Estimating model-form errors using random field models

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Model-form error in a partial differential equation (PDE) model represents the physical processes that the model does not capture. In a PDE model, the error is often modeled as an additive or scaling variable that varies in space and time. The model-form error is estimated using observational data or high-fidelity simulations that contain the physical processes missing from PDE model.

The estimation of the model-form error is challenging. In many cases, a parameterized error model is postulated in terms of the local state, and model-form error quantification is reduced to estimating the parameters of the error model. However, there are no rules to guide what the postulated model should be.

Alternatively, if there is sufficient data e.g., from a high-fidelity simulation, the model-form error may be estimated as a spatiotemporal field, using a random field model as a prior. The prior (or regularization) imposes smoothness on the estimated field. In this talk, we will explore the use of Markov random fields (MRF) to estimate model-form errors, and how different MRFs can lead to different model-form error estimates. A random field model can have free parameters that govern the degree of smoothness and non-stationarity that can be accommodated in the model-form error estimates, and we explore how these parameters might be set via cross-valdiation. Illustrations of the method will be provided using a canonical PDE model of 1D radiative heat transfer that can be computed quickly as part of the estimation of the model-form error.

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