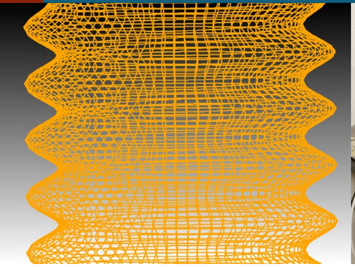
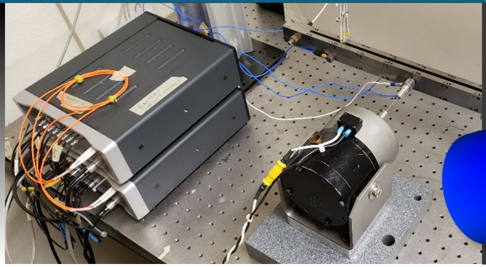
N-O-MAD Research Institute

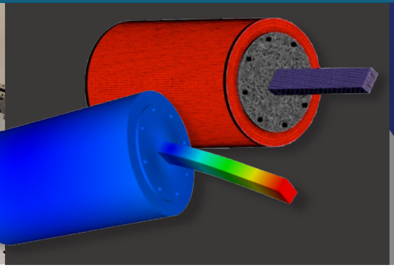
Sandia National Laboratories

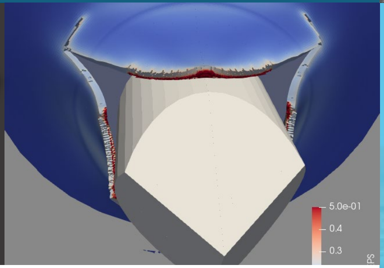
THE UNIVERSITY OF NEW MEXICO

Obtaining Fixed-Base Nonlinear Modal Models from Free Boundary Testing





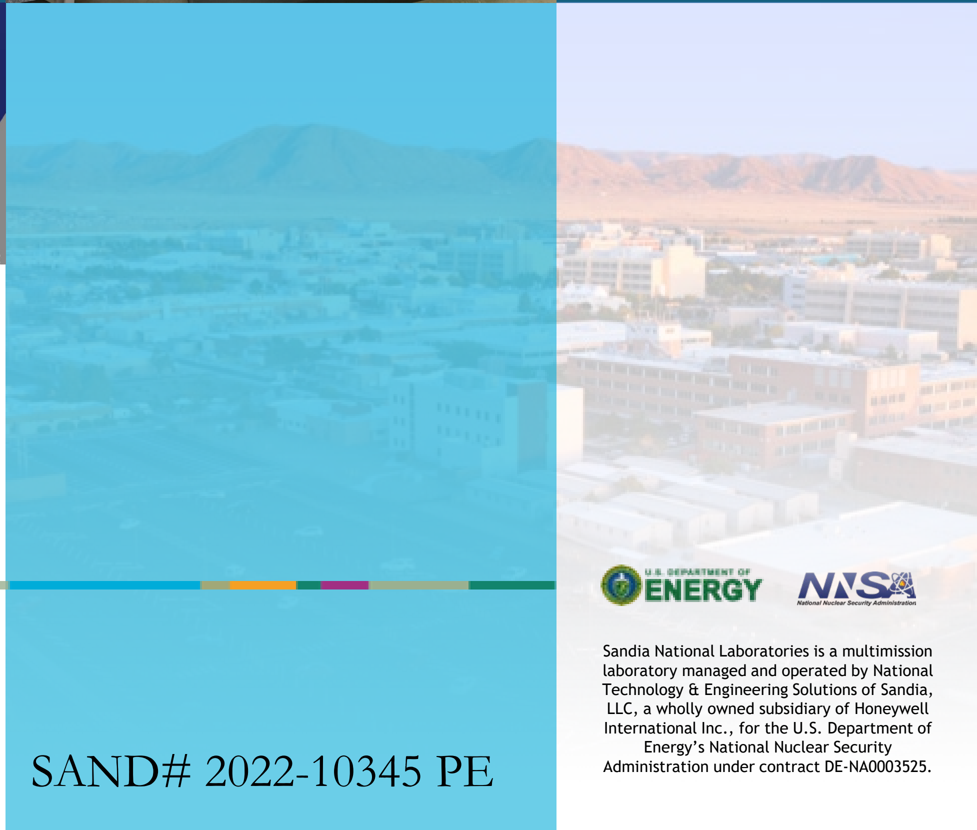




Dani Agramonte, Judith Brown, AJ Sanchez

Mentors: Matt Allen, Ben Pacini, Dan Roettgen

August 2, 2022



SAND# 2022-10345 PE

U.S. DEPARTMENT OF ENERGY NNSA National Nuclear Security Administration

Team and Motivation

Introduction

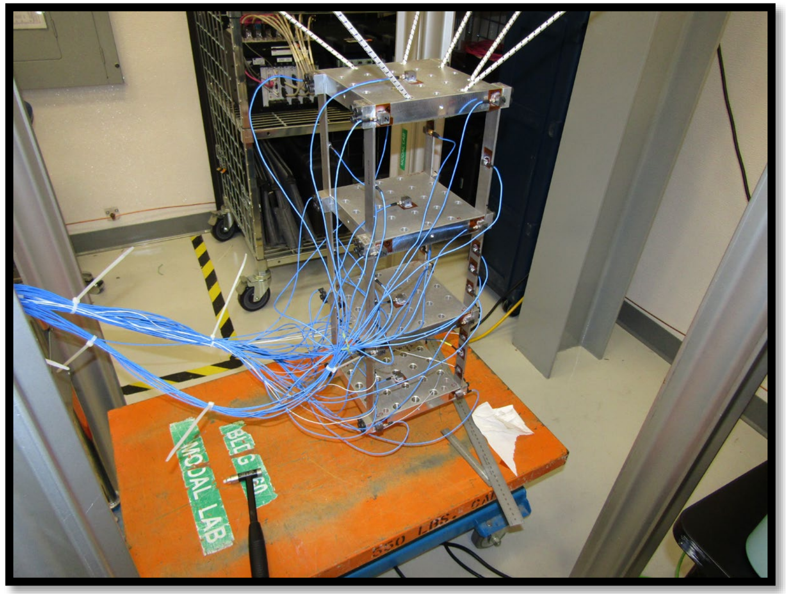
- Dani Agramonte
  - Mechanical Engineering
  - Undergraduate and Master's degree from University of Georgia
  - PhD student at Georgia Technological Institute
  - Previous research: optimization and use of piezoelectric actuators in modal tests

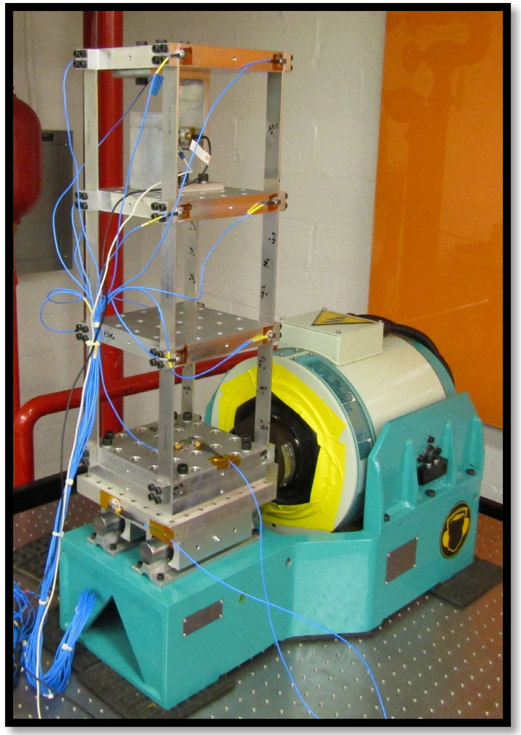
- Judith Brown
  - Mechanical Engineering
  - Undergraduate at the University of Nebraska-Lincoln
  - Previous research: vibration reduction in airplane wings using a nonlinear vibration absorber

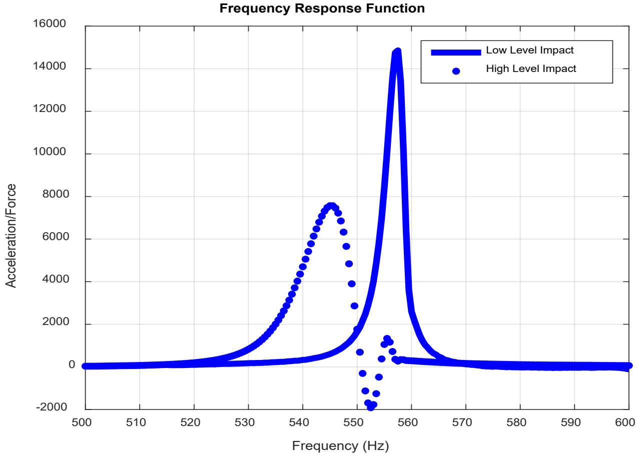
- AJ Sanchez
  - Mechanical Engineering
  - Undergraduate at the University of Texas San Antonio
  - Previous research: Created MATLAB GUI determining Frequency Response Functions (FRFs) for single and multi-degree of freedom systems from boundary conditions

Motivation

- Most model validation and updating activity at SNL is performed using free-boundary experimental modes
- Complicated jointed structures often exhibit weakly nonlinear behavior and methods to excite, identify, and simulate this response for free-boundary structures have been the focus of many studies at SNL and externally





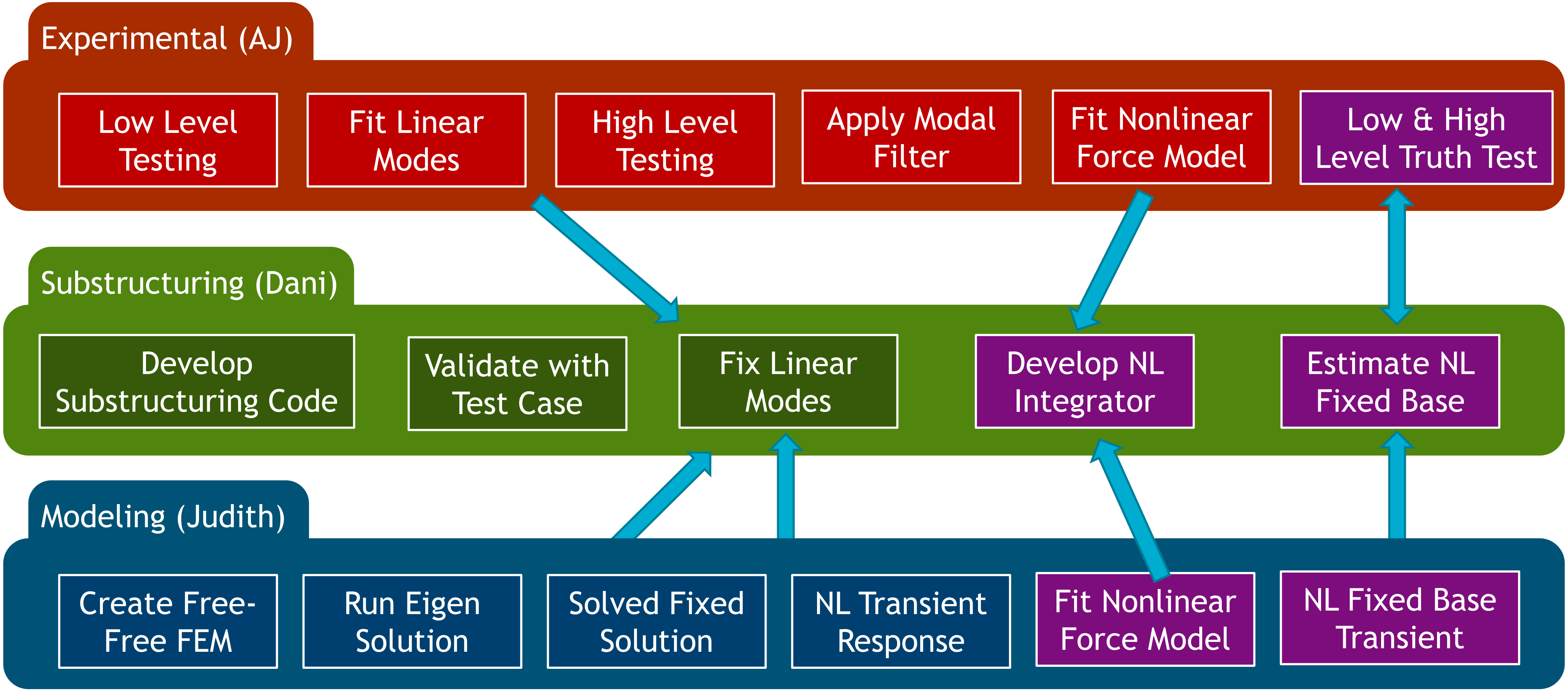


- Typically, once models are validated structures are tested on a shaker table with a new boundary condition
- **Team Challenge:** Can we use substructuring techniques to change the boundary condition of free-free modal test result? What about a nonlinear modal test result?

Project Plan

Task Division

- Extremely complicated project with lots of hand-offs



Experimental (AJ)

Low Level Testing

Fit Linear Modes

High Level Testing

Apply Modal Filter

Fit Nonlinear Force Model

Low & High Level Truth Test

Substructuring (Dani)

Develop Substructuring Code

Validate with Test Case

Fix Linear Modes

Develop NL Integrator

Estimate NL Fixed Base

Modeling (Judith)

Create Free-Free FEM

Run Eigen Solution

Solved Fixed Solution

NL Transient Response

Fit Nonlinear Force Model

NL Fixed Base Transient

Mentors scoped three NOMAD projects and we almost completed all three

Linear Predictions

Finite Element Modeling

Modeling (Judith)

Create Free-Free FEM

Run Eigen Solution

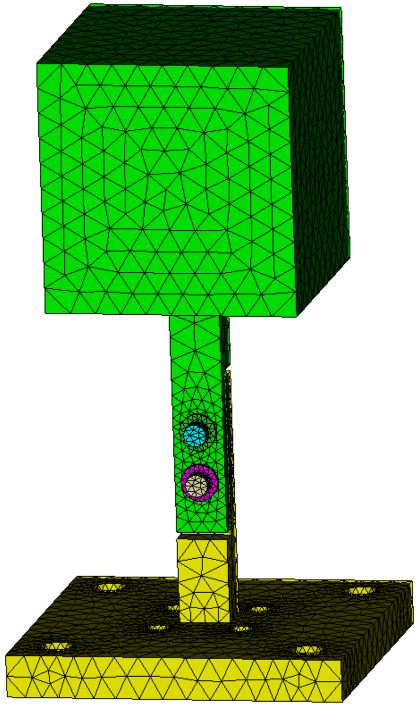
Solved Fixed Solution

NL Transient Response

Fit Nonlinear Force Model

NL Fixed Base Transient

- Hardware is from the 80's 90's with little documented history
- No known geometry
- Unknown material
- **Peak Challenge for FEM!**



Finite Element Modeling

Modeling (Judith)

Create Free-Free FEM

Run Eigen Solution

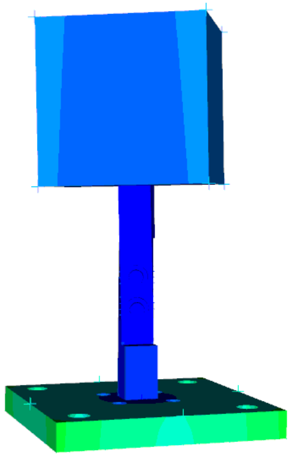
Solved Fixed Solution

NL Transient Response

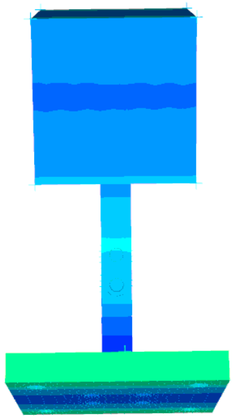
Fit Nonlinear Force Model

NL Fixed Base Transient

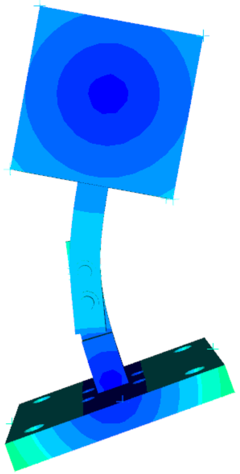
- Free-free boundary conditions
- Comparison with experimental data
- Model verification not highest priority
  - Validation of theory can work with just Sierra outputs



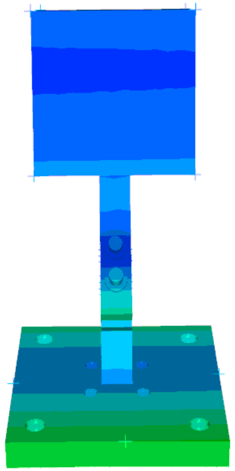
Mode 1: 166 Hz



Mode 2: 227 Hz

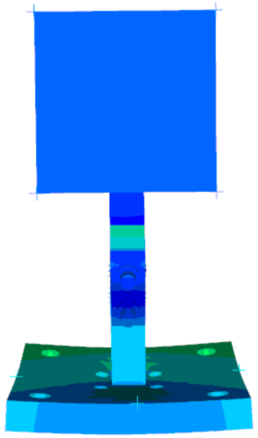


Mode 3: 351 Hz



Mode 4: 786 Hz

Mode 5: 1047 Hz



Mode 6: 1824 Hz

Finite Element Modeling

Modeling (Judith)

Create Free-Free FEM

Run Eigen Solution

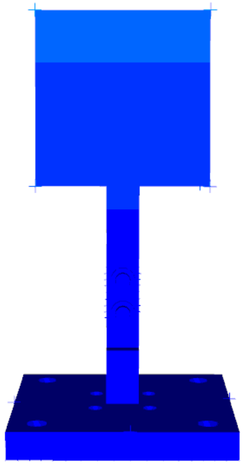
Solved Fixed Solution

NL Transient Response

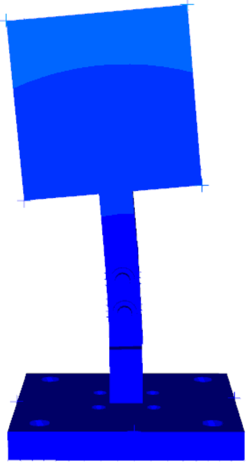
Fit Nonlinear Force Model

NL Fixed Base Transient

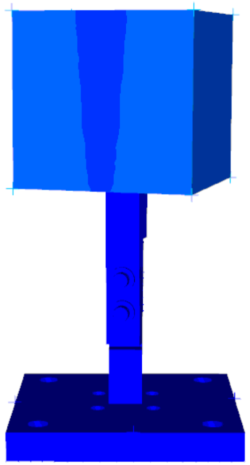
- Fixed base boundary conditions
  - Entire base fixed
- Comparison with analytical determination



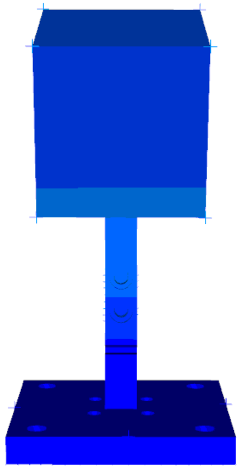
Mode 1: 38.94 Hz



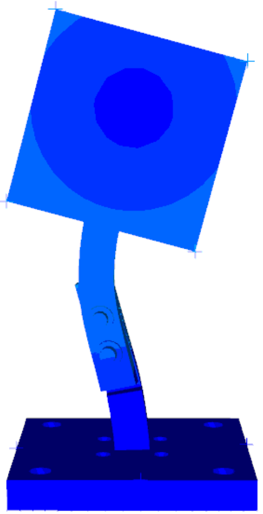
Mode 2: 58.96 Hz



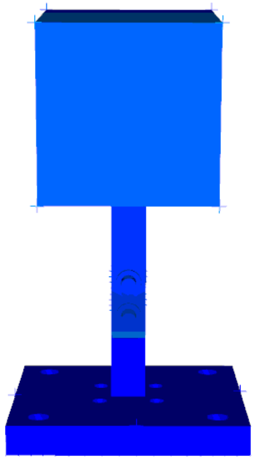
Mode 3: 100.98 Hz



Mode 4: 488.48 Hz



Mode 5: 679.56 Hz



Mode 6: 1004 Hz

Fixed-Base Theory I of III

Substructuring (Dani)

Develop Substructuring Code

Validate with Test Case

Fix Linear Modes

Develop NL Integrator

Estimate NL Fixed Base

- Assume we have free modes of a structure under test or from a model

$$\mathbf{I}\ddot{\bar{\mathbf{q}}}_{\mathbf{a}} + \mathbf{C_q}\dot{\bar{\mathbf{q}}}_{\mathbf{a}} + \mathbf{K_q}\bar{\mathbf{q}}_{\mathbf{a}} = \mathbf{u_q}.$$

- Partition the modes and degrees of freedom of the “assembly” (a) into “fixture” (f) and substructure (s)

$$\bar{\mathbf{x}}_{\mathbf{a}} = \begin{bmatrix} \bar{\mathbf{x}}_{\mathbf{af}} \\ \bar{\mathbf{x}}_{\mathbf{as}} \end{bmatrix} \qquad \boldsymbol{\Phi}_{\mathbf{a}} = \begin{bmatrix} \boldsymbol{\Phi}_{\mathbf{af}} \\ \boldsymbol{\Phi}_{\mathbf{as}} \end{bmatrix}$$

- Define fixture motion

$$\bar{\mathbf{x}}_{\mathbf{af}} = \boldsymbol{\Phi}_{\mathbf{af}}\bar{\mathbf{q}}_{\mathbf{a}}$$

Fixed-Base Theory II of III

Substructuring (Dani)

Develop Substructuring Code

Validate with Test Case

Fix Linear Modes

Develop NL Integrator

Estimate NL Fixed Base

- Approximate fixture modal response using partitioned fixture modes

$$\bar{\mathbf{q}}_\mathbf{f} \approx \mathbf{\Phi}_\mathbf{af}^{+}\bar{\mathbf{x}}_\mathbf{af}$$

- Combine equations to approximate fixture modal response from assembly modal response

$$\bar{\mathbf{q}}_\mathbf{f} \approx \mathbf{\Phi}_\mathbf{af}^{+}\mathbf{\Phi}_\mathbf{af}\bar{\mathbf{x}}_\mathbf{f}$$

- We seek a boundary change where the fixture DOFs are zero

$$\bar{\mathbf{q}}_\mathbf{f} \approx 0$$

- We can define a constraints equation in classical substructuring form

$$\mathbf{\Phi}_\mathbf{f}^{+}\mathbf{\Phi}_\mathbf{af}\bar{\mathbf{q}}_\mathbf{a} = \mathbf{B}\bar{\mathbf{q}}_\mathbf{a} = \mathbf{0}$$

Fixed-Base Theory III of III

Substructuring (Dani)

Develop Substructuring Code

Validate with Test Case

Fix Linear Modes

Develop NL Integrator

Estimate NL Fixed Base

- Transforming to new coordinates for a new boundary system we find the L must reside in the nullspace of B

$$\mathbf{B}\mathbf{L}\bar{\boldsymbol{\eta}}_{\mathbf{a}} = 0$$

- Next, transform to the new coordinates from free-boundary modal equations of motion

$$\mathbf{L}^{\mathbf{T}}\mathbf{L}\ddot{\bar{\boldsymbol{\eta}}}_{\mathbf{a}} + \mathbf{L}^{\mathbf{T}}\mathbf{C}_{\mathbf{q}}\mathbf{L}\dot{\bar{\boldsymbol{\eta}}}_{\mathbf{a}} + \mathbf{L}^{\mathbf{T}}\mathbf{K}_{\mathbf{q}}\mathbf{L}\bar{\boldsymbol{\eta}}_{\mathbf{a}} = \mathbf{L}^{\mathbf{T}}\mathbf{u}_{\mathbf{q}}$$

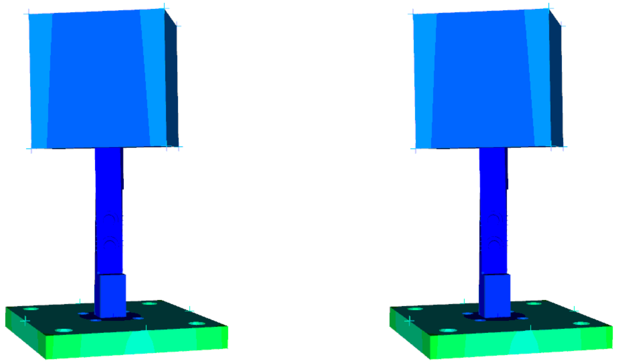
- The eigen solution of this transformed equations results in the mode shapes and natural frequencies for the new fixed base system!
- We can even (eventually) add nonlinear terms to see the system response starting from free-free nonlinear modes!

$$\mathbf{L}^{\mathbf{T}}\mathbf{L}\ddot{\bar{\boldsymbol{\eta}}}_{\mathbf{a}} + \mathbf{L}^{\mathbf{T}}\mathbf{C}_{\mathbf{q}}\mathbf{L}\dot{\bar{\boldsymbol{\eta}}}_{\mathbf{a}} + \mathbf{L}^{\mathbf{T}}\mathbf{K}_{\mathbf{q}}\mathbf{L}\bar{\boldsymbol{\eta}}_{\mathbf{a}} = \mathbf{L}^{\mathbf{T}}\mathbf{u}_{\mathbf{q}} + \mathbf{L}^{\mathbf{T}}\boldsymbol{F}_{\boldsymbol{nl}}$$

Fixed Base Substructuring Results from FEM Free Modes

Substructuring (Dani)

- Develop Substructuring Code
- Validate with Test Case
- Fix Linear Modes
- Develop NL Integrator
- Estimate NL Fixed Base



Substructuring Prediction	FEM Results	% Error
37.15	38.88	4.4%
55.73	58.91	5.4%
96.41	100.02	3.6%
462.59	486.90	5.0%
639.21	679.26	5.5%
832.33	1005.37	17.2%

Experimental Validation Sneak-Peak

Experimental Data Acquisition

Experimental (AJ)

- Low Level Testing
- Fit Linear Modes
- High Level Testing
- Apply Modal Filter
- Fit Nonlinear Force Model
- Low & High Level Truth Test

- "Bobble Head" Structure
  - Lap joint with two bolts
  - 23 degrees of freedom
  - Free-free: suspended by 2 bungees
- Impact Testing
  - 3 impact points (colored dots in pictures)
  - 6 elastic modes under 1700 Hz
  - Modal Analysis on LMS
  - Minimize non-linear response

Experimental Data Acquisition

Experimental (AJ)

Low Level Testing

Fit Linear Modes

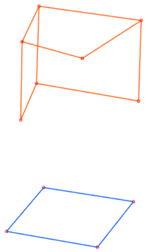
High Level Testing

Apply Modal Filter

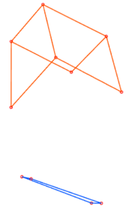
Fit Nonlinear Force Model

Low & High Level Truth Test

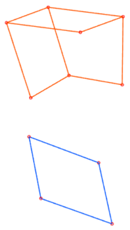
- Combined linear FRFs in LMS PolyMAX
- Used the curve fitting technique to define poles in the combined FRF
- PolyMAX takes the defined poles and defines the mode shapes at these natural frequencies



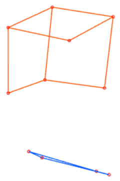
Mode 1
w_n:174 Hz
ζ: 0.04%



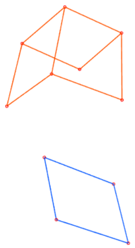
Mode 2
w_n:178 Hz
ζ: 0.36%



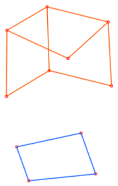
Mode 3
w_n: 347 Hz
ζ: 0.18%



Mode 4
w_n: 621 Hz
ζ: 0.20%



Mode 5
w_n: 939 Hz
ζ: 0.23%



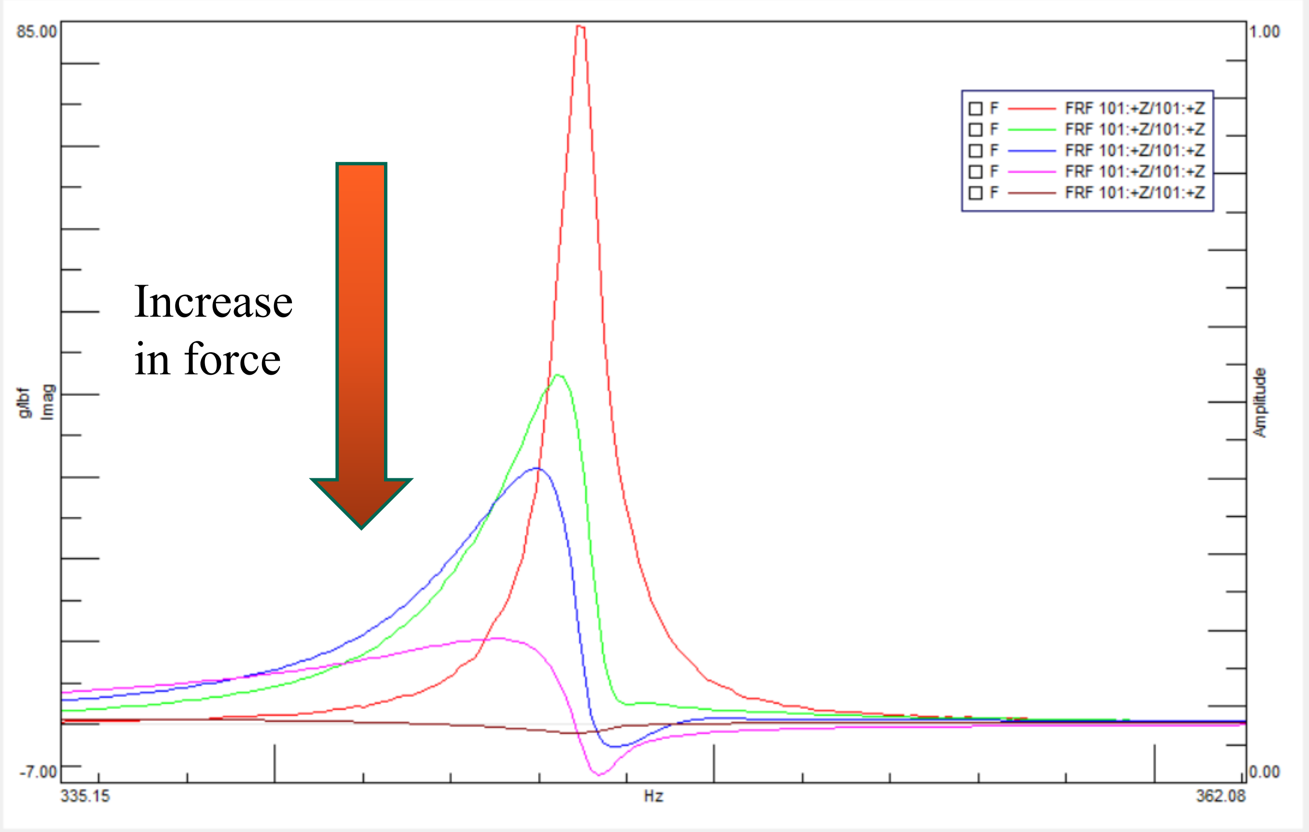
Mode 6
w_n:1695 Hz
ζ: 0.41%

Experimental Data Acquisition

Experimental (AJ)

Low Level Testing | Fit Linear Modes | **High Level Testing** | Apply Modal Filter | Fit Nonlinear Force Model | Low & High Level Truth Test

- High level Impact Testing from 5-40 lbf
- There is high degree of nonlinearity in the structure
- Compared drive point FRFs at ranging forces to see non-linearity in the data
- Took time history data from forces ranging 0-40 lbf



Increase in force

Experimental Non-Linear Data

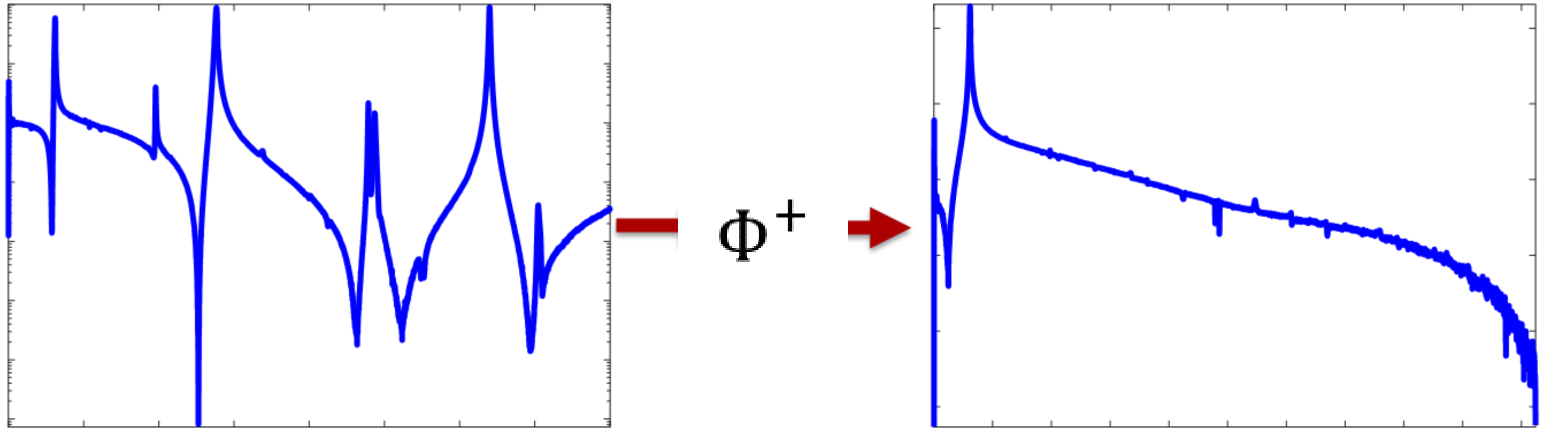
Experimental (AJ)

- Low Level Testing
- Fit Linear Modes
- High Level Testing
- Apply Modal Filter
- Fit Nonlinear Force Model
- Low & High Level Truth Test

- Took the time history data from the 3 impact points and split up each mode into individual shapes

$$x = \Phi q$$
$$q = \Phi^{+} x$$

Modal Filter



Experimental Non-Linear Data

Experimental (AJ)

Low Level Testing

Fit Linear Modes

High Level Testing

Apply Modal Filter

Fit Nonlinear Force Model

Low & High Level Truth Test

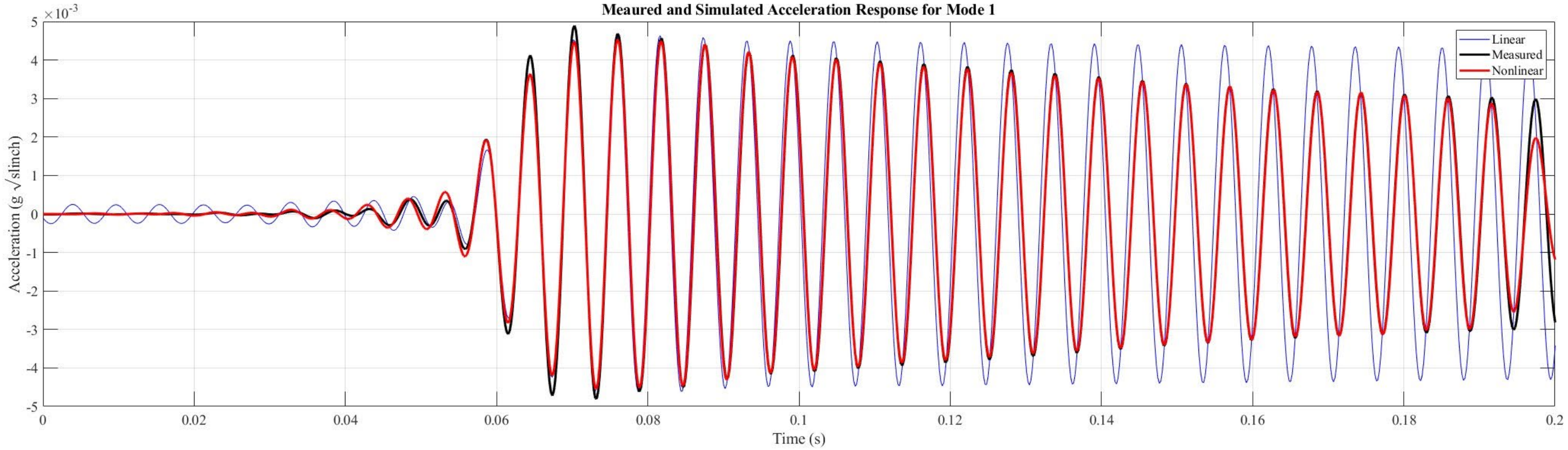
- Assumed cubic polynomial for stiffness of damping
- $\ddot{q}(t) + c_0\dot{q}(t) + c_1|\dot{q}(t)|\dot{q}(t) + c_2\dot{q}^3(t) + k_0q(t) + k_1|q(t)|q(t) + k_2q^3(t)(q(t),\dot{q}(t)) = F(t)$
  - Modal equation of motion including non-linear terms

$$\begin{bmatrix} \dot{\bar{q}}|\dot{\bar{q}}| & \dot{\bar{q}}^3 & \bar{q}|\bar{q}| & \bar{q}^3 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ k_1 \\ k_2 \end{Bmatrix} = \bar{F}_r - c_0\dot{\bar{q}} - k_0\bar{q}$$

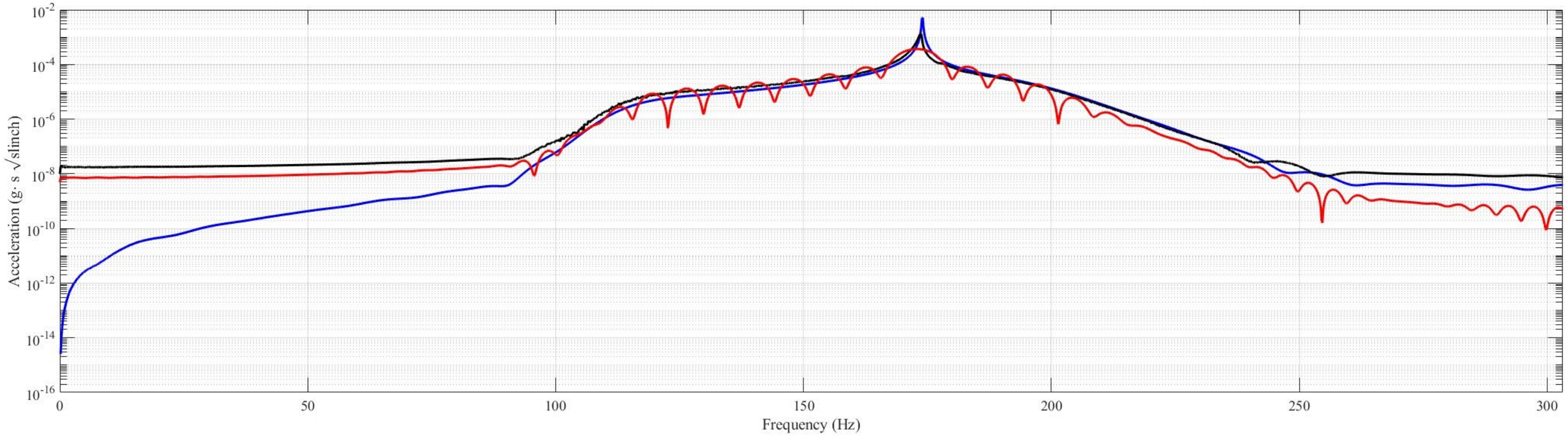
Known Data

Unknown Coefficients

Closer look into Modal Response Simulation



Meaured and Simulated Acceleration Response for Mode 1

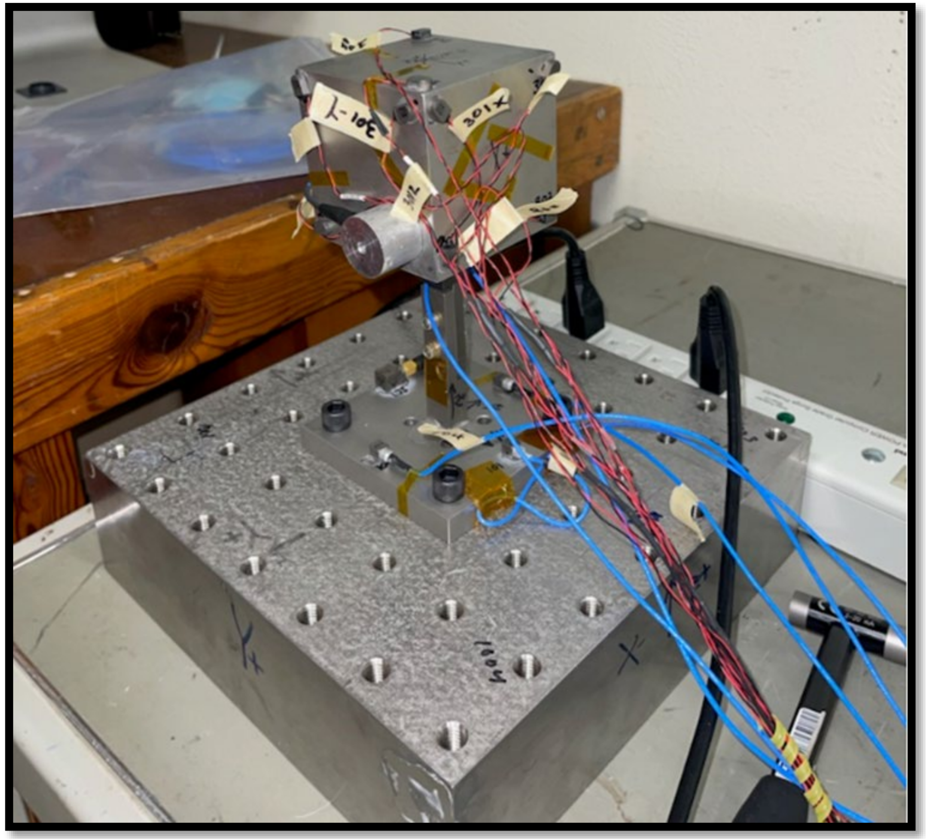


Next Steps

- Year-round internship
  - Truth data – attach bobblehead to larger seismic mass
    - Smaller one has movement, isn't true fixed base
  - Compare truth data to finite element analysis predictions
- Pass nonlinear transient data through Sierra SM and fit nonlinear modal models similar to experiment
- Predict nonlinear fixed-base response by integrating and adding forcing term

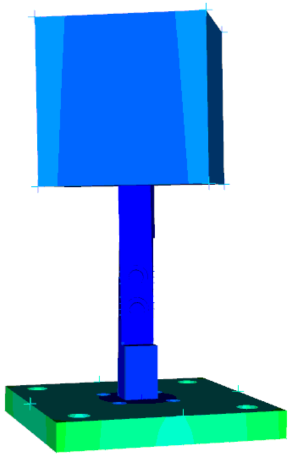
$$\mathbf{L}^{\mathbf{T}}\mathbf{L}\ddot{\bar{\boldsymbol{\eta}}}_{\mathbf{a}} + \mathbf{L}^{\mathbf{T}}\mathbf{C}_{\mathbf{q}}\mathbf{L}\dot{\bar{\boldsymbol{\eta}}}_{\mathbf{a}} + \mathbf{L}^{\mathbf{T}}\mathbf{K}_{\mathbf{q}}\mathbf{L}\bar{\boldsymbol{\eta}}_{\mathbf{a}} = \mathbf{L}^{\mathbf{T}}\mathbf{u}_{\mathbf{q}} + \boxed{\mathbf{L}^{\mathbf{T}}\boldsymbol{F}_{\boldsymbol{nl}}}$$

- Present final results internal at Sandia and at SEM IMAC



Conclusion

- Analytical determination of fixed base response
- Experimental model
  - Truth data
  - Nonlinear coefficients
- Finite element model
  - Linear modes
  - Rigid body modes
  - Nonlinear transient data
- Analytical substructuring
  - Creation of substructing code
  - Combination of data
- Thank you! Questions?



Thank you!

Dan Roettgen

Ben Pacini

Matt Allen

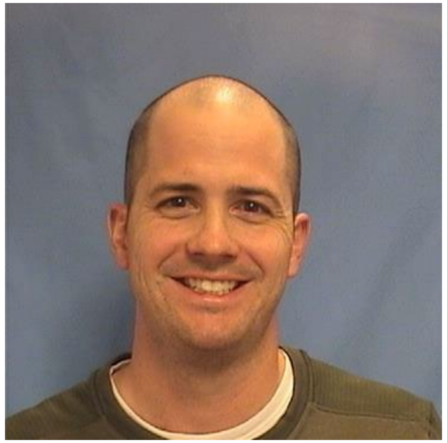
Rob Kuether

Debby Fowler

Brooke Allensworth

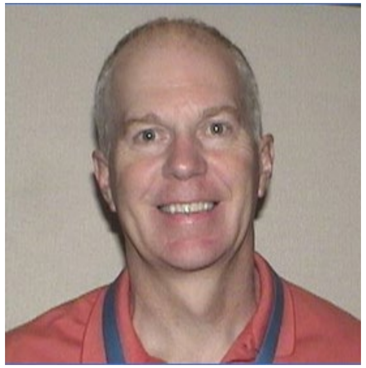
Tariq Khraishi

Joe Bishop







References

R. Mayes, B. Pacini and D. Roettgen, "A Modal Model to Simulate Typical Structural Dynamic Nonlinearity," in 34th International Modal Analysis Conference, 2016.

B. Pacini, R. Mayes, B. Owens and R. Schutlz, "Nonlinear Finite Element Model Updating Part I: Experimental Techniques and Nonlinear Modal Model Parameter Extraction," Dynamics of Coupled Structures, Proceedings from the 35th IMAC, pp. 165-178, 2017.

R. Mayes, "A Modal Craig-Bampton Substructure for Experiments, Analysis, Control and Specifications," in 33rd International Modal Analysis Conference, 2015.