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An Assessment of the Laminar Hypersonic Double-Cone Experiments in the LENS-XX Tunnel

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What is this talk about?

- A cautionary tale on decision-making using surrogate models
- Surrogate models: Data-driven, fast running proxies of CFD solvers
 - Widely used in UQ and Bayesian inverse problems but they are approximate
 - Results e.g., point estimates from inverse problems can be wrong
 - Ditto, wrong decisions based on *point* estimates
- Application: Checking the consistency of experimental datasets from LENS-XX expansion tunnel
 - Laminar flow over double cone. Never modeled successfully
 - Questions about inaccurate inlet boundary conditions (BCs)
- Basic idea: Infer inlet BC by inverting measurements on the cone
 - To be safe, Bayesian inference; BCs as probability density functions (PDFs)
 - But what if the surrogate models of the Navier-Stokes simulator are approximate?



Thanks, Kieweg et al, SciTech 2019 (a)





Recap – the experiments

- We have a double-cone in hypersonic flow
 - LENS-XX expansion tunnel, low temperatures, thermochemical equilibrium in freestream (in principle)
 - Freestream errors: 3 % (U, T); 7% (ρ)
 - 6 experiments, $H_0 = [5.4, 21.8] MJ/kg$
 - Mild vibrational non-equilibrium to widespread dissociation
- Laminar, attached flow on the fore-cone; simple physics
 - Shock interactions, separation bubble on the aft cone
- Experiments of interest
 - Case 1: H₀ = 5.4 MJ/kg; mild vibrational non-equilibrium
 - Case 4: H₀ = 21.8 MJ/kg; vibrational & chemical non-equilibrium







Recap – validation studies

- LENS-XX experiments never modeled successfully. Symptoms
 - Models underpredict heating on the fore cone
 - Ditto, the size of the separation zone
- Two potential causes investigated
 - Cause 1: Thermochemical models are inaccurate
 - Many thermochemical models & assumptions tried; changed the size of separation but not enough, or too much
 - Cause 2: The inlet BC is mis-specified (outside expt. error bounds)
 - Bayesian inference of (ρ , U, T, T_v) from measurements on fore-cone (laminar, attached flow)
 - Case 1 (low enthalpy): BCs barely within experimental error bounds i.e., OK
 - Case 4 (high enthalpy): BCs well outside error bounds; possibly mis-specified





Problem statement

- Aim: Decide if the BC mis-specification is dependent on flow enthalpy
 - Or was that an artefact of using surrogate models?
- Consequence: Did thermochemical non-equilibrium (inside and upstream of LENS-XX test-section) play a role in BC (mis-) specification?
- How to address the problem?
 - Pose and solve an inverse problem for inlet BC.
 - Use Navier-Stokes CFD solver, not surrogates
 - Data: p(x), q(x), H₀ and P_{pitot}, but only on the forecone, before separation







Technical approach

- Formulation: Deterministic inversion optimization of a cost function J
 - Gradients (sensitivities w.r.t. optimization variables) computed using adjoints

•
$$\min_{\Theta} J = \alpha \left\| q^{(obs)} - q(\Theta) \right\|_{2}^{2} + \beta \left\| p^{(obs)} - p(\Theta) \right\|_{2}^{2} + \gamma \left\| H_{o}^{(obs)} - H(\Theta) \right\|_{2}^{2} + \delta \left\| P_{Pitot}^{(obs)} - P(\Theta)_{Pitot} \right\|_{2}^{2}$$

• Uncertainty bounds on Θ^* under assumed Gaussian posterior distribution

•
$$\Theta \sim N(\Theta^*, \Gamma), \Gamma = H^{-1}, H = \left| \frac{\partial^2 J}{\partial \theta_i \partial \theta_j} \right|$$

- Outstanding questions:
 - $\Theta = \{ \text{density, velocity,} \}$
 - Is the Gaussian assumption for the posterior distribution valid?







Bayesian comparison

- BC estimation done via Bayesian inversion and surrogate models
 - Data: Used the same measurements on the fore-cone
 - Method: MCMC
 - Exact posterior PDFs for (ρ , U, T, T_v); quantified the uncertainty in the estimates
 - Also MAP values (most probable or maximum a posteriori)
- Findings:
 - PDFs for T, T_v too wide can't be estimated from data
 - Case 1's BC (MAP values) barely inside experimental error bounds
 - Case 4's BC (MAP values) outside experimental error bounds
- Reliance on MAP values and surrogate models to make decisions is a deadly combination
 - So check it remove surrogates, for starters





Case 4

Local sensitivity analysis

- Sensitivity = $\frac{\partial A}{\partial B}$ at the nominal inlet conditions
 - A = {surface pressure, surface heat flux} on double cone
 - $B = \{\rho, U, T, T_v\}$
- Findings:
 - Sensitivity w.r.t. {T, T_v } too small won't be able to estimate from data
 - Expected hypersonic flow energies are kinetic, not thermal
 - So, Θ = {ρ, U}
 - Hold T, T_{ν} at the values specified in the experimental dataset





Validation with Run 35

- Low enthalpy (3.71 MJ/kg) flow; LENS-I shock tunnel
- Modeled successfully by Nompelis in 2003
 - Bayesian inference (Ray et al, 2020) established that inferred (ρ, U) lay within experimental error bounds
- Findings: Deterministic & Bayesian agree
 - Θ^* is close to $\Theta^{(MAP)}$ and $\Theta^{(Nom)}$ (experimental data)
 - Gaussian posterior is too wide
 - Conclusion: Method works

| | _@ (Nom) (uncertainty limits) | Θ* | ⊖(MAP) | Density |
|----------|--|-------|--------|---------|
| ρ [g/m³] | 0.5848 (0.5429, 0.6257) | 0.589 | 0.5737 | |
| U [m/s] | 2545.0 (2468.6, 2621.4) | 2506 | 2490.0 | |







Test – Case 1

- LENS-XX; low enthalpy (5.44 M/kg); mild vibrational non-eq
- Bayesian method (w/ surrogates): Inferred (ρ, U) within experimental error bounds
- Finding: Deterministic & Bayesian *disagree*
 - Θ^* is not close to $\Theta^{(MAP)}$ and $\Theta^{(Nom)}$
 - Gaussian posterior too wide

| | _{@(Nom)} (uncertainty limits) | Θ* | ⊖(MAP) |
|----------|---|-------|--------|
| ρ [g/m³] | 0.499 (0.4641, 0.5339) | 0.433 | 0.4897 |
| U [m/s] | 3246 (3148.6, 3343.4) | 3540 | 3340 |







4.5

5.0

Case 1 – multi-start check

- Case 1 BC also seem to be mis-specified
 - Deterministic & Bayesian inference disagree
- Did we fall into a local minima?
 - Redo inference starting from different guesses
 - Guesses outside the experimental error bounds
 - All converge to the same estimate of (p, U)
- Conclusion
 - The estimate of (ρ, U) is correct
 - The approximate surrogate models & MCMC did not find the MAP estimate
 - Case 1 inlet BC may be mis-specified



Density [kg/m^3]

11

Did we improve predictive skill?



- Fore-cone predictions are good but separation zone shortened; RMS errors increased
- Inlet BC are definitely only part of the cause



Test – Case 4

- LENS-XX; high enthalpy (21.77 MJ/kg); vibrational & chemical non-eq
- Bayesian method (w/ surrogates): Inferred (ρ, U) outside experimental error bounds
- Finding: Deterministic & Bayesian *agree*
 - Θ^* is close to $\Theta^{(MAP)}$ and far from $\Theta^{(Nom)}$
 - Gaussian posterior too wide

| | _(Nom) (uncertainty limits) | Θ* | (MAP) |
|----------|--|--------|--------|
| ρ [g/m3] | 0.9840 (0.9151, 1.0529) | 0.8619 | 0.8608 |
| U [m/s] | 6479 (6284.6, 6673.4) | 6950 | 7060.0 |







MCMC

20

25

15

--- Det.

Gaussian MAP

Cross-validation of Gaussian assumption

- The Gaussian posteriors are wrong and not useful
 - Discovered when we compared with MCMC posteriors
 - Test for Gaussian assumption?
- Cross-validation
 - Generate estimates (ρ, U)_k, k = 1 ...K using random subsets of the observations
 - Scatter in (ρ, U)_k wider than the (unknown) exact posterior
 - Because $(\rho, U)_k$ drawn on fewer data / information
 - "Upper bound" on true posterior / uncertainty
 - If scatter is narrower than Gaussian posterior, assumption not justified
- Our case (using Run 35): Not justified
 - Scatter commensurate with MCMC posterior
 - Scatter too narrow compared to Gaussian posterior





Conclusions

- Developed a deterministic inference method to check inflow BC for LENS-XX experiments
 - They have never been modeled successfully
 - Method uses the Navier-Stokes CFD solver, not surrogates
 - Also computed uncertainty bounds using a Gaussian assumption
- Findings:
 - Both LENS-XX experiments (low & high-enthalpy) have an inflow BC that is inconsistent with double-cone surface measurements
 - But correcting the BC does not fix the problem. There are other causes behind the model / experiment mismatch
 - Gaussian posterior is easy to calculate, but the Gaussian assumption may not be valid
 - Constructed cross-validation checks to test the validity of Gaussian assumption
 - Gaussian assumption inappropriate; corroborated with MCMC posterior distributions

