Optimizing Test Setup Parameters for Force Appropriation Testing

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Current Problem to Address

• Dynamic Nonlinearities Widespread in Mechanical Systems

• Better methods needed to test nonlinear structures
  • Material nonlinearities (stress and strain relationship is nonlinear)
  • Contact nonlinearities such as Bolted Joints (stick, slip, open)

Current Experimental Methods to Test Nonlinear Parameters

• Nonlinear swept sine testing
• Nonlinear Resonant Decay
• Force appropriation testing

OUR FOCUS
Nonlinear Normal Modes

Resonant frequency changes based on energy in the system. The resonant frequencies of a nonlinear system depend on energy, so experimentally the resonant frequency must be found at each energy step.
**Force Appropriation Procedure**

- Constraint 90 degree phase difference between excitation force and response to identify resonance – allows measurement of nonlinear mode
- Control system used to
  - Excite at constant voltage
  - Sweep frequency looking for resonance
  - Once resonance is found, step up the signal voltage and repeat

**Force Appropriation Challenges**

- Outcomes highly dependent on excitation location

- Excite at high-amplitude point results in large shaker-structure interaction which reaches maximum force of shaker

**Where is the Best Excitation Location?**

\[ 90^\circ = \frac{1}{4} \text{ cycle} \]
Project Description

• Compare two simulation methods to predict the optimal force input location
  • Optimal = Maximize modal response for input voltage and force our equipment can support
  • Confirm simulated predictions with experimental data and determine best method

Method 1: Linearized Frequency Response Functions (FRF)

Linearize nonlinear contact elements and compute frequency response functions between response and voltage

Method 2: Harmonic Balance

Utilize harmonic balance to simulate a force appropriation test by enforcing a phase constraint and using a nonlinear model. Compare outcomes at several input locations
Simulation Methods-Linearized System about Preloaded State

• Shaker-structure combined system EOM linearized about preloaded equilibrium

\[ M\ddot{y} + C\dot{y} + \bar{K}y = f_{ext}(t), \]

Given:

\[ y = x - x_0, \quad (x_0 \text{ is a preloaded equilibrium point}) \]

\[ \bar{K} = K + \frac{\partial f_{nl}}{\partial x} \bigg|_{x=x_0} \quad \text{(linearized stiffness)} \]

\[ f_{nl} \text{ corresponds to the nonlinear contact forces} \]

\[ f_{ext}(t) \text{ corresponds to the external excitation (harmonic voltage input to amplifier)} \]

• From the EOM, we can further derive the linearized FRF Matrix as

\[ H = [\bar{K} - \omega^2 M + j\omega C]^{-1} \]
Multi-Harmonic Balance (MHB) - Theory

Multi-harmonic balance is a method to solve nonlinear equations of motion of the form

\[
M \ddot{x} + C \dot{x} + K x + f_{nl}(x, \dot{x}) = f_{pre} + f_{ext}(t)
\]

The response vectors \((\ddot{x}, \dot{x}, x)\) as well as the forcing \((f_{ext}(t))\) are unknown.

Assuming a periodic steady state response, the Fourier series are written as,

\[
x(t) = \frac{c_0^x}{\sqrt{2}} + \sum_{k=1}^{N_h} \left[ s_k^x \sin(k\omega t) + c_k^x \cos(k\omega t) \right]
\]

\[
f_{nl}(x, \dot{x}) = \frac{c_0^{nl}}{\sqrt{2}} + \sum_{k=1}^{N_h} \left[ s_k^{nl} \sin(k\omega t) + c_k^{nl} \cos(k\omega t) \right]
\]

\[
f_{ext}(t) = \frac{c_0^f}{\sqrt{2}} + \sum_{k=1}^{N_h} \left[ s_k^f \sin(k\omega t) + c_k^f \cos(k\omega t) \right]
\]

The Fourier coefficients are combined to form the harmonic balance equations of motion

\[
r(z, b_{ext}, \omega) = A(\omega)z + b(z) - b_{pre} - b_{ext}
\]

With solutions of the form

\[
y = [z \ b_{ext} \ \omega]^T
\]
Multi-Harmonic Balance – Force Resonant Constraint

At a given \(i\)th excitation location, the force and response are described using the Fourier coefficients of the first harmonic

\[
 f_{ext,1,i}(t) = s_{1,i}^{f} \sin(\omega t) + c_{1,i}^{f} \cos(\omega t)
\]

\[
 x_{1,i}(t) = s_{1,i}^{x} \sin(\omega t) + c_{1,i}^{x} \cos(\omega t)
\]

The phase resonance constraint requires the phase lag to be 90°, resulting in

\[
 \Delta \varphi_i = -\tan^{-1} \left( \frac{s_{1,i}^{f}}{c_{1,i}^{f}} \right) + \tan^{-1} \left( \frac{s_{1,i}^{x}}{c_{1,i}^{x}} \right) - \frac{\pi}{2} = 0
\]

To form a solvable system, monophase excitation is assumed

\[
 f_{ext,k,i}(t) = s_{1,i}^{f} \sin(k\omega t) + c_{1,i}^{f} \cos(k\omega t)
\]

\[
 x_{k,i}(t) = s_{1,i}^{x} \sin(k\omega t) + c_{1,i}^{x} \cos(k\omega t)
\]
Test Structure: “C-Beam”

- Two beams held together with bolts at both ends
- When excited at high enough energy levels the bolted joint area exhibits nonlinearity
- A model of this system has been developed with nonlinear Jenkins elements connecting the joints to simulate the nonlinear contact

Shaker Characterization

Important Why is this important?

We model the shaker as a series of masses connected by springs and dampers.

\[
M_{skr} = \begin{bmatrix} M_1 & 0 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 \\ 0 & 0 & M_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad C_{skr} = \begin{bmatrix} (c_{12} + c_{13}) & -c_{12} & -c_{13} & 0 & 0 \\ -c_{12} & c_{12} & 0 & 0 & 0 \\ -c_{13} & 0 & c_{13} & 0 & 0 \\ BL & -BL & 0 & L_e & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{k_a} \end{bmatrix}, \quad K_{shk} = \begin{bmatrix} (k_{12} + k_{13}) & -k_{12} & -k_{13} & -BL & 0 \\ -k_{12} & k_{12} & 0 & BL & 0 \\ -k_{13} & 0 & k_{13} & 0 & 0 \\ 0 & 0 & 0 & R_e & -1 \\ 0 & 0 & 0 & 0 & \frac{\omega_b}{k_a} \end{bmatrix}
\]


Shaker Characterization

- The resistance and three masses of the system are known
- The spring constants, damping coefficients, and inductance can be curve fit using the known impedance FRFs
Linear Modal Analysis

- To confirm the linear model was representative of our test system, modes from the linear modal test were compared to a finite element model using a Modal Assurance Criterion (MAC).
- All MAC values were above 98% which demonstrates a high correlation between the finite element model and the test structure.
- This indicates that simulations using this model are representative of the test at the linear level.

<table>
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<th>Sim. Mode</th>
<th>Freq. (Hz)</th>
<th>Exp. Mode</th>
<th>Freq. (Hz)</th>
<th>Diff. (%)</th>
<th>MAC (%)</th>
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</tr>
</tbody>
</table>
Model Mode Shapes of “C-Beam” Structure

Mode 1: 282 Hz

Mode 2: 352 Hz

Mode 3: 505 Hz

Mode 4: 593 Hz

Mode 5: 776 Hz

Mode 6: 926 Hz
Omitting shaker model indicates anti-node as solution.
Simulation Method 1 – Linearized FRF Method

Response at Midpoint of the beam for unit force input vs. Excitation Location (No Shaker)

Optimal shaker location 25% from either end

Including shaker model indicates anti-node is non-optimal
Simulation Method 2 – Harmonic Balance Method

Response at Anti-Node vs. Input Voltage

Response at Anti-Node vs. Excitation Frequency

Response at Anti-Node vs. Applied Force
Force Appropriation – Test Setup
Force Appropriation – Control System

• A control system operated the shaker at the second elastic mode and gradually increased the input voltage while varying frequency to maintain phase quadrature between the force and response.

• While in phase quadrature the system is in resonance at each step of increased voltage.
Experimental Results and Comparison

**Experimental Results**

- Peak Displacement Amplitude vs. Frequency
- Peak Displacement Amplitude vs. Volts

**Simulated Results**

- % Beam Length: 2.3%, 11.4%, 30.3%, 36.6%, 42.9%, 49.2%
- Peak Displacement Amplitude vs. Frequency

**Mode 2**

- Excitation Location Along Beam $x_e/L$
Results and Conclusion

• Both the shaker model and the structure model were correlated and updated to experimental data

• Both simulation approaches produced similar location predictions
  • Linearized FRF method is more computationally efficient but large variations in damping may result in more ambiguous outcomes
  • Harmonic balance method is more computationally demanding but incorporates nonlinear effects including nonlinear damping

• Experimental data corresponds well to predicted results
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References


THANK YOU