Unfolding a neutron energy spectrum using a trendfilter

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Abstract: Estimating the distribution of energies of neutrons from observations captured at a detector (called "spectrum unfolding") is a longstanding problem in nuclear physics. The probability distribution over energies is usually smooth, but its shape is usually not known. In this talk, we show numerical experiments in unfolding a neutron spectrum using a *trendfilter,* a sparsity-enforcing statistical technique that develops a piece-wise linear approximation of the spectrum. Unlike existing unfolding techniques like MLEM which have poor stopping criteria, trendfilters are numerically and theoretically sound (i.e., they iterate to an error tolerance). However, the trendfilter must be augmented to ensure non-negativity of the unfolded spectrum. The original trendfilter was developed for data with Gaussian noise and we develop an approximate formulation to accommodate the Poisson (counting) noise in our detector readings. Here we show unfolding results from a U235 Watt spectrum, with good agreement with the true spectrum.

Introduction

The problem of estimating the energy distribution of neutrons emanating from a source, from detector readings, is a fundamental one and has practical uses. The detector reading takes the form of a vector $F^{(obs)}$, which is a discretization over the range of energies observed by detector after the neutrons scatter in the detector; each element of the vector records the number of neutrons in an energy bin. Typically, one has 1024 bins. The spectrum $\hat{P}$ is a vector defined over the discretized energy range of the incident neutrons and contains the probability density function (PDF) of neutron energies; it is estimated by solving a linear inverse problem $F = [\mathcal{R}]P$. This is an under-constrained system, and the estimation is performed by injecting prior beliefs or an assumption of smoothness, introduced via regularization.

There are many iterative methods to unfold P [1]; however, they suffer from the lack of a rigorous stopping criterion and overfitting. There have been attempts to use sparsity concepts borrowed from compressive sensing [2], using basis sets obtained from a singular value decomposition of prior realizations of P or the Krylov subspace of $[\mathcal{R}]$; see Ref. [3]. Trendfilter [4] is a sparsity-enforcing method for solving regression problems of the form $F = [\mathcal{R}]P$, where (F, P) are vectors, under the assumption that P varies smoothly. This is done by penalizing the curvature of P, with the result that one obtains a piece-wise linear approximation $\hat{P}$. The cost function is

$$\min_{P_i} \left\| F^{(obs)} - [\mathcal{R}]P \right\|_2^2 + \lambda_1 \|DP\|_1, \tag{1}$$

where D is a tridiagonal matrix that computes the central difference approximation of the second derivative of P and λ_1 is a penalty that determines smoothness. Typically, λ_1 is determined by cross-validation. Inherent in Eq. 1 is the assumption that $F^{(obs)} = [\mathcal{R}]P + \epsilon, \epsilon \sim N(0, \sigma^2)$, which is not true for us.

Methodology

We first adapt Eq. 1 to Poisson counting statistics. Starting with a Poisson likelihood for observing $F^{(obs)}$ given an energy-specific mean $r = [\mathcal{R}]P$, and expanding it to second order, we see that $\|:\|_2^2$ in Eq. 1 can be replaced with $\sum_i \left(\left(F_i^{(obs)} - r_i\right) / \sqrt{F_i^{(obs)}} \right)_2^2$ where i iterates over energy bins. To ensure that $\hat{P}$ is non-negative, we adopt a predictor-corrector construction. In the "predictor" step, we solve Eq. 1 (but with the Poisson approximation) to obtain P', some of whose elements can be negative. The negative elements of P' are clipped and replaced with "guesses" linearly interpolated from adjoining non-zero values, to form P''. Typically, only a few elements (<10) are negative and need to be corrected. In the "corrector" step, we

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solve a nonlinear optimization problem in $\eta = \log P''$ with Tikhonov regularization. Optimizing in η ensure non-negativity. We solve Eq. 1 using the "`genlasso`" package in R [5].

Results

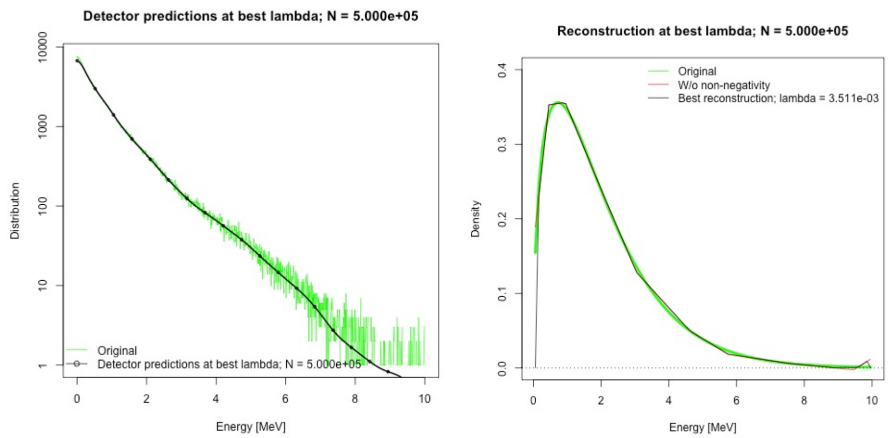

Figure 1 (left): Noisy $F^{(obs)}$ (green line) and F (black line) as reconstructed from the unfolded spectrum. Right: The unfolded spectrum (black) and the true one (green).

Figure 1 shows the unfolded Watt spectrum from a U235 source. On the left, in green, we see the noisy distribution of measured event energies observed in the detector. On the right, in black, we see the unfolded spectrum $\hat{P}$ described over 100 energy bins; it is very close to the true one (in green). This unfolded spectrum is used to predict the detector readings (plotted in black); it is a good fit with the green. Thus, we see that the "predictor-corrector" algorithm is working. We have repeated this with a (α, n) spectrum too.

Conclusions

We have a new method, based on trendfiltering, which creates piece-wise linear approximations to neutron energy spectra by imposing sparsity. With synthetic data, is it seen to work well. In the future, we will test the algorithm with experimental measurements.

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