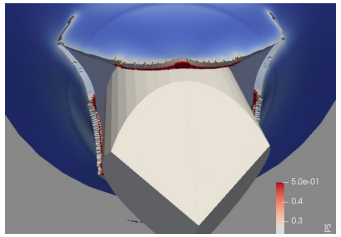
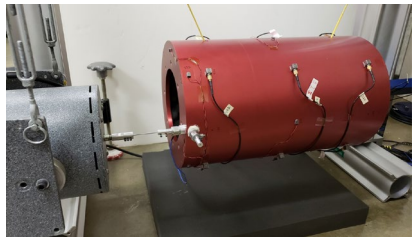
Neural Network Informed Uncertainty Quantification for Structural Dynamics Reduced Order Models





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Goals

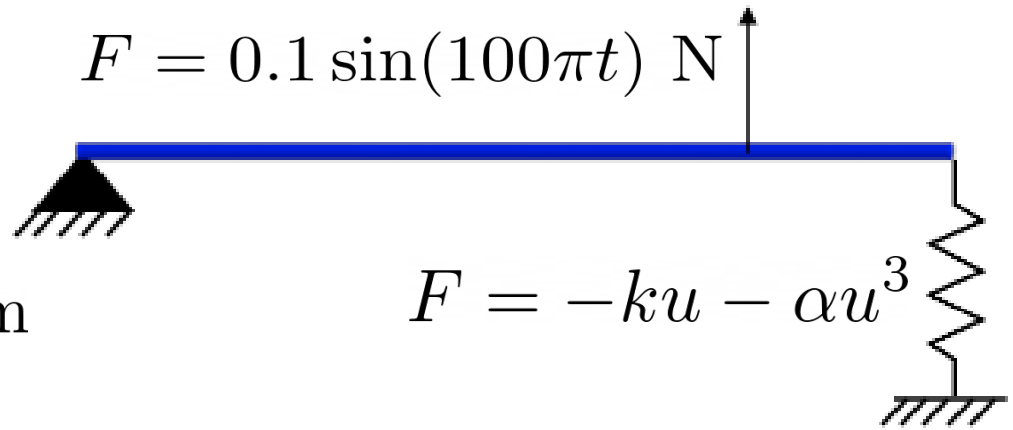
Goals

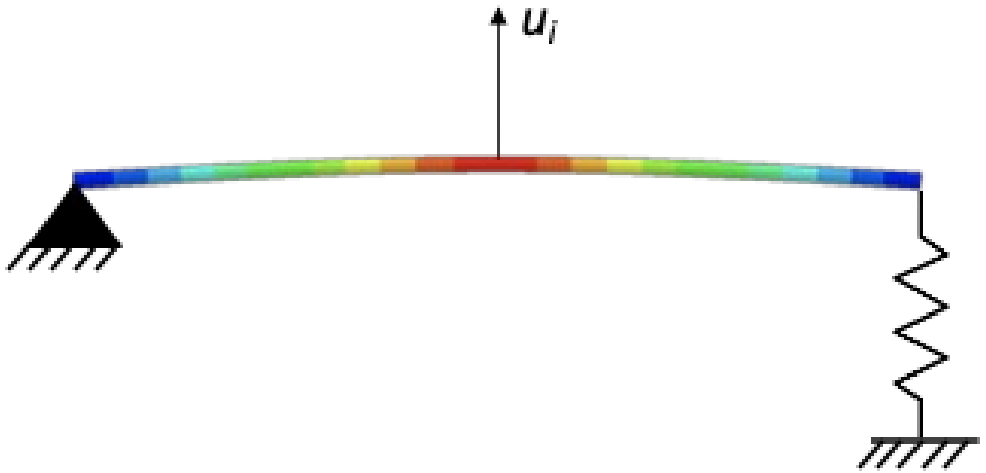
- Use neural networks to augment reduced-order models (ROMs) for improved prediction
- Learn error behavior to allow for future-time error prediction, either deterministic or statistical
- Use real-time ROM augmentation to more accurately simulate and predict extreme events in physical systems

Problem Description

Physical Problem

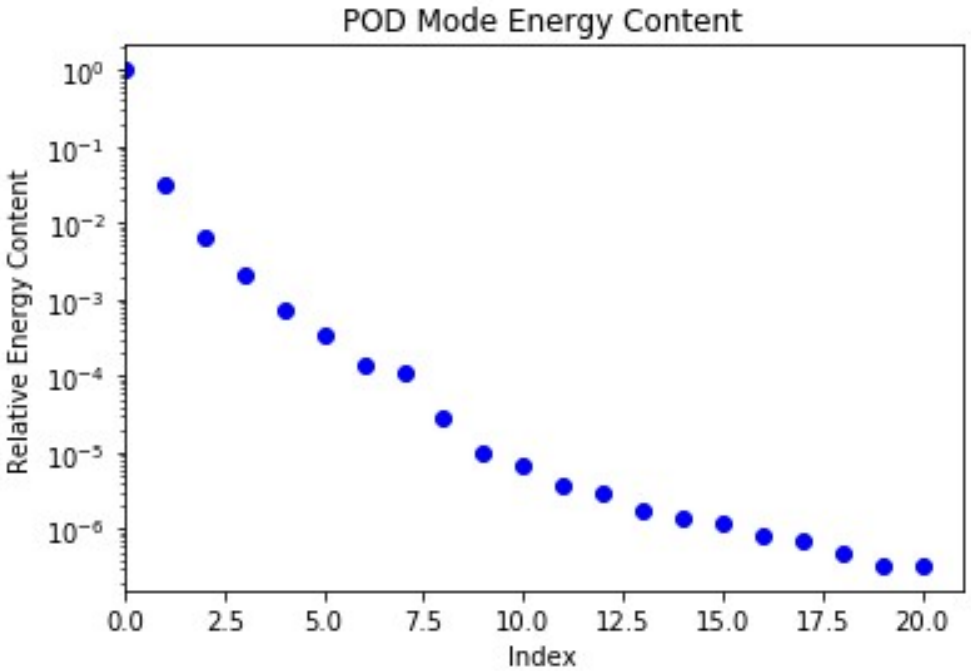
- Euler-Bernoulli beam ($L = 1\text{m}$)
- Harmonic forcing at $x = 0.75\text{m}$
- Initial displacement prescribed at $x = 0.5\text{m}$
- 10 equidistant 'sensor' locations
- 4 datasets were generated varying initial conditions and spring constants

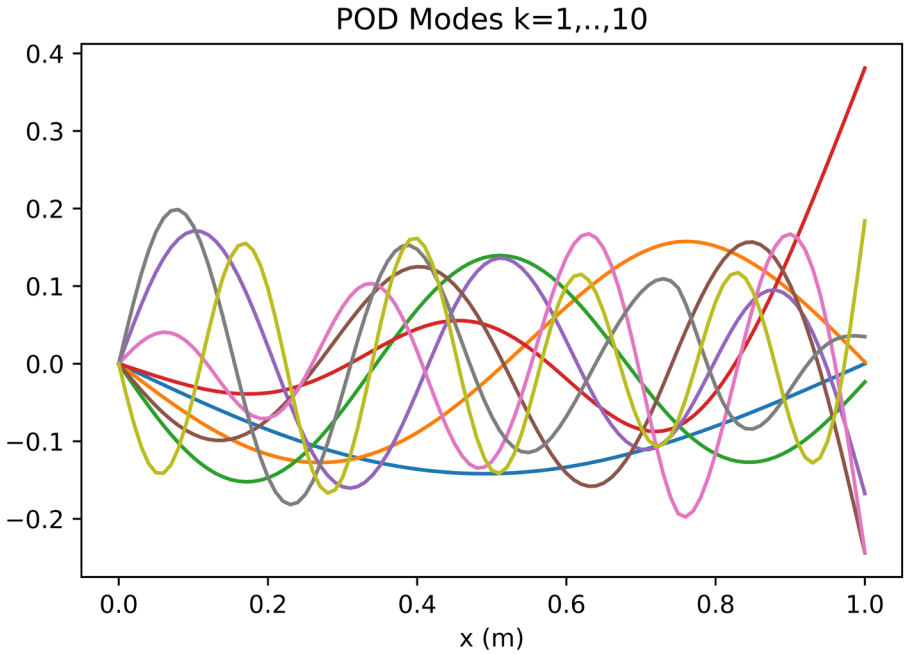




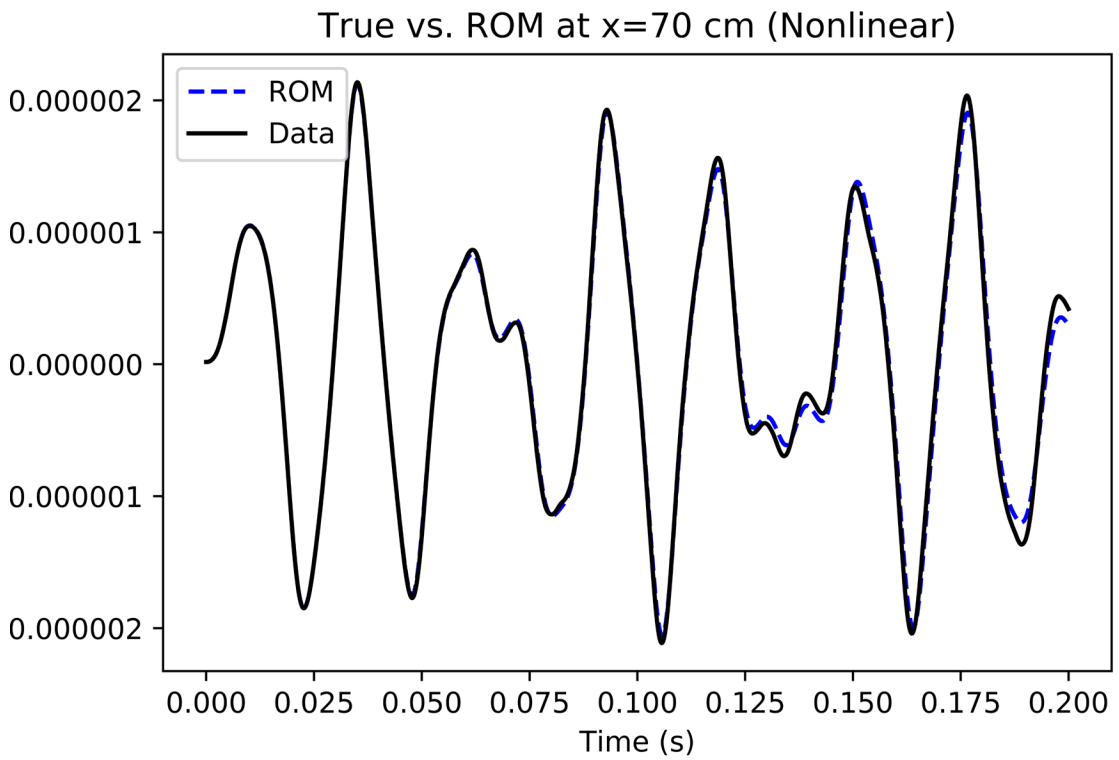
Reduced Order Model

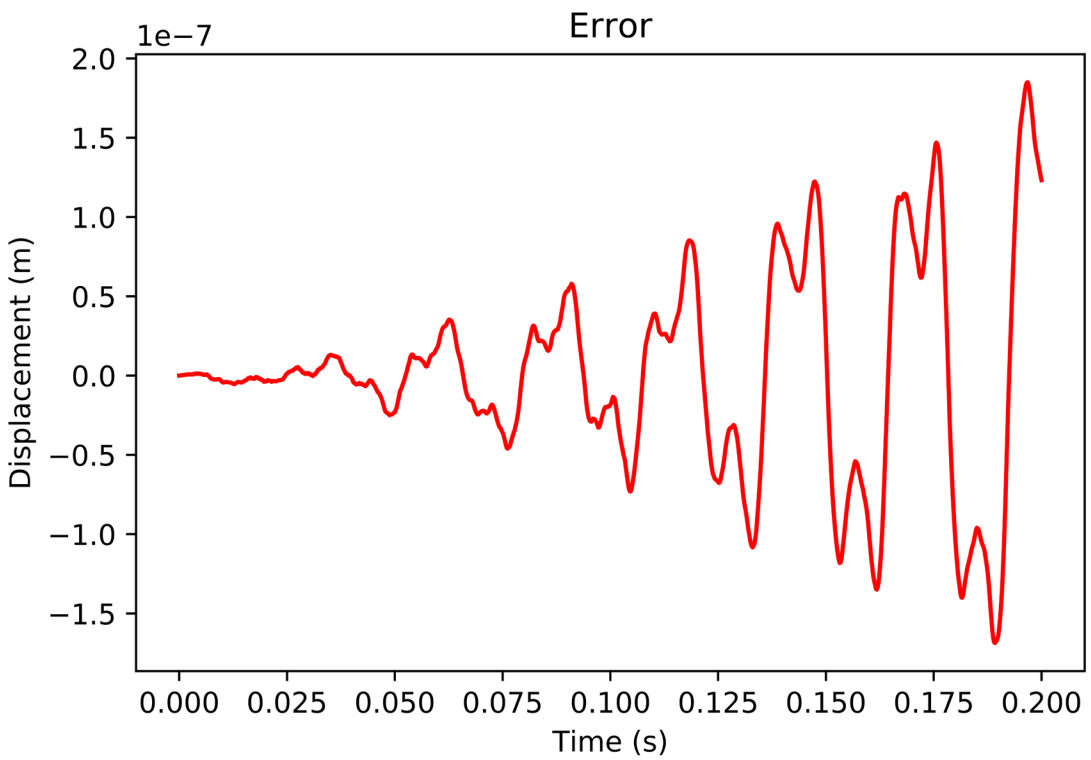
- Galerkin projection onto low-rank basis
- POD basis generated from simulations with varying parameters and initial conditions
- 10 modes retained for each ROM basis
- Fast integration using implicit Runge-Kutta methods ($\approx 0.5\mathrm{s}$ each)





ROM Errors


True vs. ROM at x=70 cm (Nonlinear)
ROM
Data
0.000002
0.000001
0.000000
0.000001
0.000002
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
Time (s)

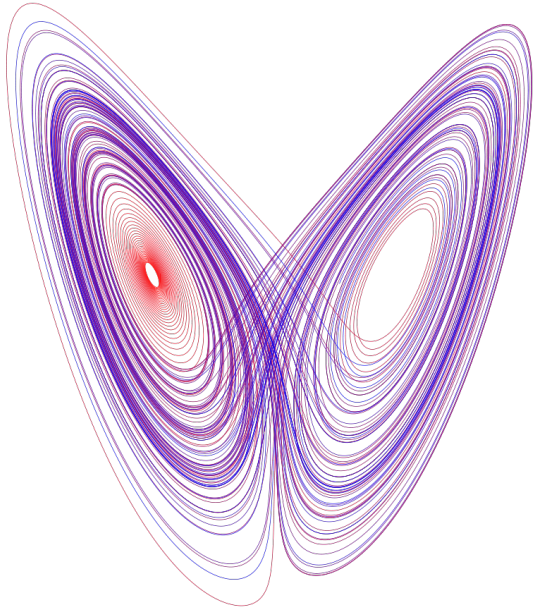

Error
1e−7
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Displacement (m)
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
Time (s)

Dynamic Prediction Framework

Instead of learning the map $t \to x(t)$, learn the map $x(t_n) \to x(t_{n+1})$.

- Often better posed for long time extrapolation
- Easier to characterize growth/decay/oscillations

We use this framework to learn the dynamics of the ROM error



Complex

vs.

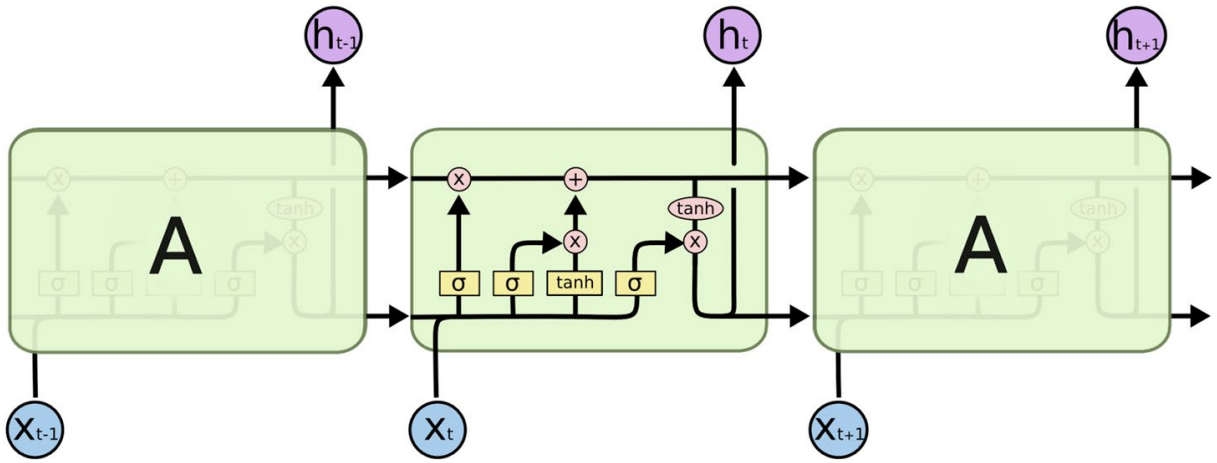
$$x' = \sigma(y - x)$$
$$y' = x(\rho - z) - y$$
$$z' = xy - \beta z$$

Simple

Network Architectures

Recurrent Neural Nets (RNNs) & Long-Short Term Memory (LSTM)

- RNNs with modified structure which enable them to learn long term dependencies.
- Natural architecture for parsing sequenced inputs or outputs (or both).
- LSTMs exhibit state-dependent context; i.e., they can look back in time a variable number of steps



The Koopman Operator

Dynamical system

$$x(t_{n+1}) = f(x(t_n))$$

Koopman Operator

$$K\varphi(x(t_n)) = \varphi(x(t_{n+1}))$$

$$x(t_{n+1}) = \varphi^{-1}(K\varphi(x(t_n))$$

Koopman Eigenfunctions

$$K\varphi_\lambda(\cdot) = \lambda\varphi_\lambda(\cdot)$$

$$x(t_{n+1}) = \varphi_\lambda^{-1}(\lambda\varphi_\lambda(x(t_n))$$

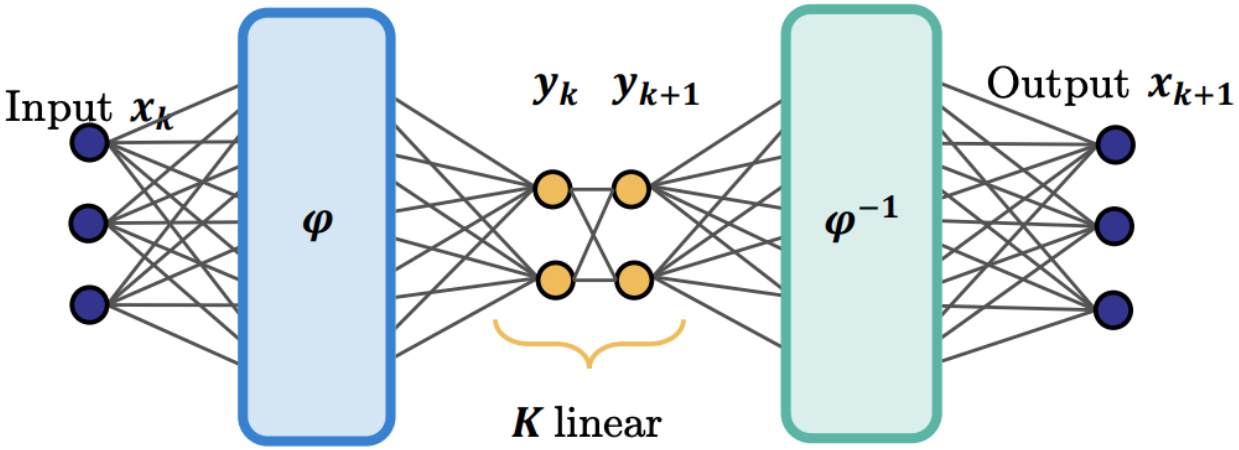
Estimating the Koopman operator generates **linear** dynamics in right coordinates, even for nonlinear systems

Koopman eigenfunctions are an efficient choice of embedding, and have physical significance (Mezić 2016)

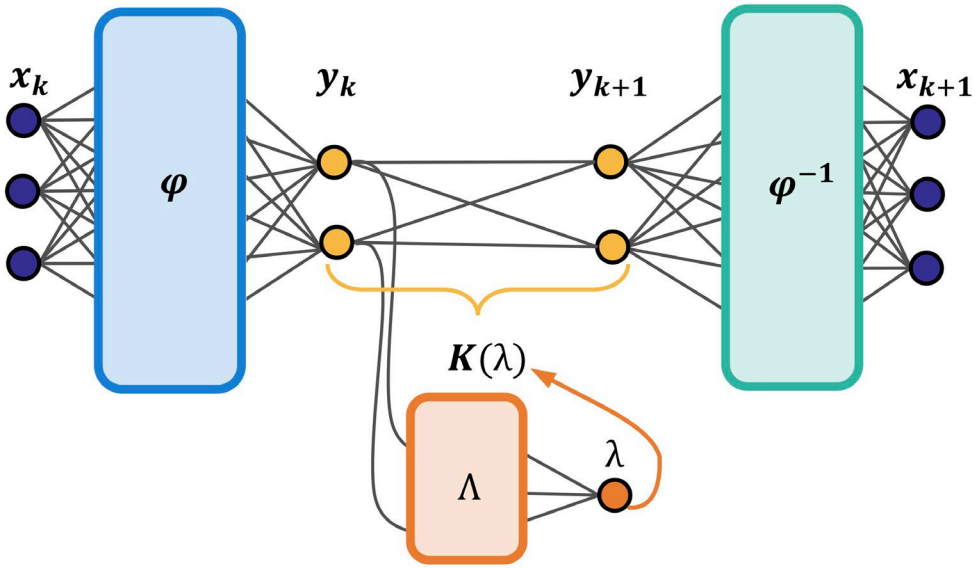
Deep Koopman Network

- Developed by Lusch et al. to learn Koopman eigenfunctions and use them for predictions
- Full version includes auxiliary network to account for nonlinear adjustments to Koopman eigenvalues corresponding to continuous spectra

Linear Predictive Autoencoder



Deep Koopman Network

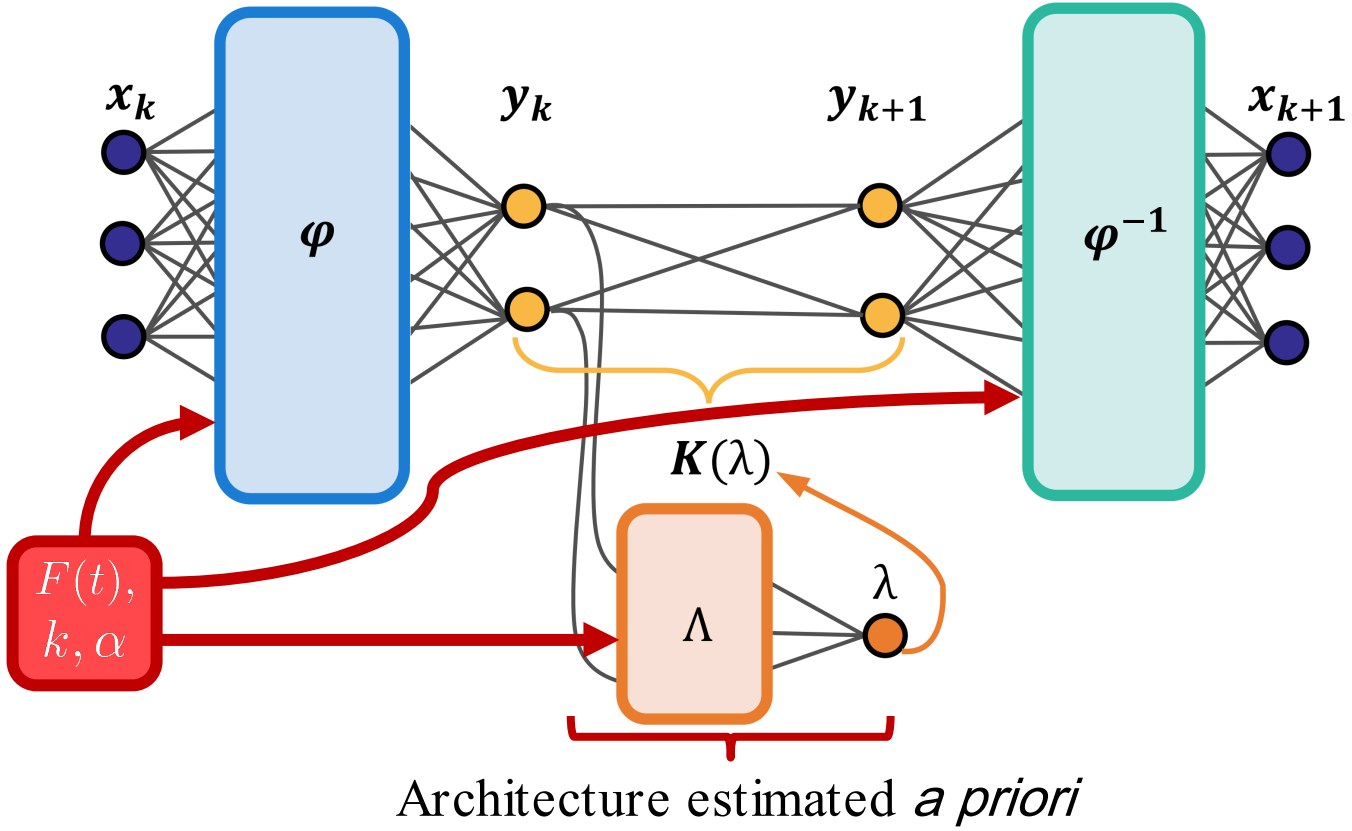


Images from Lusch et al., 2016

Modified Deep Koopman Network

Our problem is more complex than those considered in Lusch et al.:

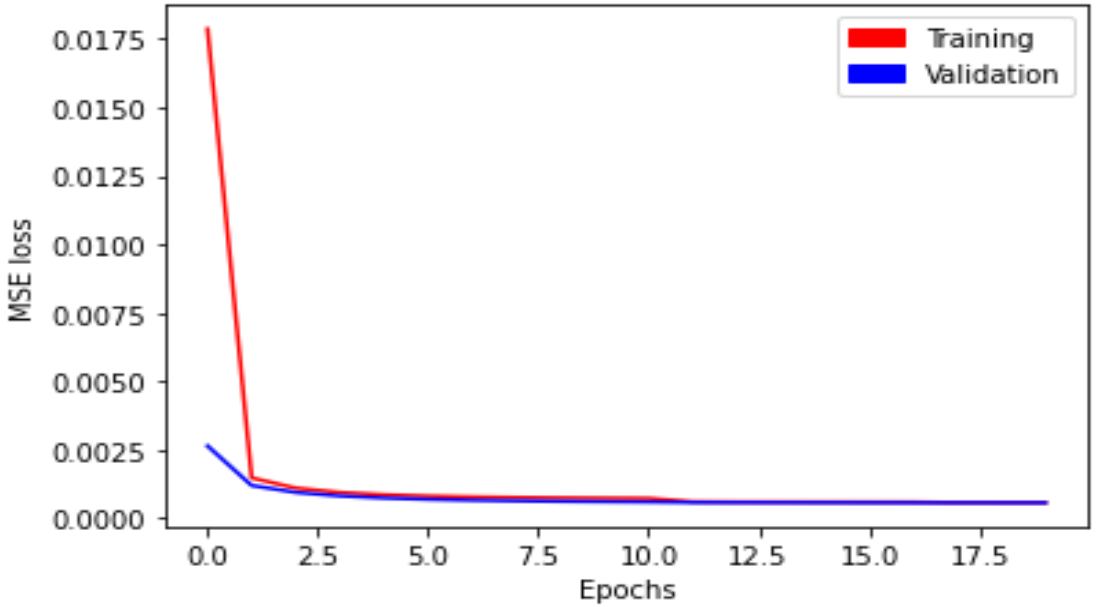
- Time dependent forcing (non-autonomous dynamics)
- Varying physical parameters (k, α)
- Unknown dynamics, requiring *a priori* estimation of parts of network architecture



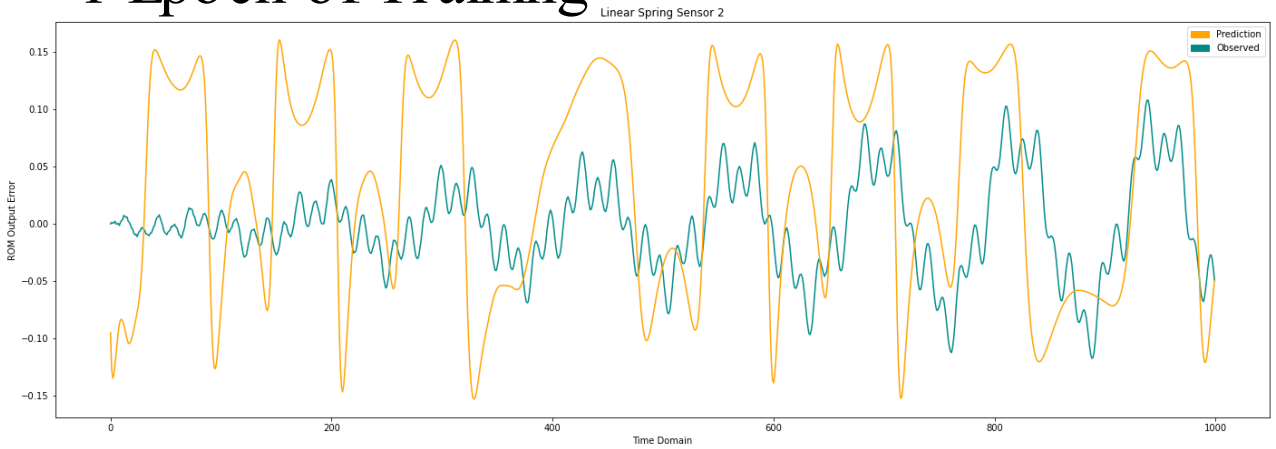
Results

LSTM Training

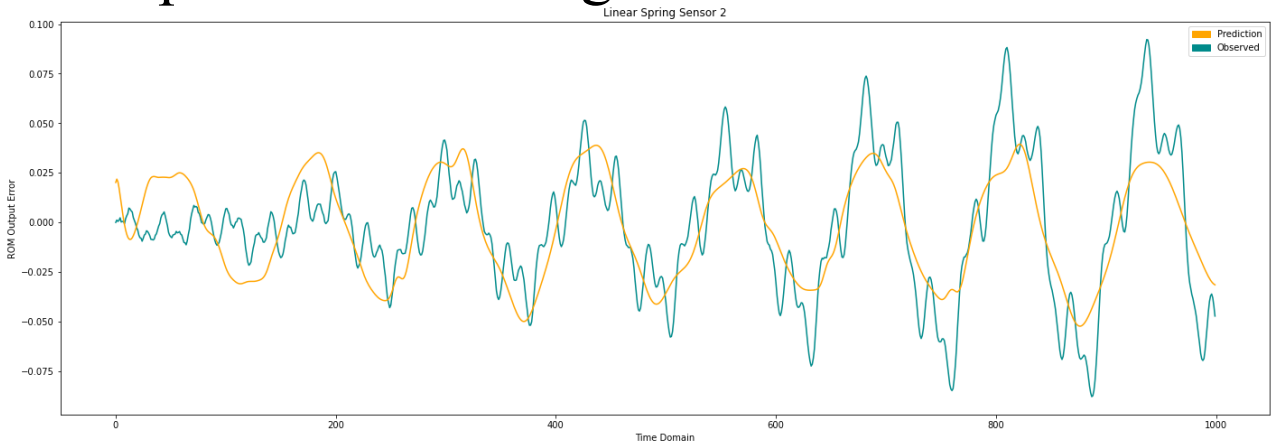
- Predicts error time series given ROM displacement time series
- Very fast training (~2s per epoch)
- Completely agnostic to parameter and forcing dependence



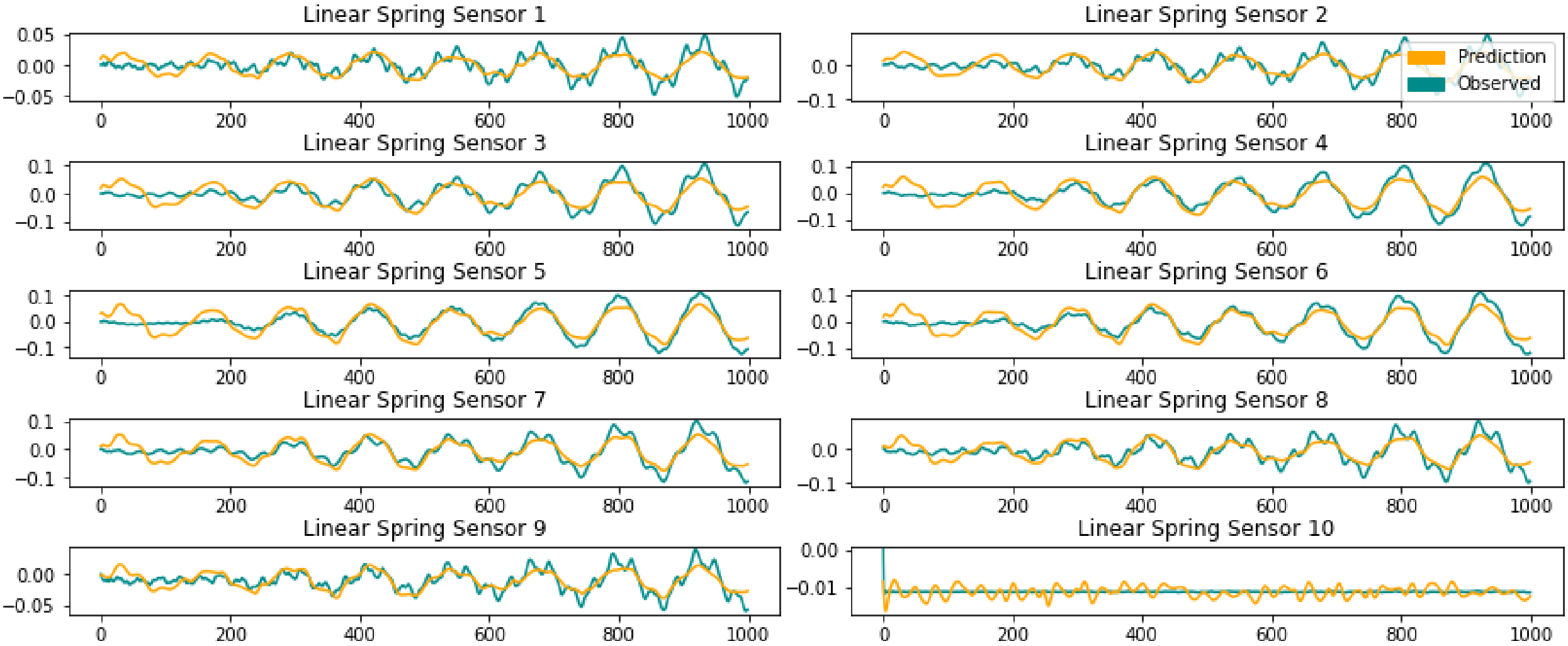
1 Epoch of Training



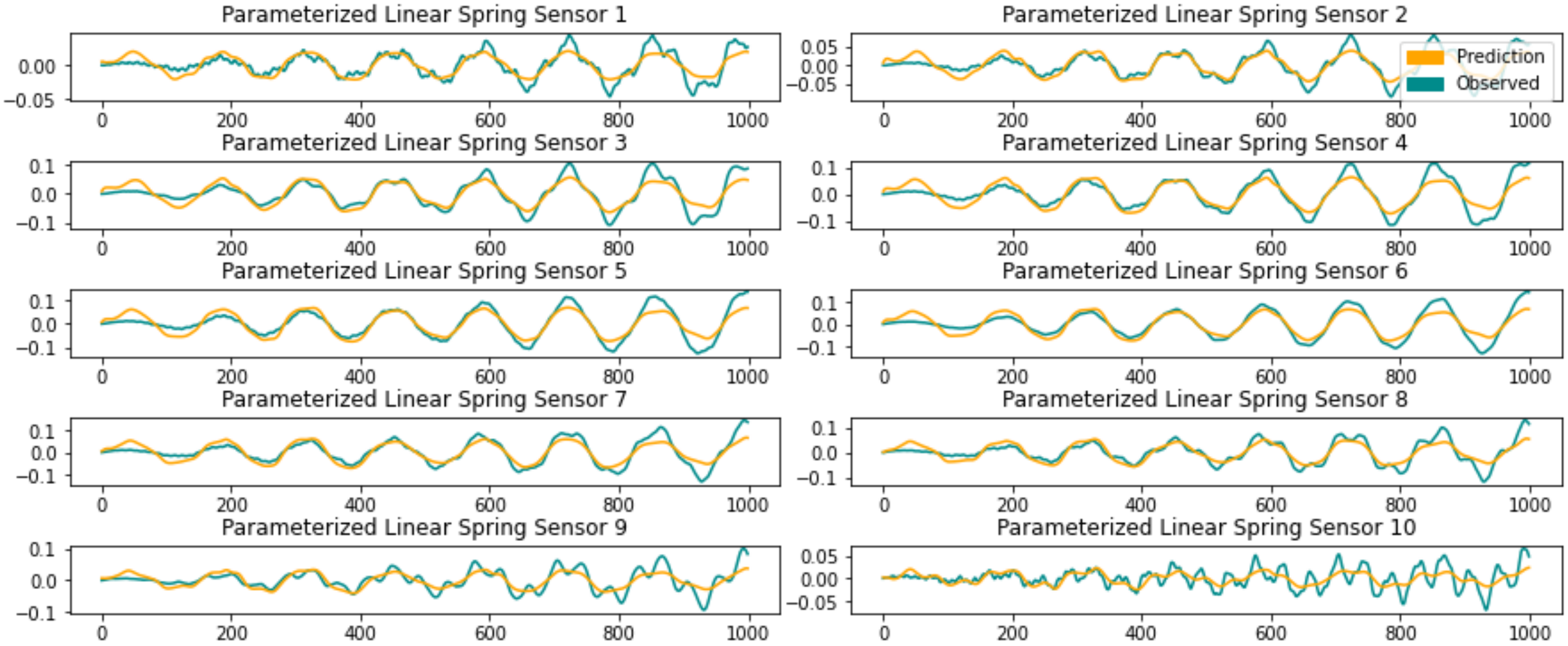
5 Epochs of Training



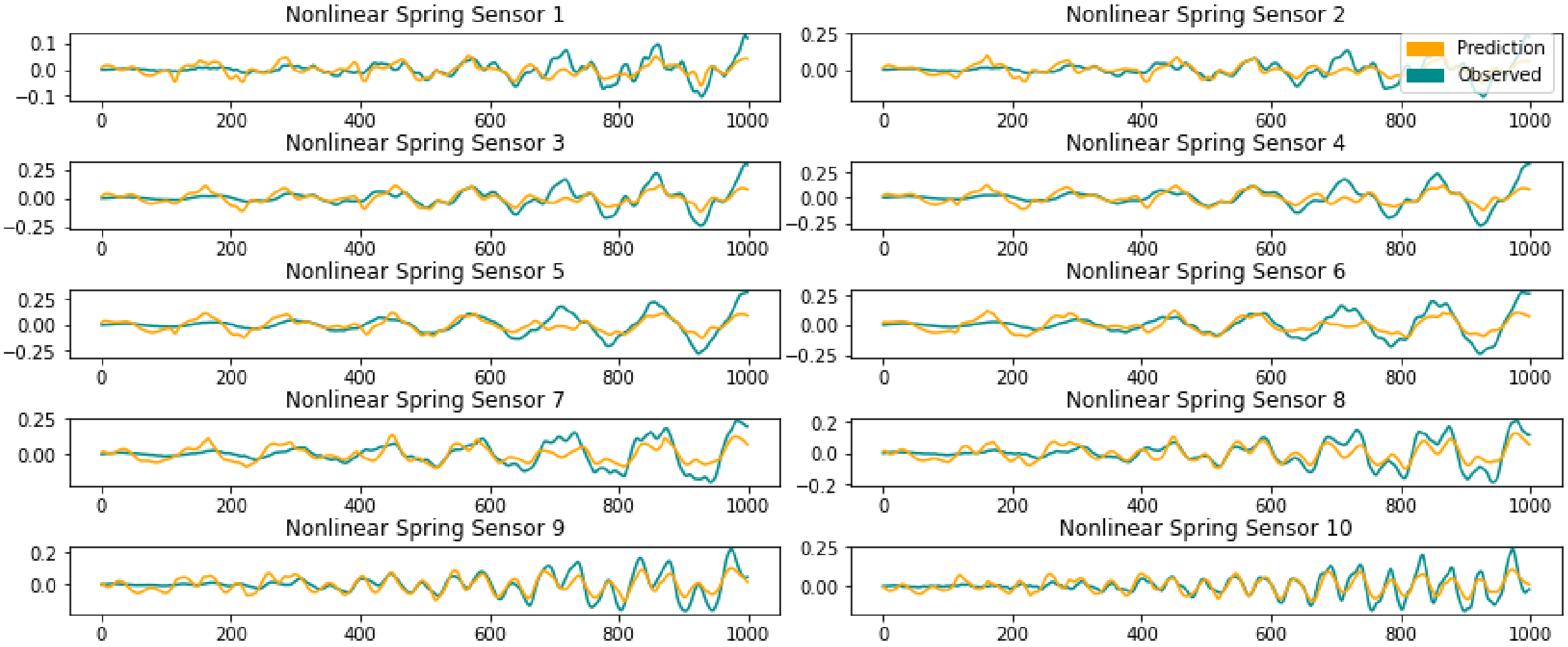
LSTM Results: Linear Spring



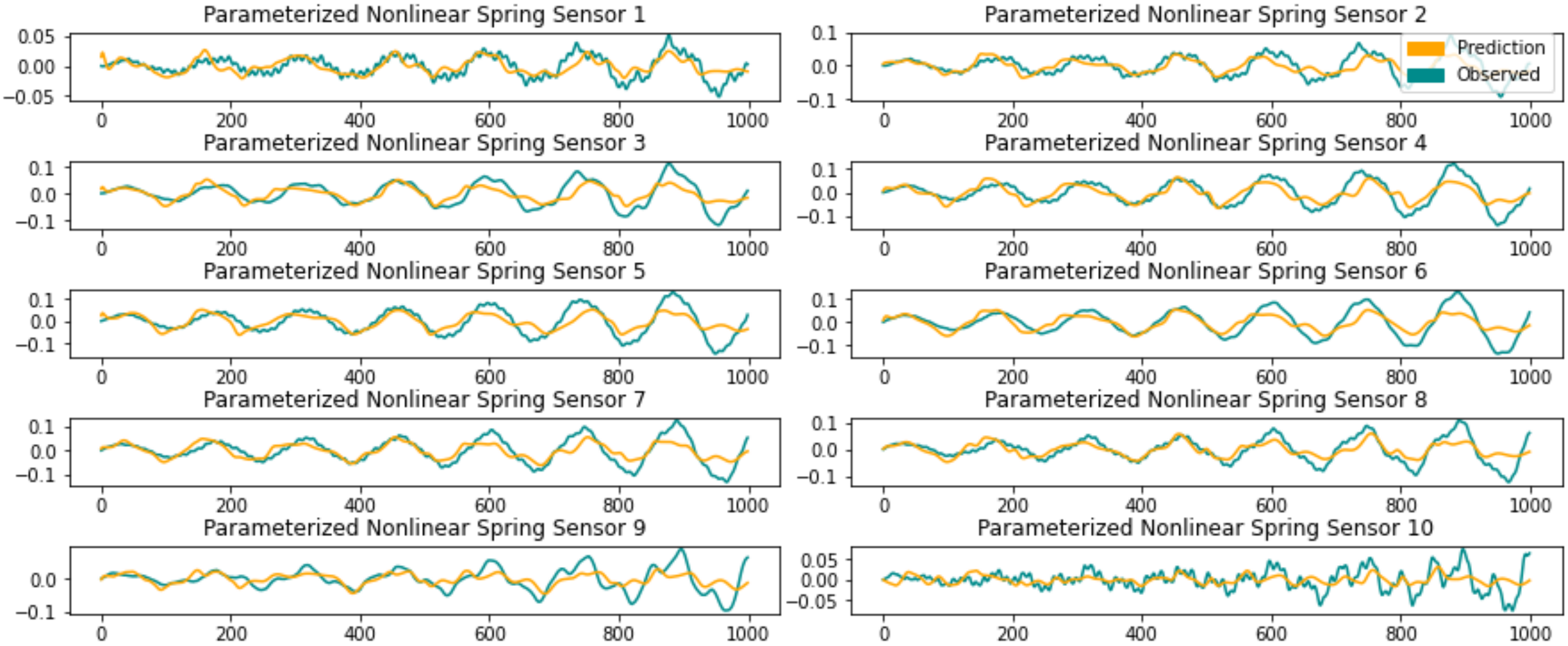
LSTM Results: Linear Spring; Varying k_i



LSTM Results: Nonlinear Spring

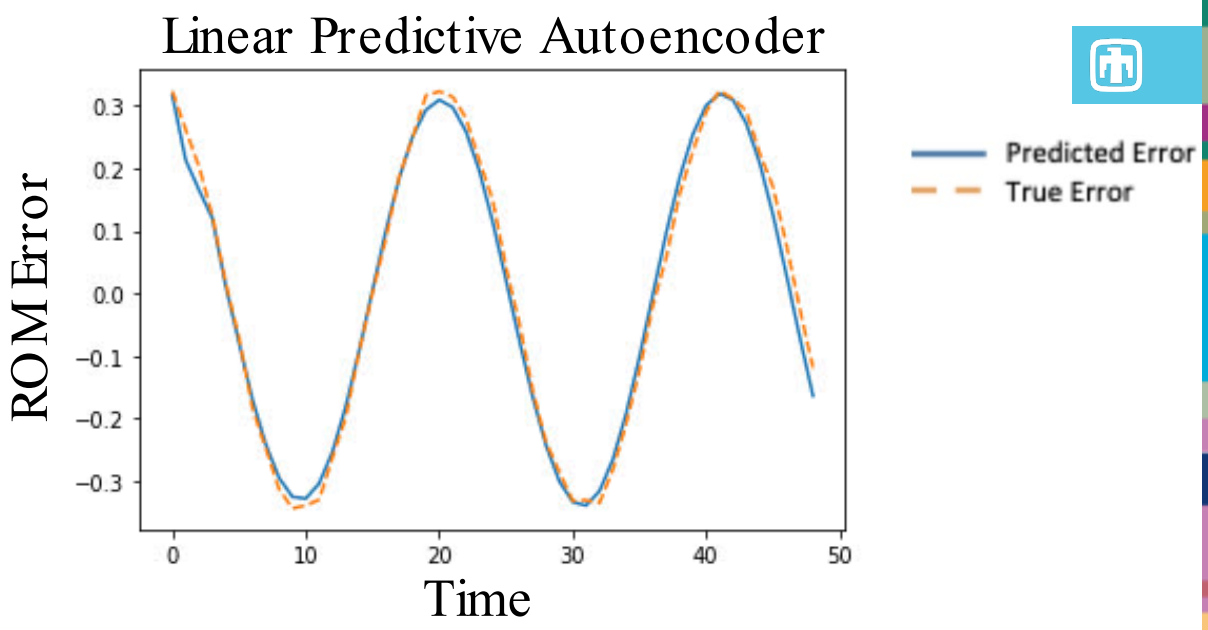


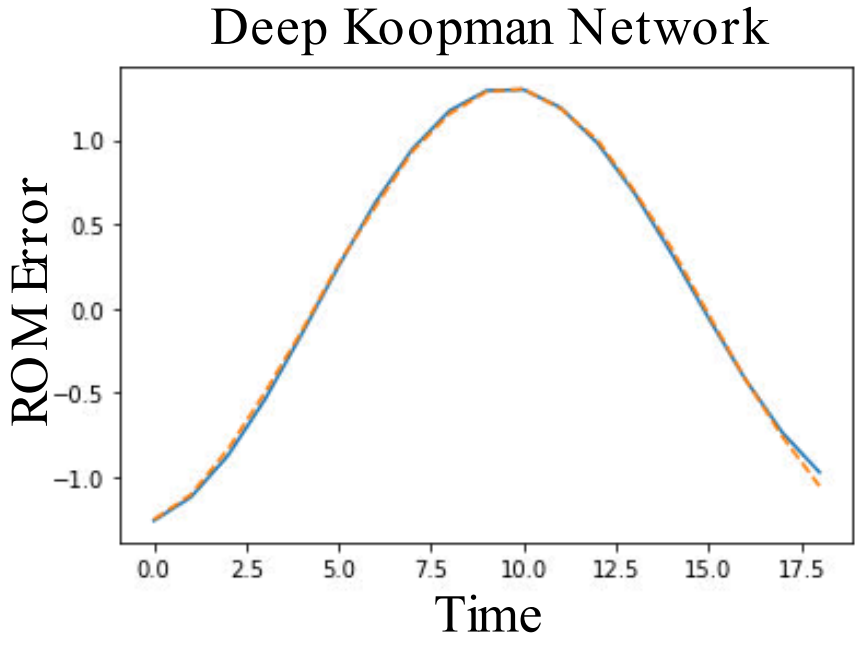
LSTM Results: Nonlinear Spring; Varying γ



Koopman Results

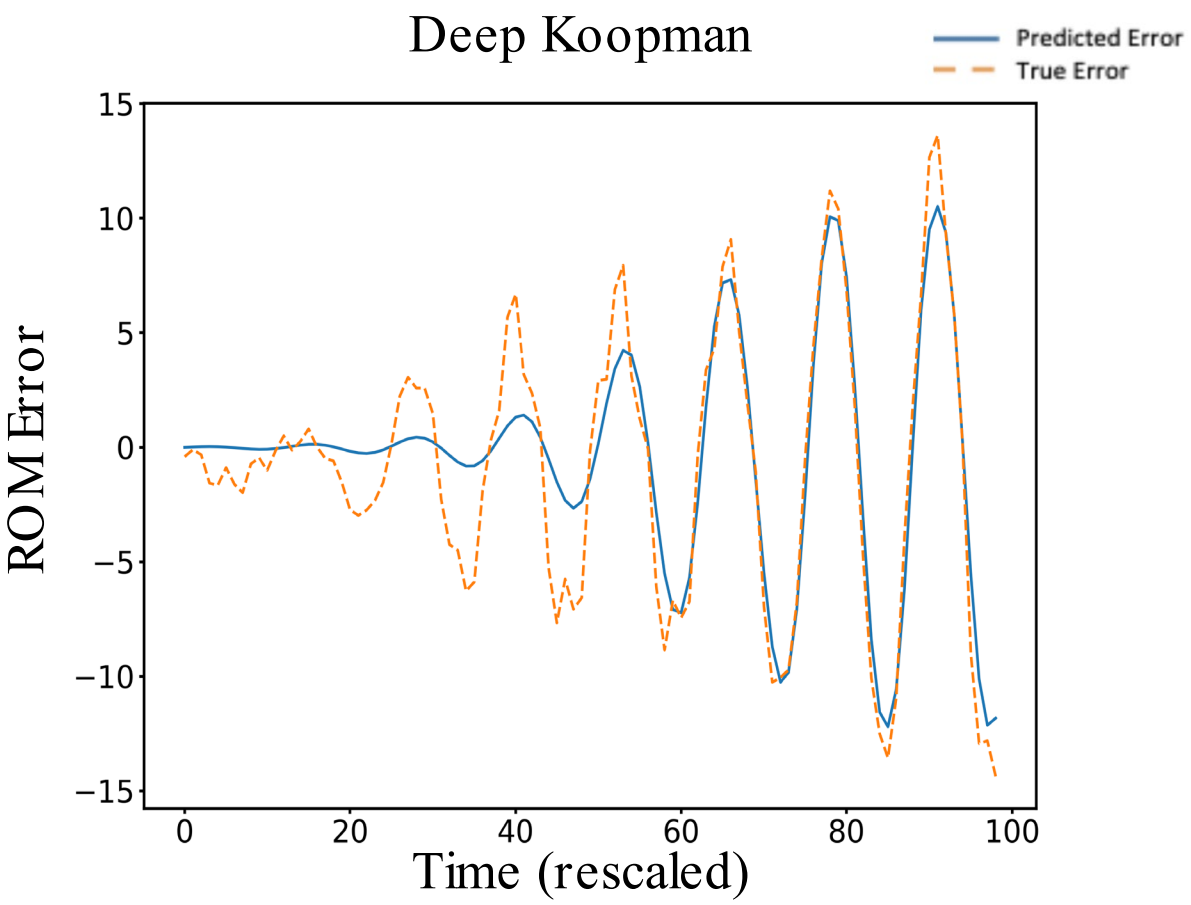
- Preliminary training on smooth sinusoidal time series
- Downsampling required to increase training speed
- Extensive hyperparameter tuning required to train effectively
- Numerical stability issues





Koopman Results cont'd

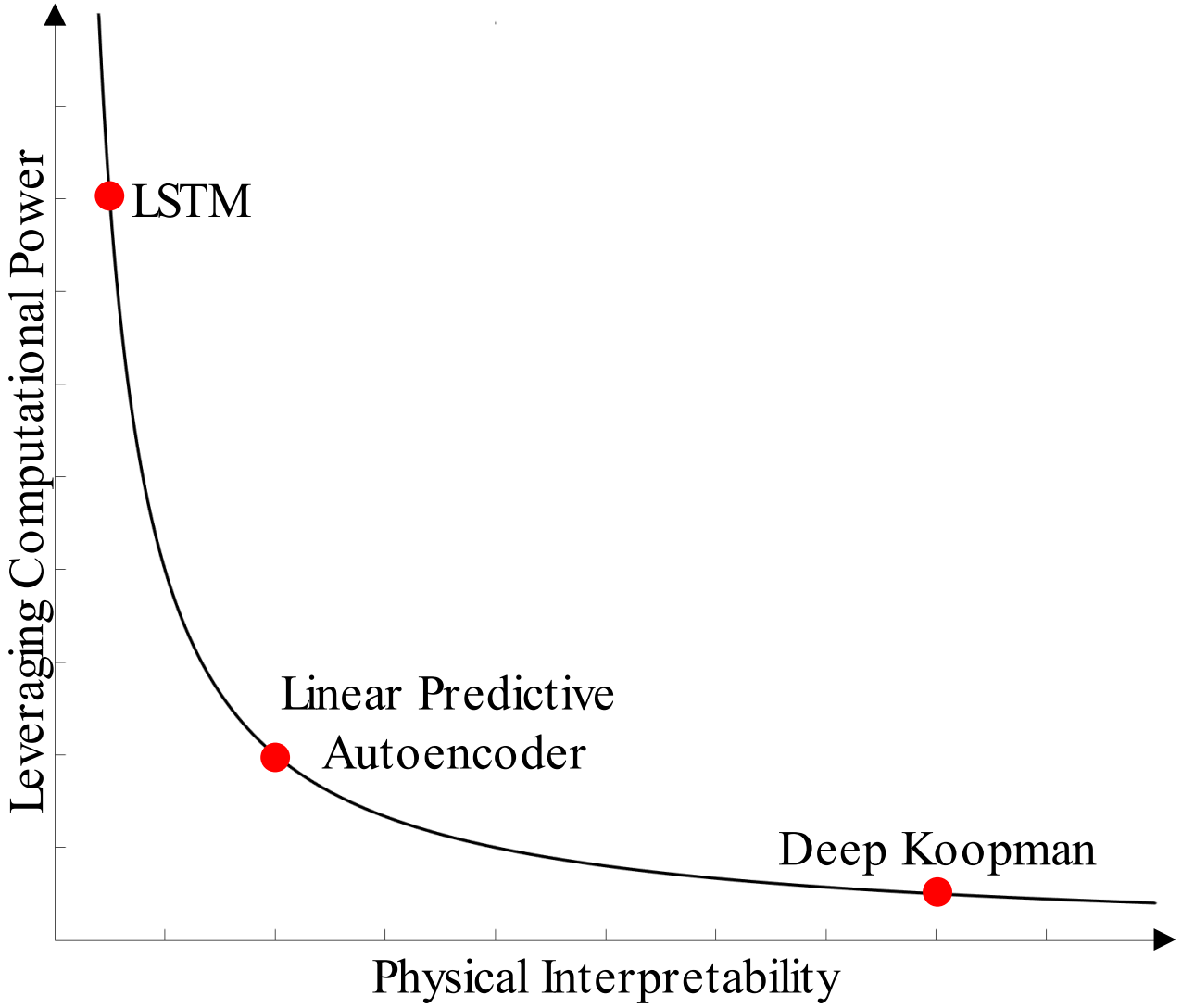
- The network struggles to learn true ROM error
- Sequential network means we can't take advantage of parallelism
- All errors start near zero, so the model has trouble reconstructing unique trajectories from each IC



Conclusions

Conclusions

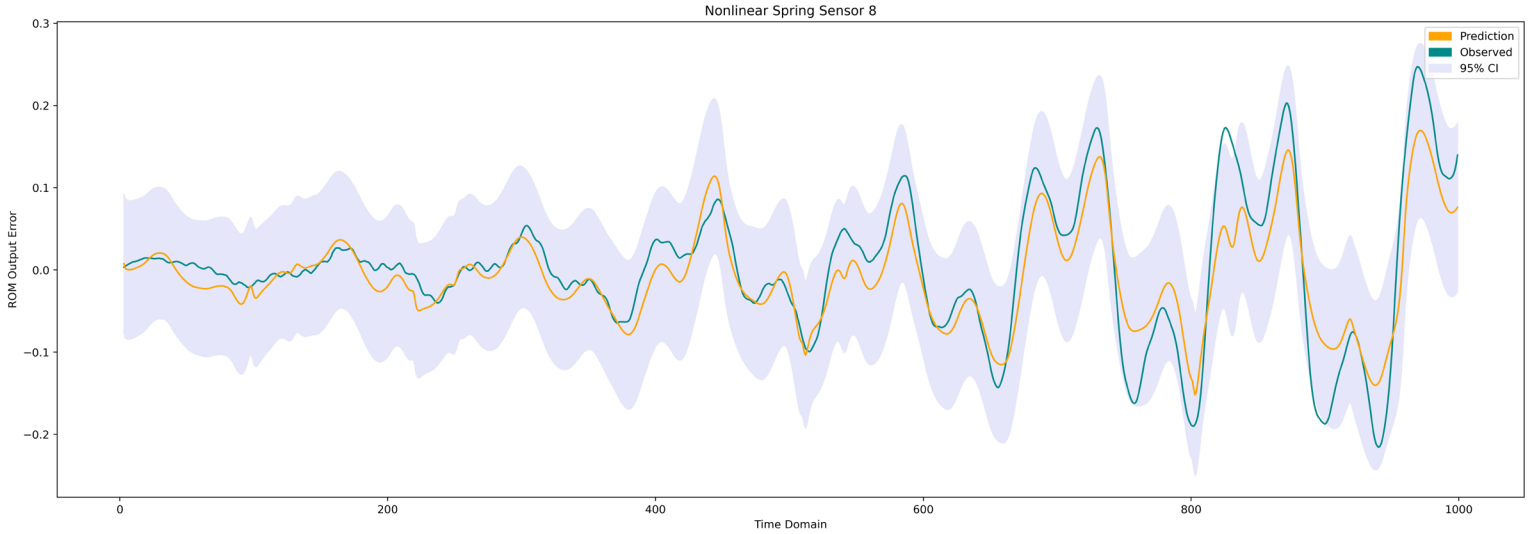
- Deep learning is an effective method for learning and predicting ROM error at coarse frequency scales
- Well-established architectures that maximize use of computational power often scale and train well
- Specialized architecture that aids in physical interpretability led to extreme sensitivity to hyperparameters and long train times



Future work

Future Work

- Investigate sensitivity of training to breadth vs. length of time series for use in experiment/simulation design
- Incorporation of ROM-specific features; i.e., dual-weighted residuals, into network
- Implementation with alternate modes of operation; i.e., control, data fusion, etc.
- Addition of statistical outputs or Bayesian training/prediction for real time uncertainty quantification and error statistics
- Determine how well error predictions enhance the ability of ROM predictions to account for extreme events



Acknowledgements

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