VORONOI OR HEXAHEDRAL MESHING FOR SIMULATION ACCURACY AND SPEED?

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INTRODUCTION:

In simulations of fluid flow through porous media it is frequently necessary to flex meshes to conform to geological features. However, the poor quality of the resulting mesh can negatively impact the accuracy of the simulation. Structured meshes can also systematically bias the calculated flow field. One alternative is to simulate using unstructured Voronoi meshes. Voronoi meshes have orthogonal fluxes between cells, and unstructured meshes do not bias the flow field, reducing two sources of numerical error. Unfortunately, simulating on unstructured polyhedral meshes can significantly increase simulation time. A series of analytical benchmark problems are simulated on three-dimensional hexahedral, flexed-hexahedral, and Voronoi meshes using the finite volume simulator PFLOTRAN. The accuracy of simulated results and computation times are compared. Finally, simulations of experiments of unstable two-phase flow are compared.

ERROR METRICS:

Several L2 error metrics are used

Q = pressure, concentration or saturation

$Q_{an,i}$ analytical value at i

$Q_{sim,i}$ simulated value at i

Pointwise Absolute Error (EA)

$$EA = (Q_{an,i} - Q_{sim,i})^2$$

Global error

$$RMS_{dom} = \sqrt{\frac{\sum_{i=1}^{N} (Q_{an,i} - Q_{sim,i})^2}{N}}$$

Error at point i

$$RMS_{i,t} = \sqrt{\frac{\sum_{t=1}^{T} (Q_{an,i}(t) - Q_{sim,i}(t))^2}{T}}$$

RMS_{dom} = root mean-squared error at all N grid points in domain

$RMS_{i,t}$ = RMS at point i over time (0,T)

EXAMPLE 1: Pressure distribution for a five-spot well pattern

Two-dimensional single-phase flow of an incompressible fluid from a point source at (0,0) to a point sink at (1,1). Three meshes are considered.

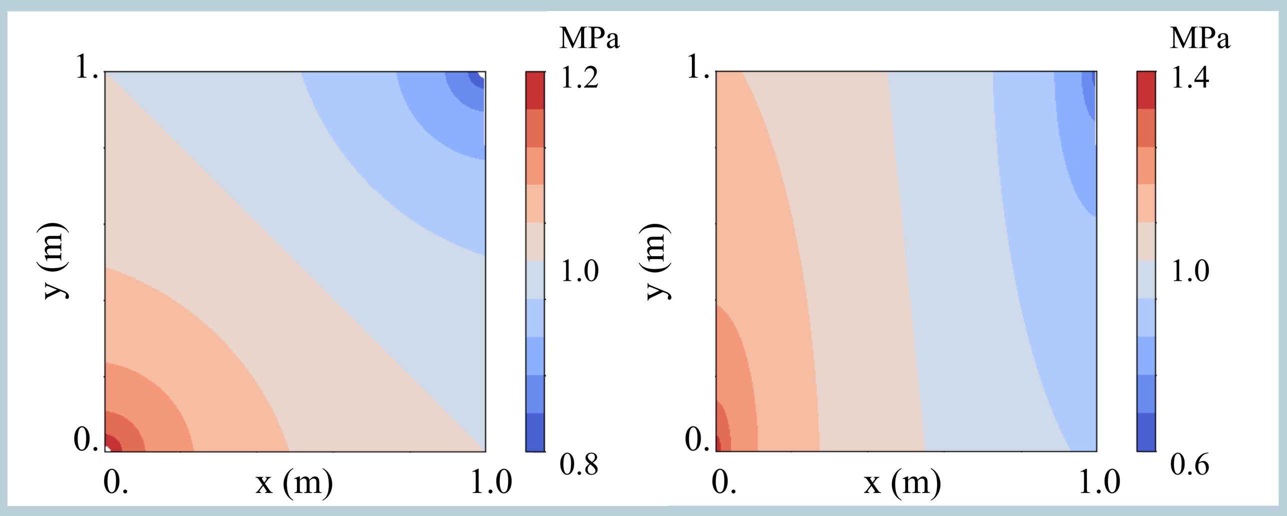

Figure 1. Left: Analytical solution for isotropic five-spot case. Right: Analytical solution for anisotropic five-spot case

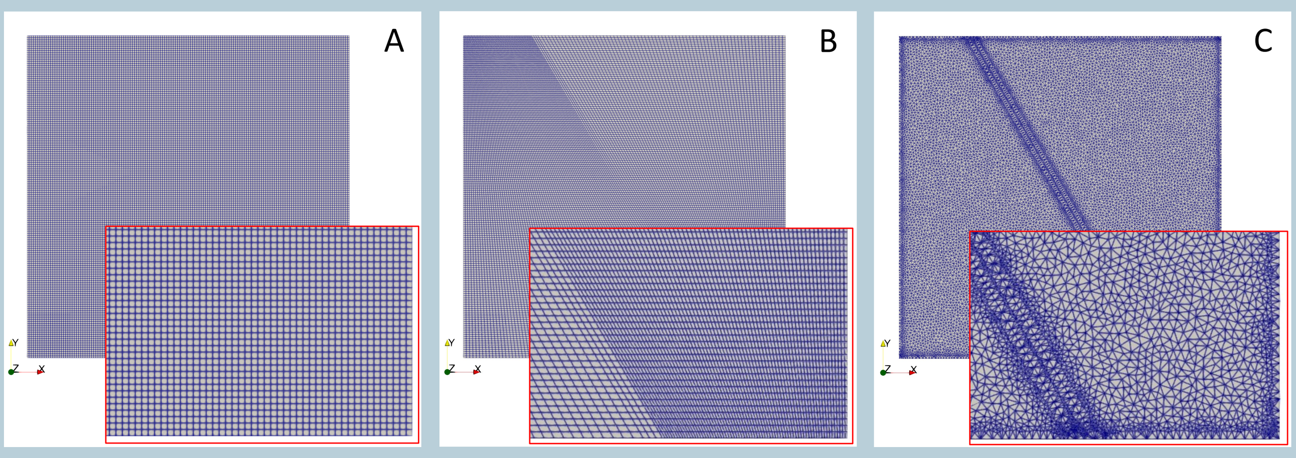

Figure 2. A) Hexahedral mesh with 67,500 cubic elements. B) Hexahedral mesh flexed to a fracture with 68,400 elements. C) Voronoi mesh with interior fracture with 94,720. Red boxes show detail of surface mesh.

Isotropic simulations Flexing the hexahedral mesh improves the simulated result because it aligns grid cell faces with the flow direction. Voronoi and hexahedral meshes have comparable RMS_{dom} error, but the Voronoi simulation takes 23-38 times longer.

Anisotropic simulations Flexed mesh has regions of high error near the source and sink. The Voronoi mesh has somewhat lower RMS_{dom} error than the hexahedral mesh, but Voronoi simulation takes 27-32 times longer.

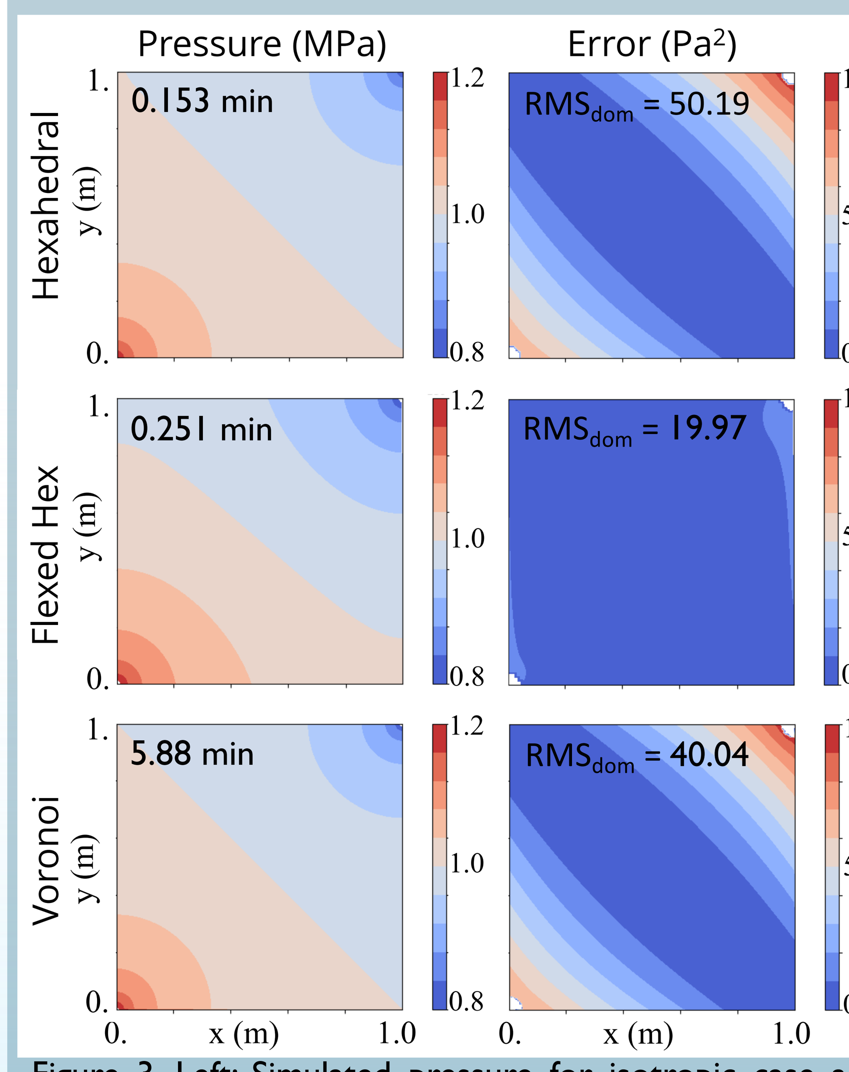

Figure 3. Left: Simulated pressure for isotropic case and simulation time. Right: Pointwise and RMS_{dom} error.

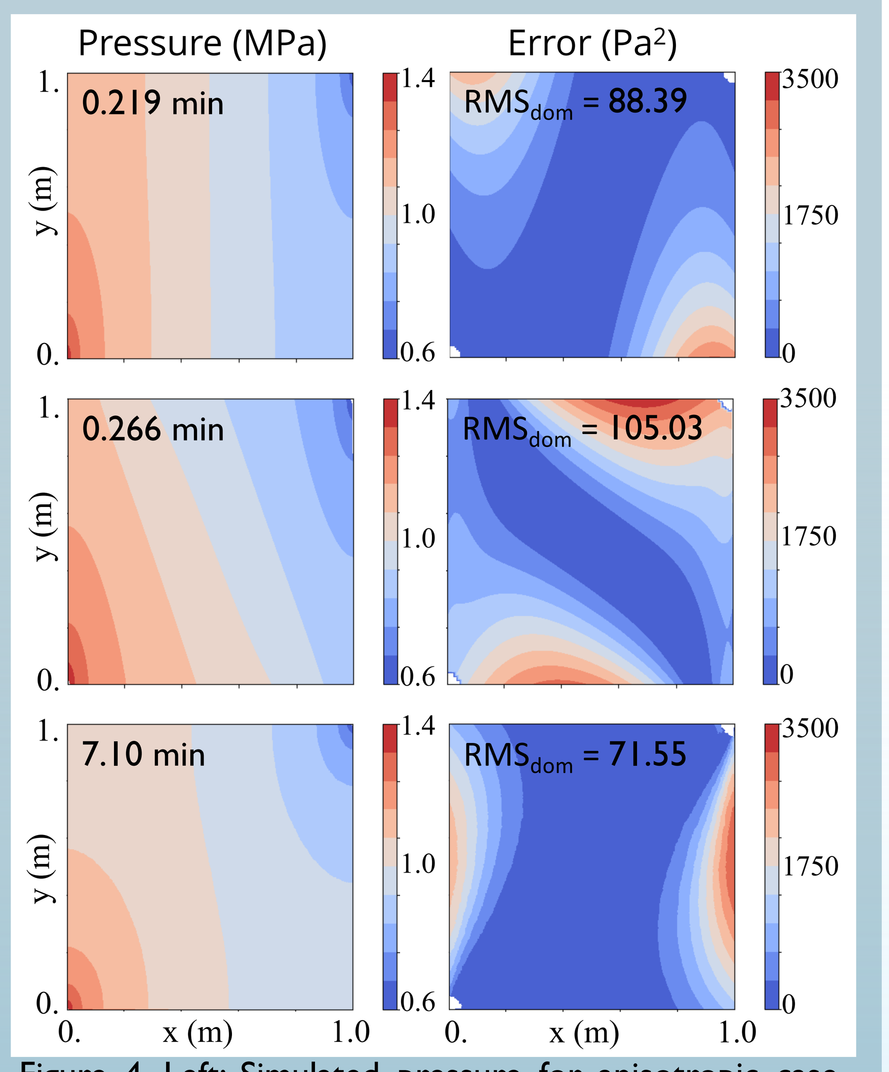

Figure 4. Left: Simulated pressure for anisotropic case and simulation time. Right: Pointwise and RMS_{dom} error.

EXAMPLE 2: Tracer release from a line source

Batu (2006) presented an analytical solution for two-dimensional tracer transport from a line source with constant background fluid flow on a domain that is finite y and infinite in x. The domain is y = (0,2010) m and tracer is released at constant rate from y = (1000,1010) m.

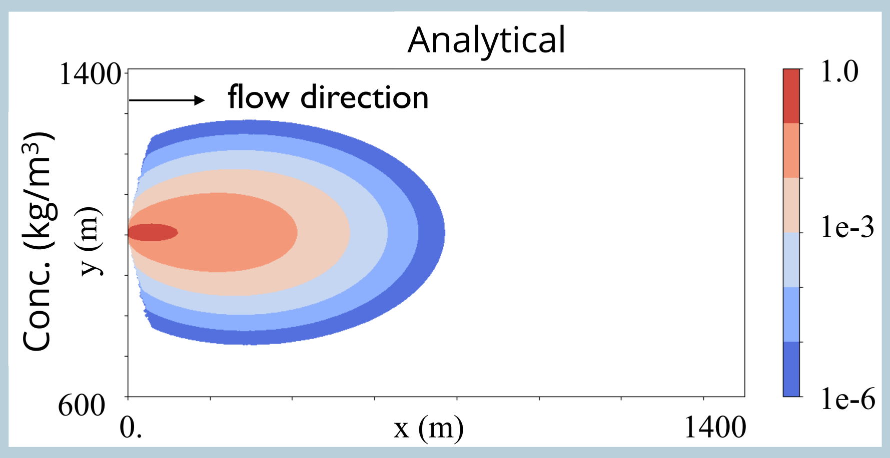

Figure 5. Tracer concentration at t = 5 years. Diffusion has been increased to approximate numerical dispersion.

Concentration on the hexahedral mesh (429K cells) shows systematic error, overestimating plume extent and underestimating lateral diffusion, yet RMS_{dom} error is similar in the simulations. $RMS_{i,t}$ at observation points shows Voronoi mesh simulation (426K cells) is the superior quality result for this error metric.

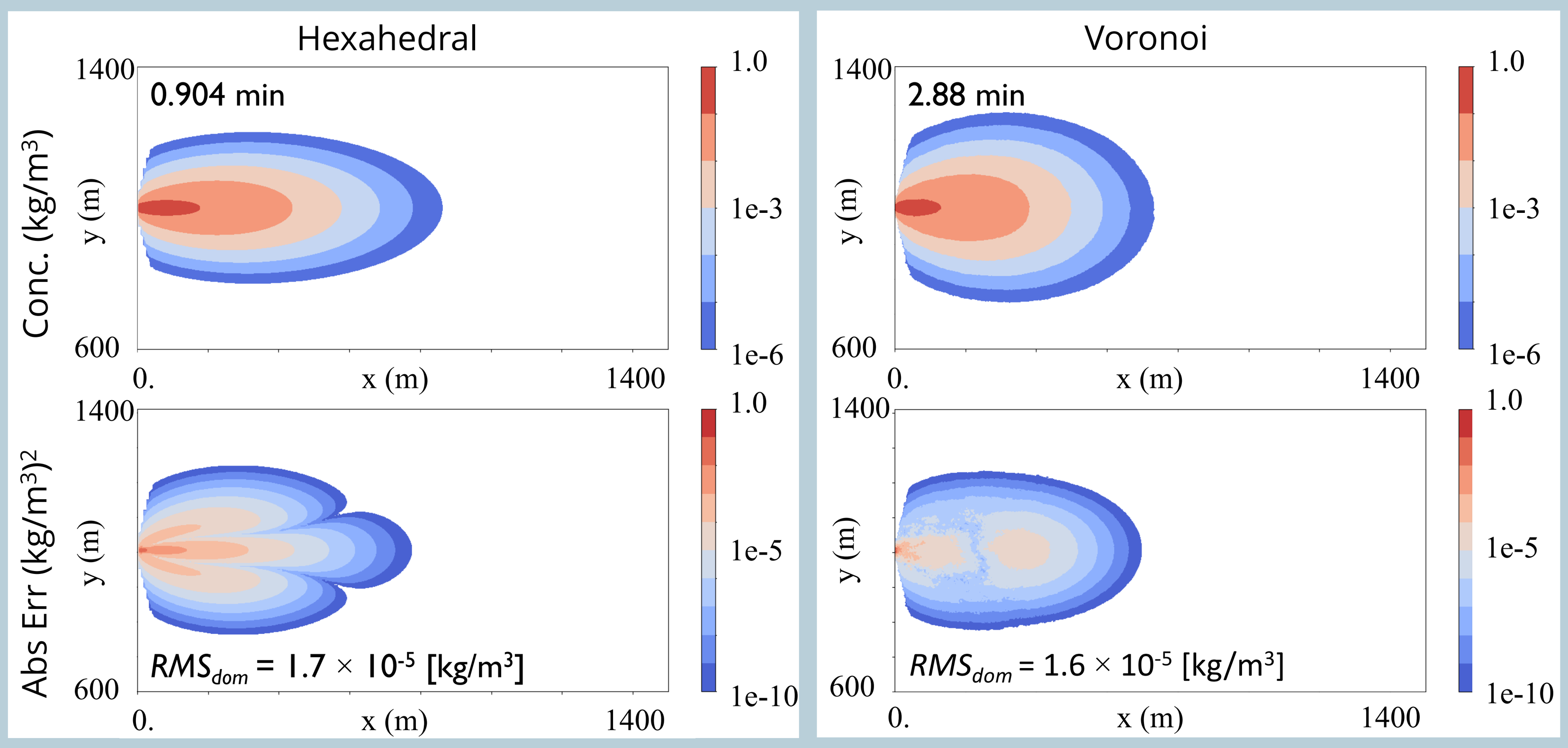

Figure 6. Top: Tracer concentration on hexahedral mesh at t = 5 years. Bottom: Absolute pointwise error.

Figure 7. Top: Tracer concentration on Voronoi mesh at t = 5 years. Bottom: Absolute pointwise error.

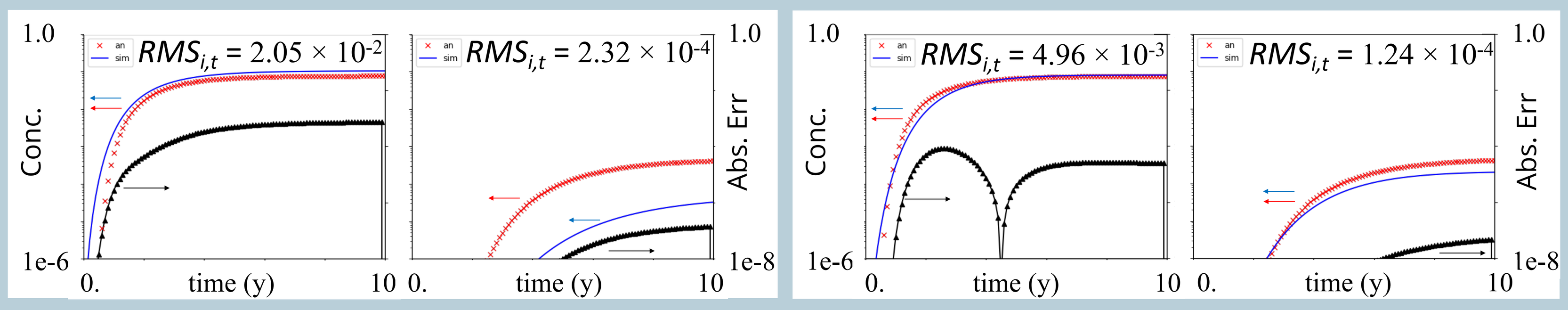

Figure 8. Tracer concentration (kg/m³) and error on hexahedral mesh as a function of time. Left: At (200,1005) m. Right: At (200,1200) m.

Figure 9. Tracer concentration and error on Voronoi mesh as a function of time. Left: At (200,1005) m. Right: At (200,1200) m.

EXAMPLE 3: Unstable gas flow experiment

Wang et al. (2013) conducted a series of experiments of CO_2 injection in a micromodel in capillary and viscous fingering flow regimes, a notoriously difficult problem to simulate using structured meshes. Three of the experiments are simulated here on meshes with around 40,000 grid cells. The hexahedral mesh simulations are unable to capture the fingering phenomena, as expected. Fingering patterns occur on the Voronoi meshes, but the transition from viscous to capillary fingering appears to happen at a lower rate than in the experiment, likely due to the relative permeability model used or differences in CO_2 density and/or viscosity between the experiment and simulation. This is an area of future work.

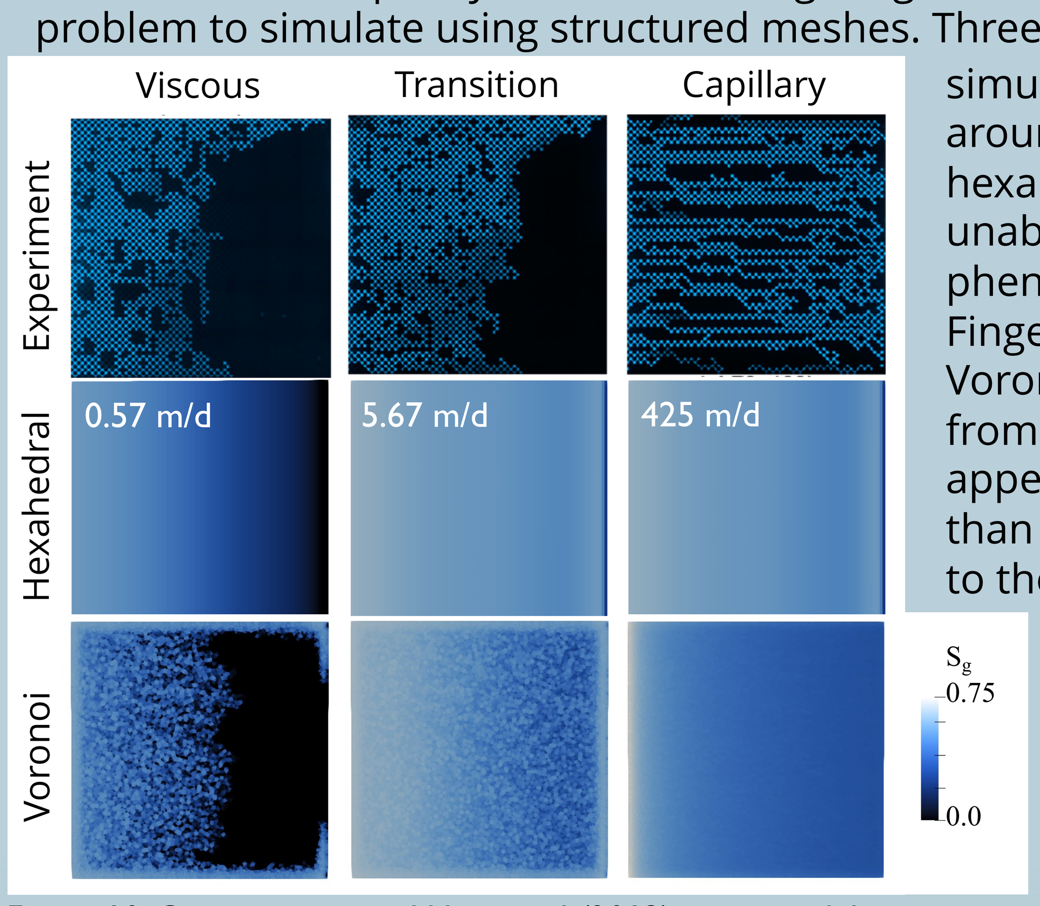

Figure 10. Gas saturation in Wang et al (2013) micromodel experiments with rate increasing to cause a transition from viscous to capillary fingering. Top: Experimental results. Middle: Hexahedral mesh. Bottom: Voronoi mesh.

CONCLUSIONS AND FUTURE WORK:

The example cases considered here indicate that there are simulations where it is imperative to use unstructured meshes to obtain an accurate solution. However in many cases, there is no one-size-fits-all solution. The most appropriate mesh depends on three considerations:

1) If the mesh must be distorted to capture geometric features of the domain.
2) The physics and complexity of the particular simulation.
3) The quantity of interest (e.g. global vs local error).

Quantitatively match Wang et al. (2013) experimental results.

REFERENCES: Wang, Y., et al. (2013). Experimental study of crossover from capillary to viscous fingering for supercritical CO2–water displacement in a homogeneous pore network. Envt Sci & Tech, 47(1), 212-218.
Batu, V. (2006) Applied Flow and Solute Transport Modelling in Aquifers. CRC Taylor and Francis.

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