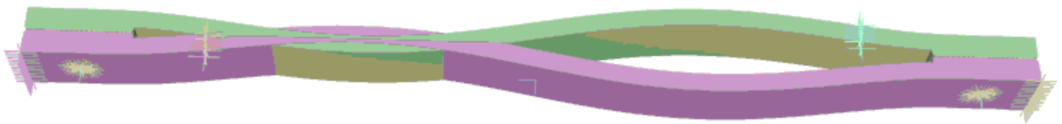
Mode 3: 505 Hz



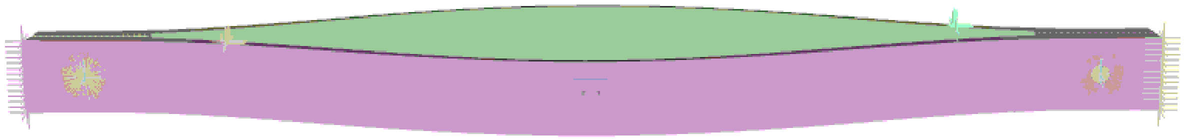
Mode 4: 593 Hz



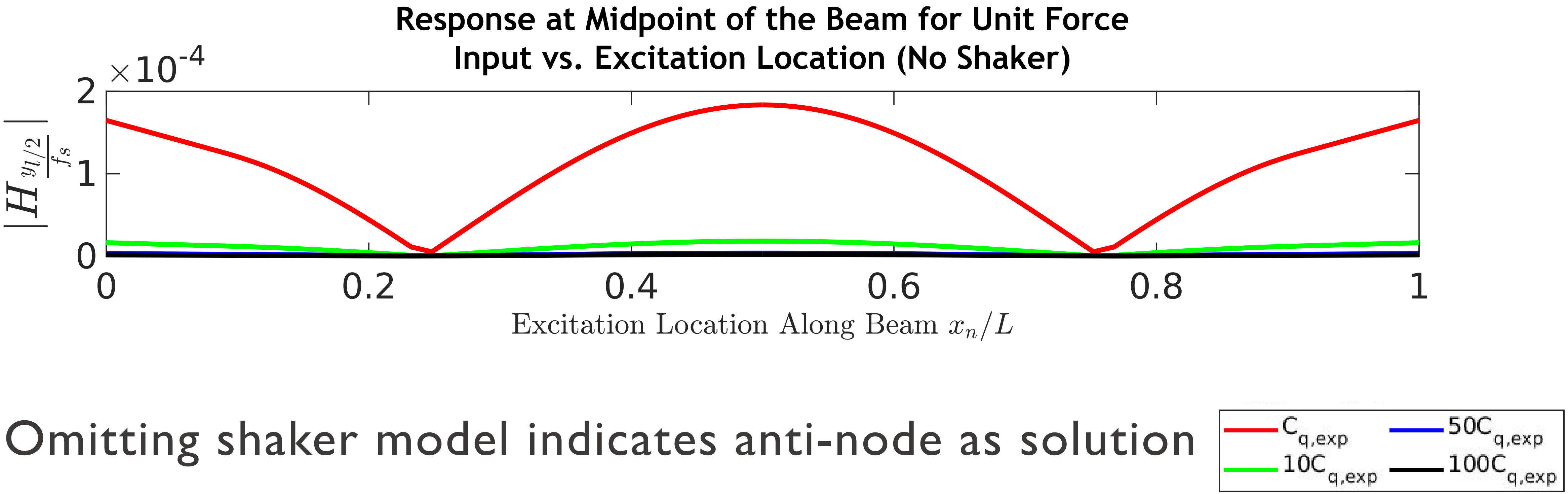
Mode 5: 776 Hz



Mode 6: 926 Hz



Simulation Method 1 -Linearized FRF Method

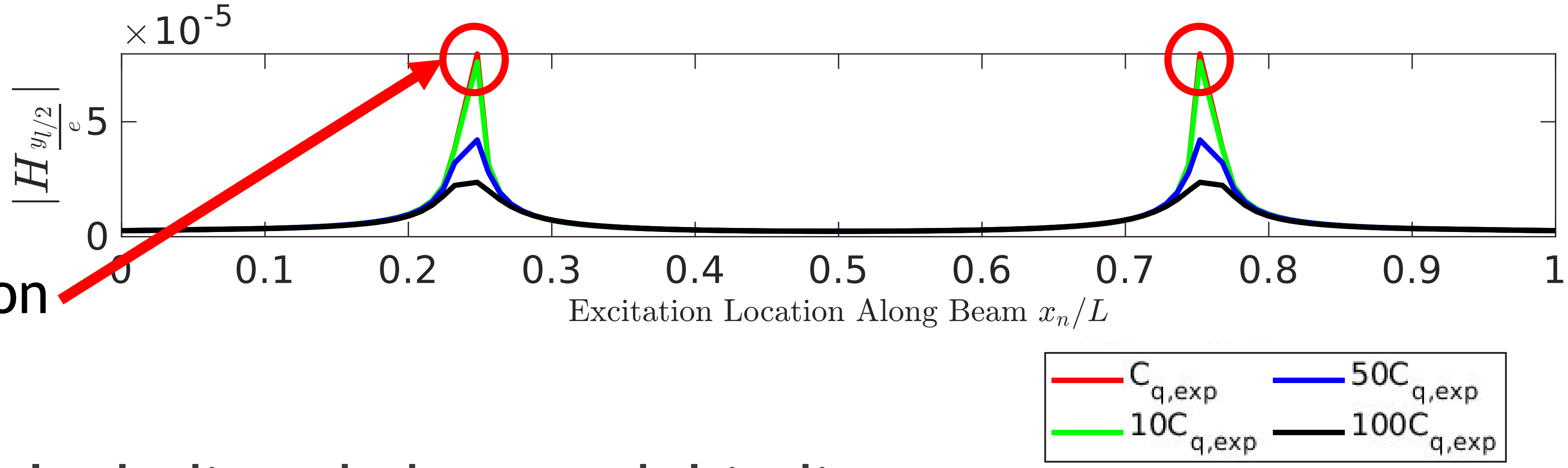


Omitting shaker model indicates anti-node as solution

Simulation Method 1 – Linearized FRF Method

Response at Midpoint of the beam for unit force input vs. Excitation Location (No Shaker)

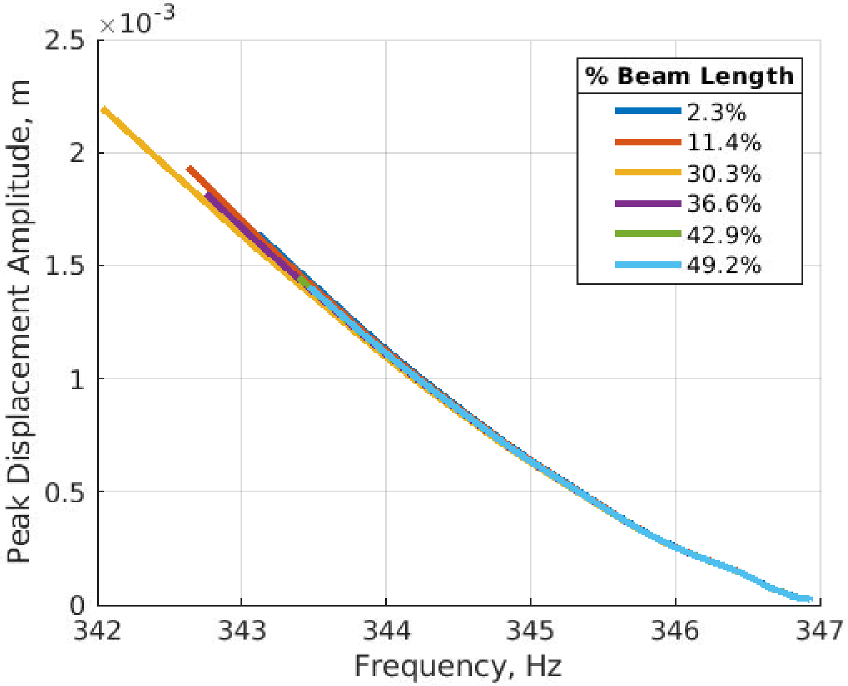
Response at Midpoint of the Beam for Unit Voltage Input vs. Excitation Location (With Shaker)



Optimal shaker location 25% from either end

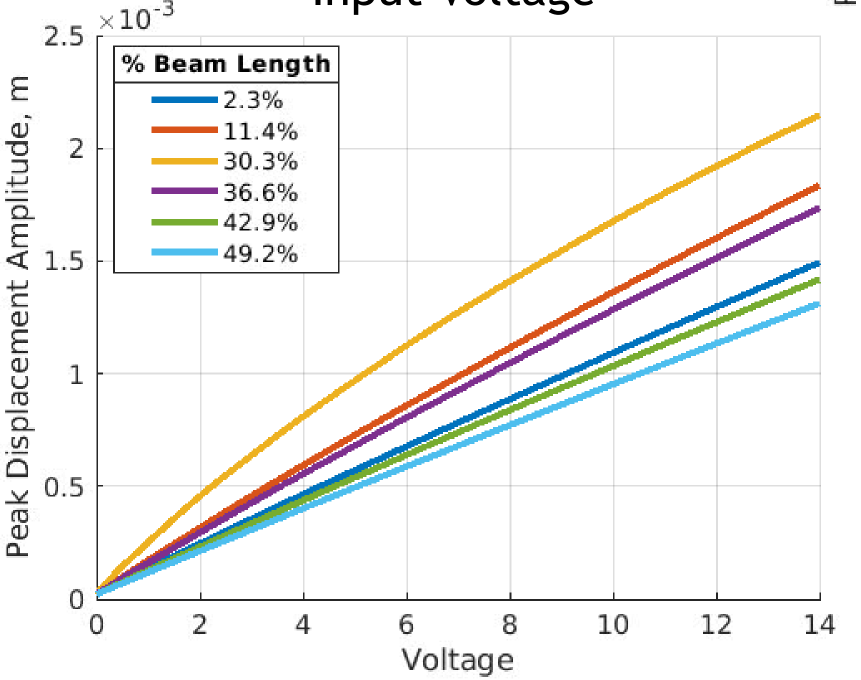
Including shaker model indicates anti-node is non-optimal

Simulation Method 2 – Harmonic Balance Method

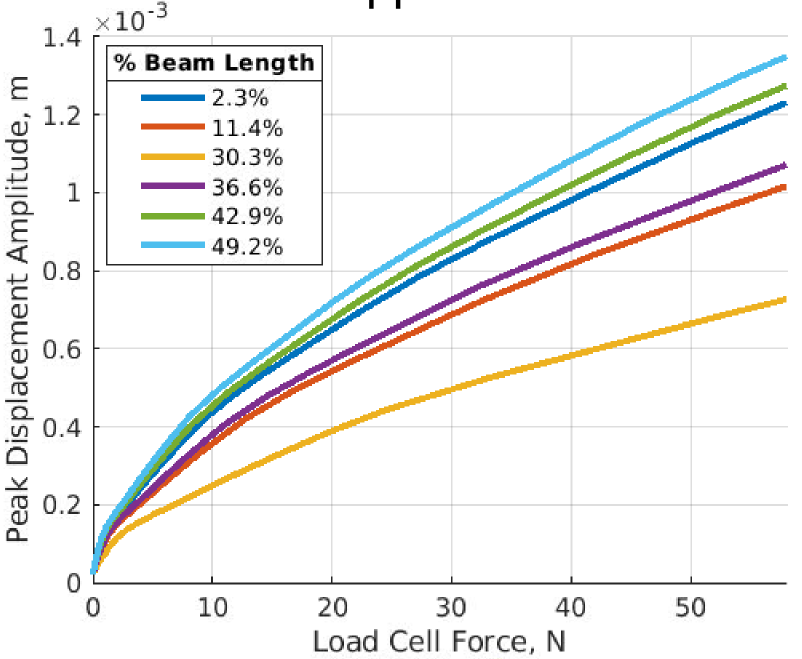


Response at Anti-Node vs. Excitation Frequency

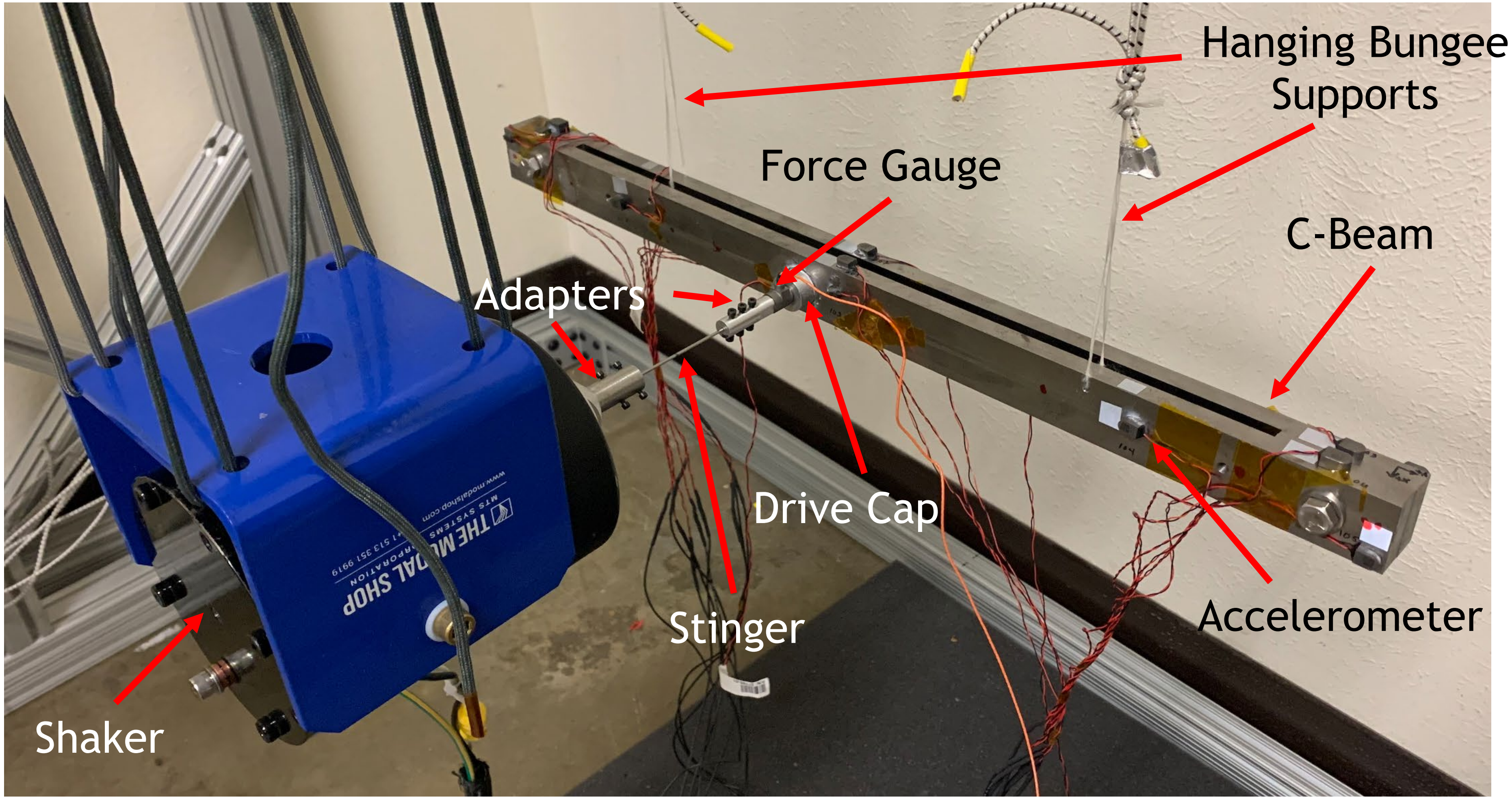
Response at Anti-Node vs. Input Voltage



Response at Anti-Node vs. Applied Force

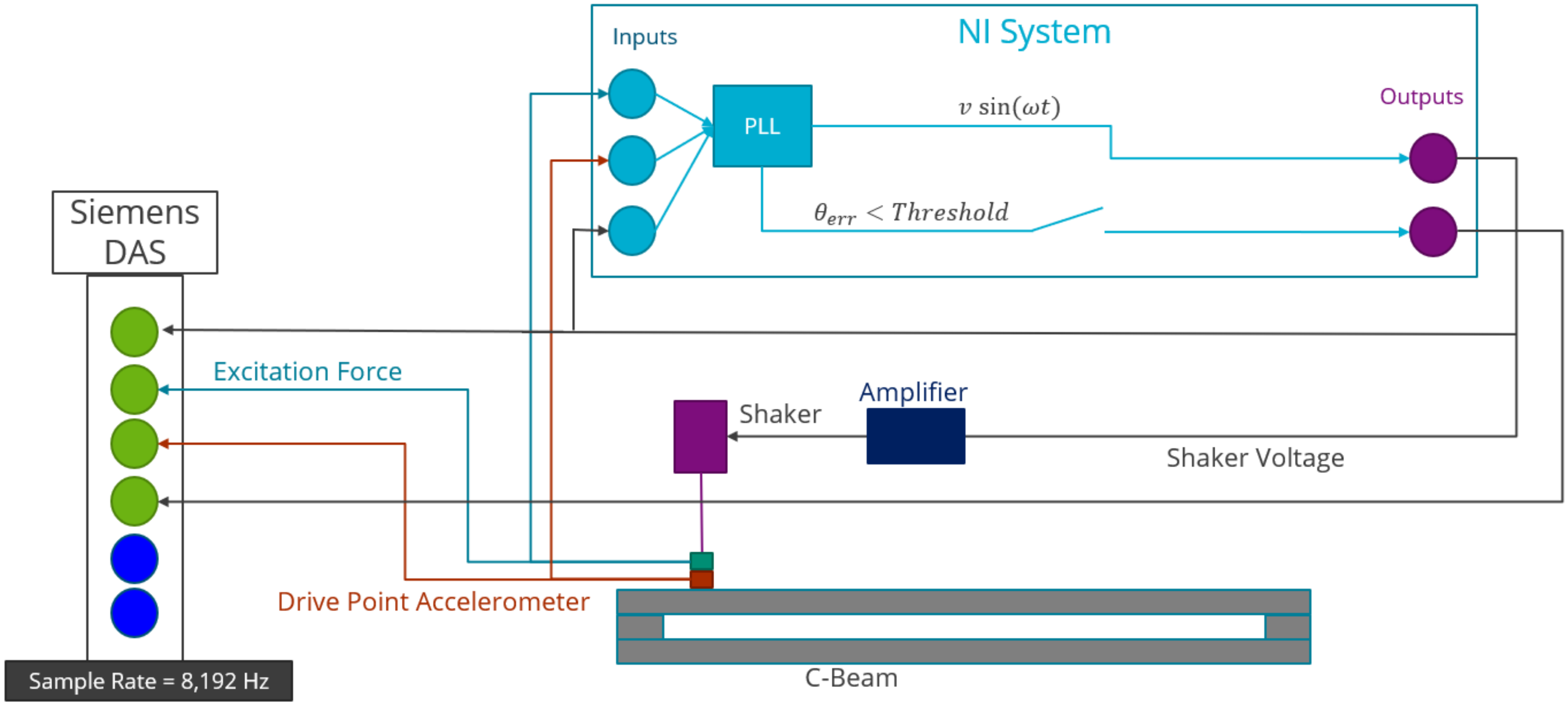


Force Appropriation – Test Setup



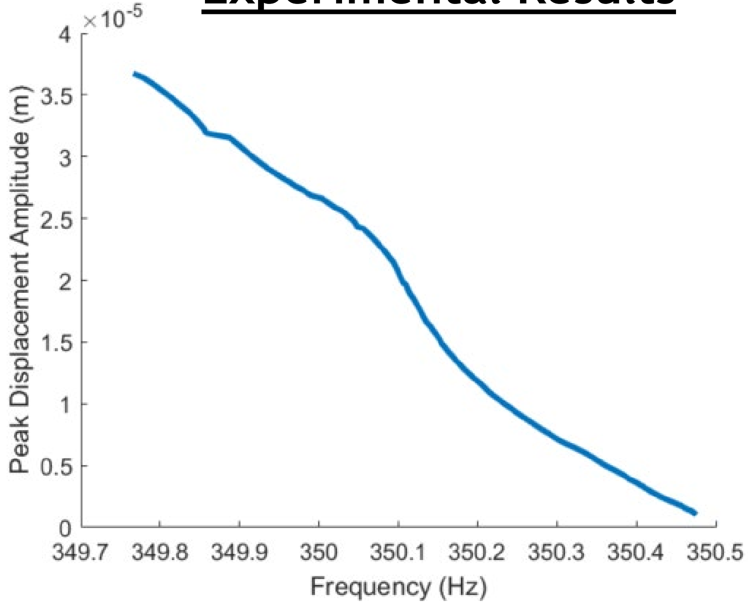
Force Appropriation – Control System

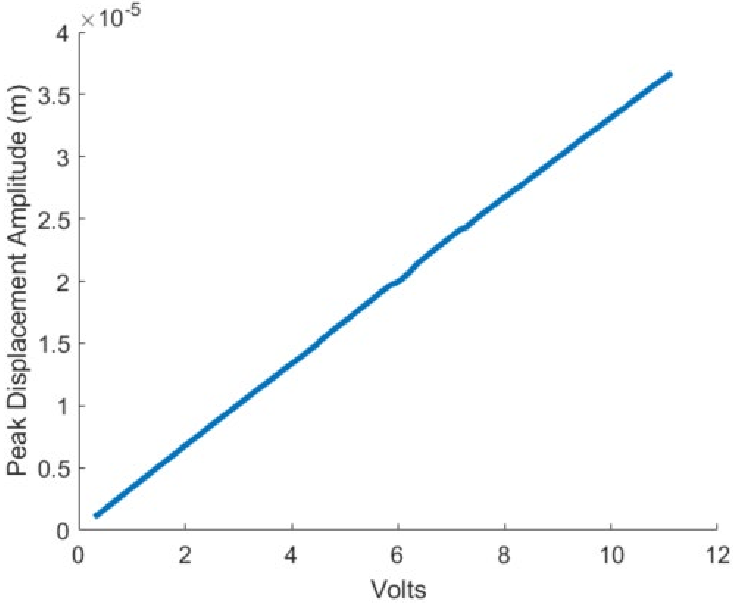
- A control system operated the shaker at the second elastic mode and gradually increased the input voltage while varying frequency to maintain phase quadrature between the force and response
- While in phase quadrature the system is in resonance at each step of increased voltage



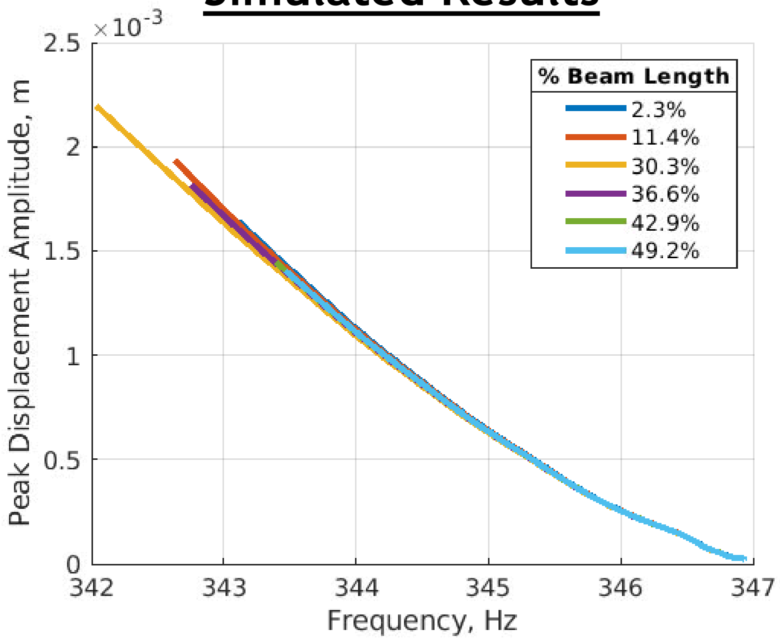
Experimental Results and Comparison

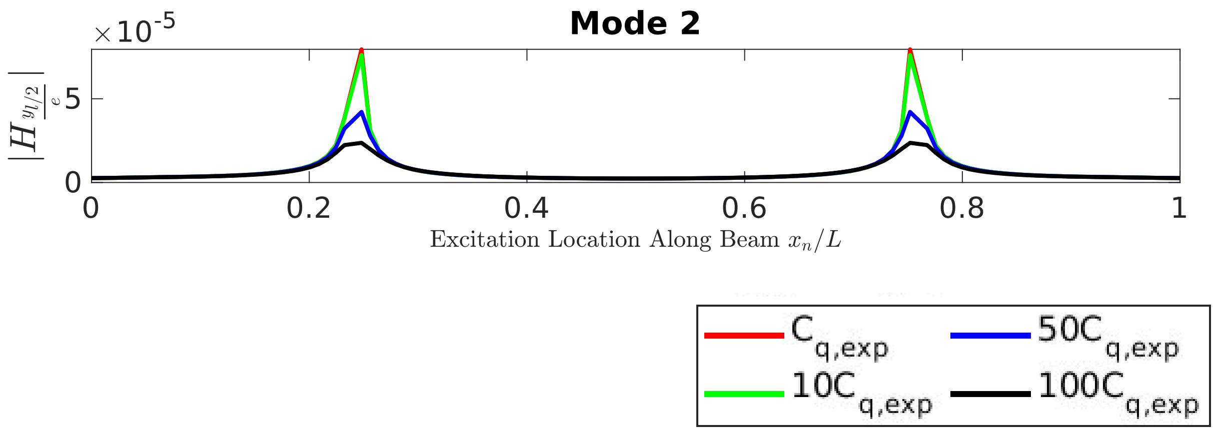
Experimental Results





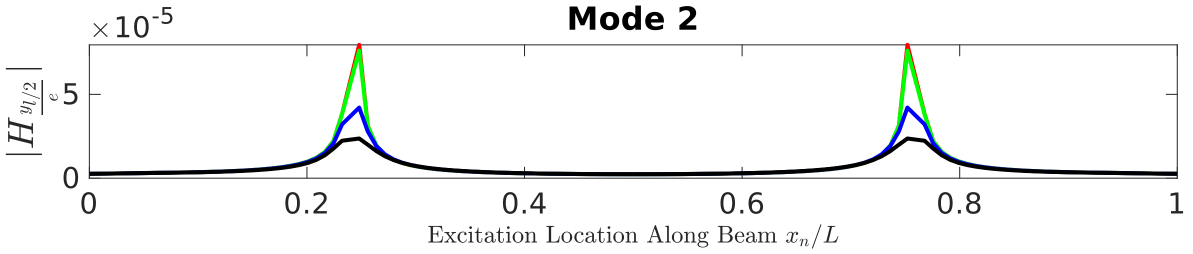
Simulated Results

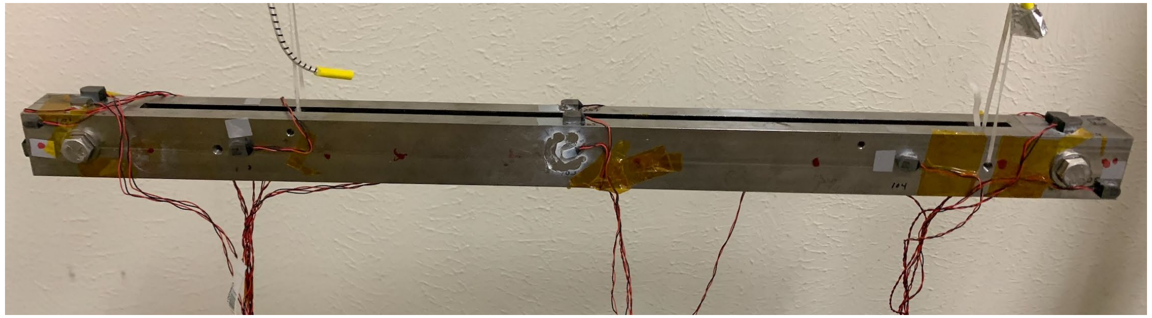




Results and Conclusion

- Both the shaker model and the structure model were correlated and updated to experimental data
- Both simulation approaches produced similar location predictions
  - Linearized FRF method is more computationally efficient but large variations in damping may result in more ambiguous outcomes
  - Harmonic balance method is more computationally demanding but incorporates nonlinear effects including nonlinear damping
- Experimental data corresponds well to predicted results





Acknowledgements

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References

[1] Gross, J., et al. "A numerical round robin for the prediction of the dynamics of jointed structures." *Dynamics of Coupled Structures, Volume 4*. Springer, Cham, 2016. 195-211.

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