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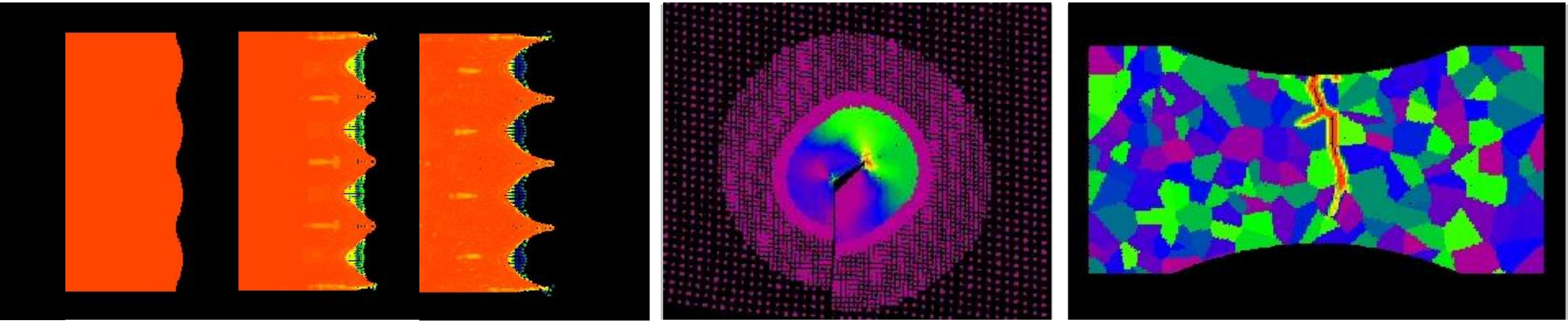

Variable length scale in a peridynamic body

Stewart Silling

Sandia National Laboratories

Pablo Seleson

University of Texas, Austin

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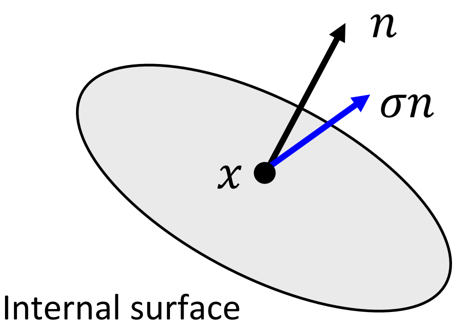
Outline

- Peridynamics background
  - States, horizon
- Rescaling a material model (at a point)
- Variable length scale (over a region)
- Partial stress
- Local-nonlocal coupling examples

Peridynamics basics: The nature of internal forces

Standard theory

Stress tensor field
(assumes contact forces and smooth deformation)

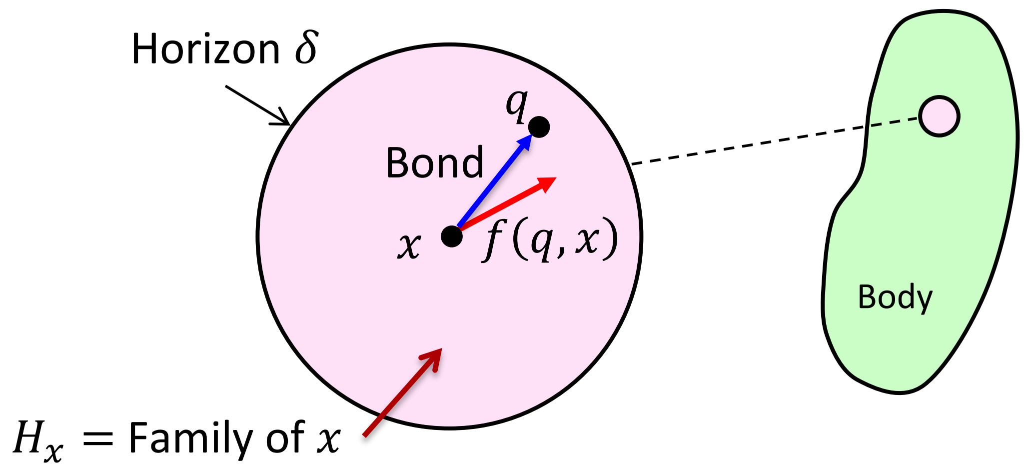


$$\rho \ddot{u}(x,t) = \nabla \cdot \sigma(x,t) + b(x,t)$$

Differentiation of contact forces

Peridynamics

Bond forces within small neighborhoods
(allow discontinuity)



$$\rho \ddot{u}(x,t) = \int_{H_x} f(q,x)\,dV_q + b(x,t)$$

Summation over bond forces

Peridynamics basics: Deformation state and force state

- The deformation state maps each bond to its deformed image.

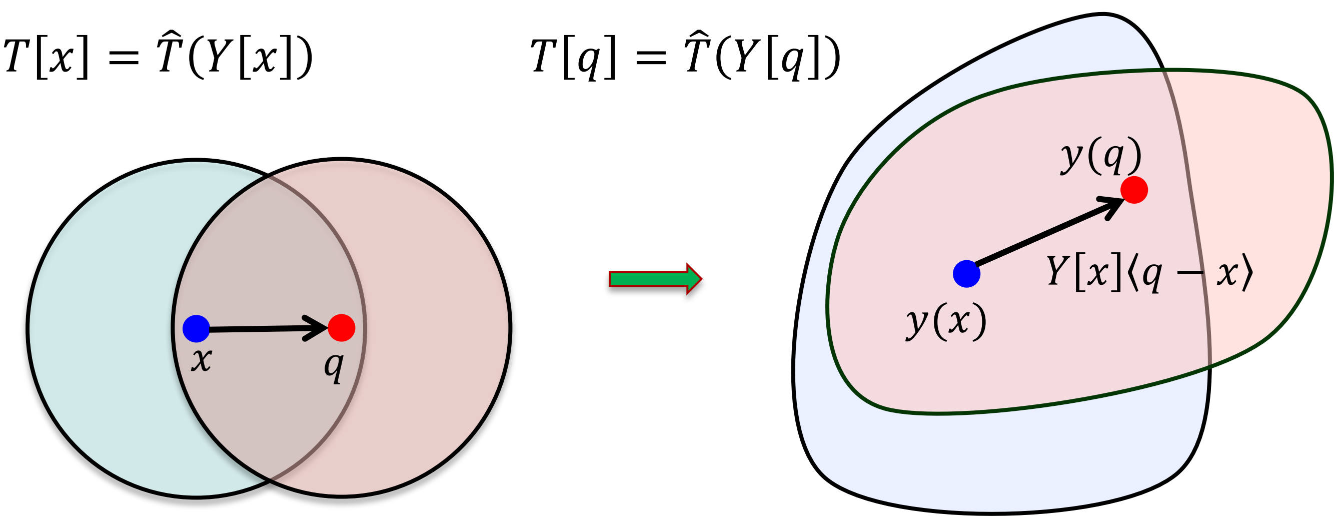
$$Y[x]\langle q - x\rangle = y(q) - y(x)$$

- The force state maps bonds to bond forces according to the constitutive model.

$$f(q,x) = T[x]\langle q - x\rangle - T[q]\langle x - q\rangle$$

- The constitutive model maps deformation states to force states.

$$T[x] = \hat{T}(Y[x]) \qquad T[q] = \hat{T}(Y[q])$$



Scaling of a material model at a point

- Let ϵ and δ be two horizons. Denote by ξ_ϵ and ξ_δ bonds within each family.
- Suppose we have a material model with horizon ϵ. Find a rescaled model with δ.
- Map the bonds (undeformed and deformed):

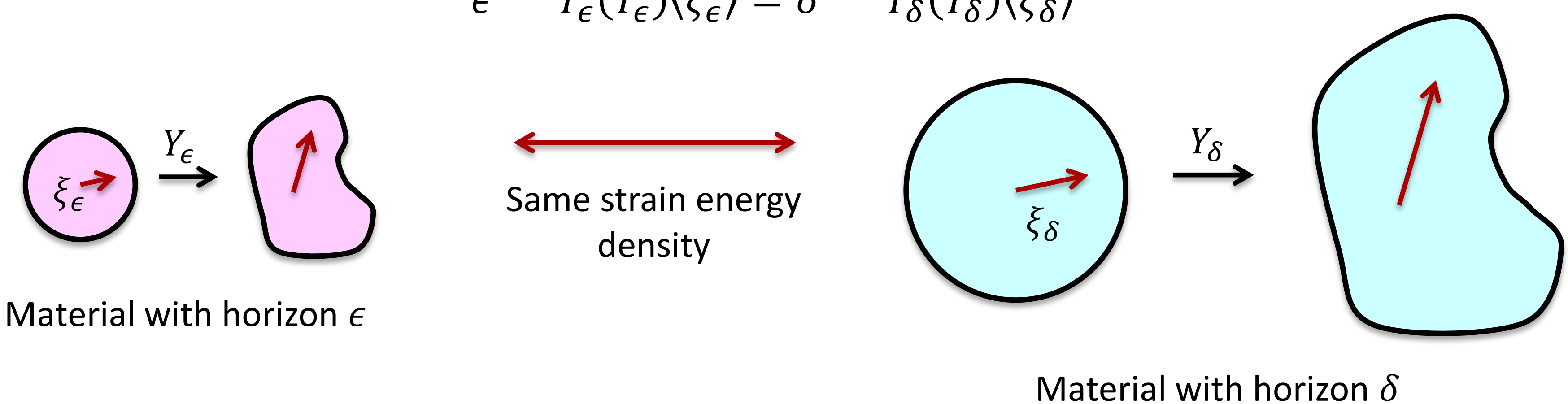
$$\frac{\xi_\epsilon}{\epsilon} = \frac{\xi_\delta}{\delta}, \qquad \frac{Y_\epsilon\langle\xi_\epsilon\rangle}{\epsilon} = \frac{Y_\delta\langle\xi_\delta\rangle}{\delta}$$

- Require

$$W_\epsilon(Y_\epsilon) = W_\delta(Y_\delta)$$

- It follows from definition of Frechet derivative that the force state scales according to

$$\epsilon^{d+1} T_\epsilon(Y_\epsilon)\langle\xi_\epsilon\rangle = \delta^{d+1} T_\delta(Y_\delta)\langle\xi_\delta\rangle$$



Rescaling works fine if the horizon is independent of position

- Example: uniform strain in a 1D homogeneous bar ($d = 1,\ F =$ constant):

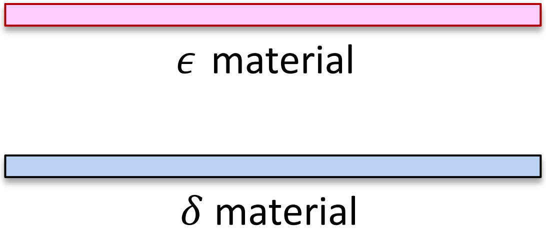
$$y = Fx$$

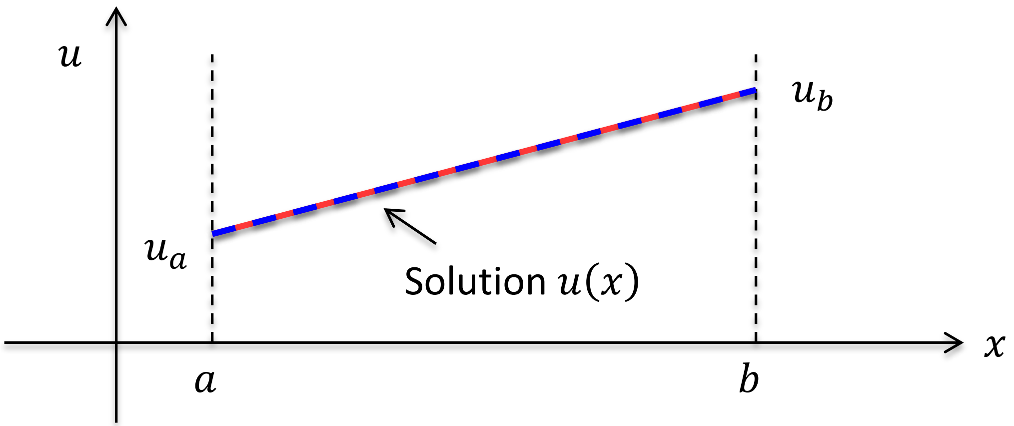
- If we scale the material model as derived above:

$$\epsilon^2 T_\epsilon(F)\langle\xi_\epsilon\rangle = \delta^2 T_\delta(F)\langle\xi_\delta\rangle$$

we are assured that the strain energy density and Young's modulus are independent of horizon.

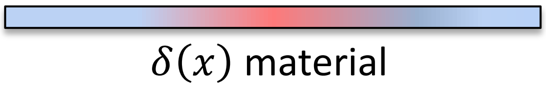
- Also the peridynamic equilibrium equation is satisfied.

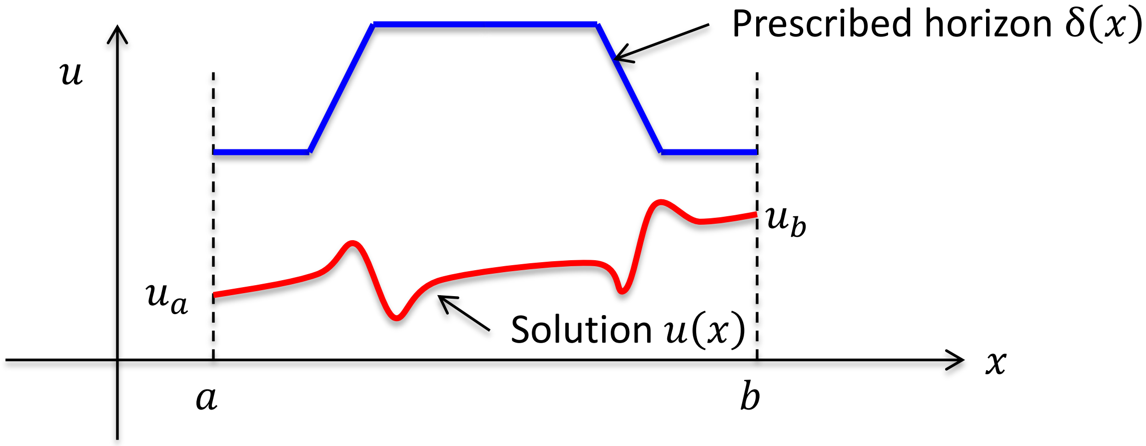




Variable horizon: the problem

- Same example: uniform strain in a 1D homogeneous bar
$$y = Fx$$
- Set $\epsilon = 1$, define $Z(F) = T_1(F)$.
- Let the horizon be given by $\delta(x)$. The scaled force state is
$$T[x]\langle\xi\rangle = \delta^{-2}(x) Z \left\langle \frac{\xi}{\delta(x)} \right\rangle$$
- From the previous discussion, we know W is independent of x.
- There's just one problem: this deformation isn't a minimizer of energy.
  - That is, the uniform strain deformation is not in equilibrium.



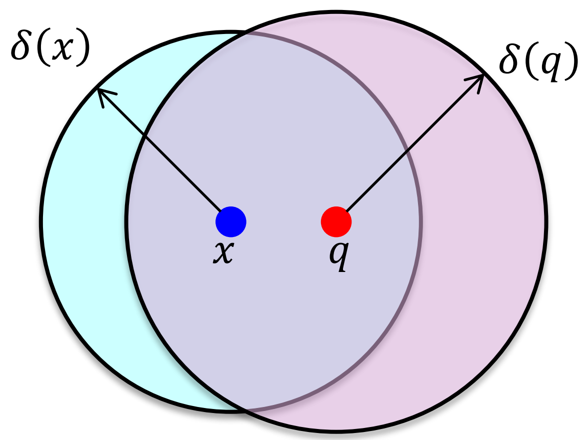


Origin of artifacts

- The peridynamic force density operator $L(x)$ involves the force state not only at x but also the force states at all points within the horizon.

$$0 = L(x) + b, \qquad L(x) = \int_{-\infty}^{\infty} \{T_{\delta(x)}[x]\langle q - x\rangle - T_{\delta(q)}[q]\langle x - q\rangle\} dq$$

so simply scaling the material model at x is not sufficient.



Variable horizon

“Patch test” requirement for a coupling method

- In a deformation of the form

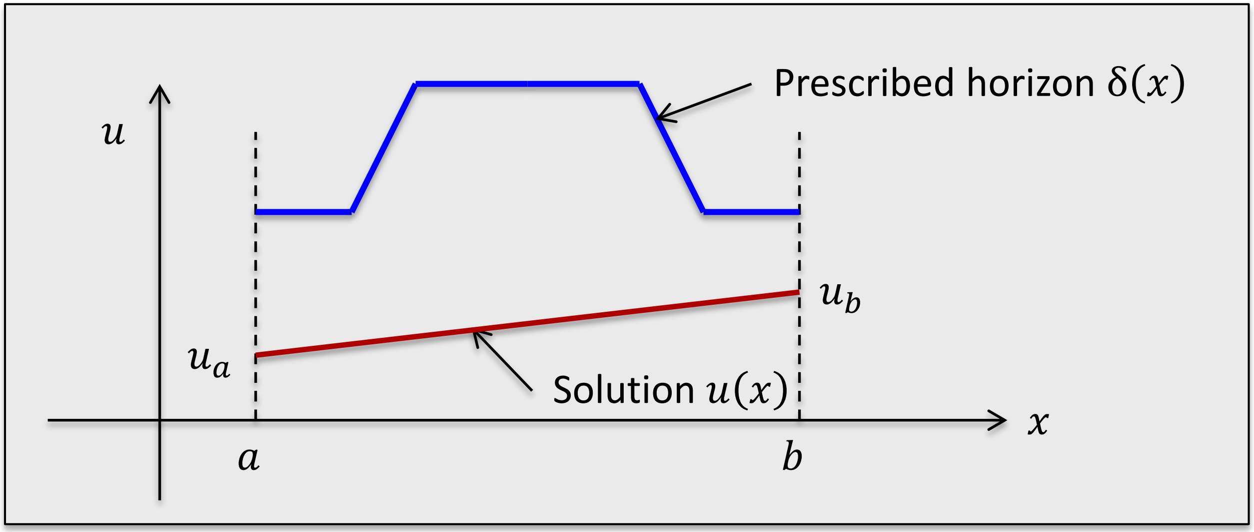
$$y(x) = a + Fx$$

where H is a constant and the material model is of the form

$$T[x]\langle\xi\rangle = \delta^{-2}(x) Z\langle\xi/\delta(x)\rangle$$

where $\delta(x)$ is a prescribed function and Z is a state that depends only on F, we require

$$L(x) = 0 \quad \text{for all } x.$$



Peridynamic stress tensor

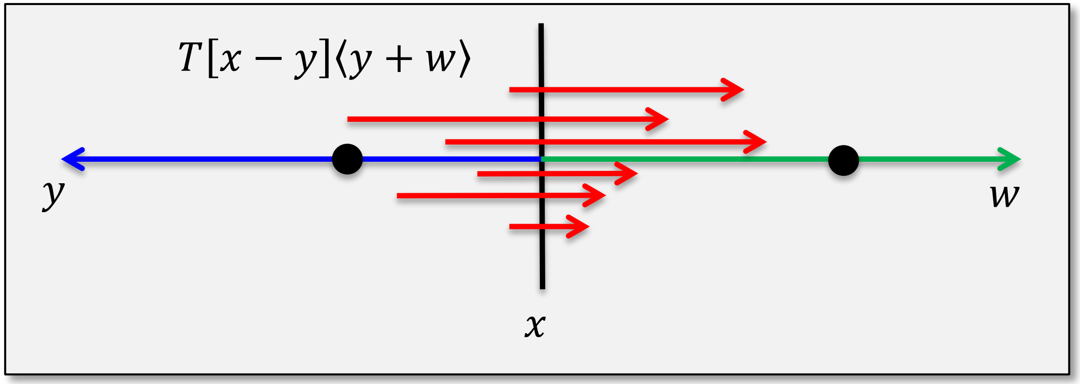
- Define the 1D peridynamic stress tensor field* by

$$v(x) = \int_0^\infty \int_0^\infty \{T[x-y]\langle y+w\rangle - T[x+y]\langle -y-w\rangle\}\, dy\, dz$$

- Identity:

$$\frac{dv}{dx} = \int_{-\infty}^{\infty} \{T[x]\langle q-x\rangle - T[q]\langle x-q\rangle\} dq$$

- $v(x)$ is the force per unit area carried by all the bonds that cross x.



*R. B. Lehoucq & SS, "Force flux and the peridynamic stress tensor," JMPS (2008)

Partial stress field

- Under our assumption that

$$T[x]\langle\xi\rangle = \delta^{-2}(x) Z\langle \xi/\delta(x)\rangle$$

one computes directly that

$$v_0(x) := \int_{-\infty}^{\infty} \xi T[x]\langle\xi\rangle \, d\xi = \int_{-\infty}^{\infty} \xi Z\langle\xi\rangle \, d\xi$$

which is independent of x, so $dv_0/dx = 0$.

- v_0 is called the ***partial stress*** field.
- Clearly the internal force density field computed from

$$L_0(x) \coloneqq dv_0/dx$$

passes the "patch test."

- This observation leads to the following idea…

Concept for coupling method

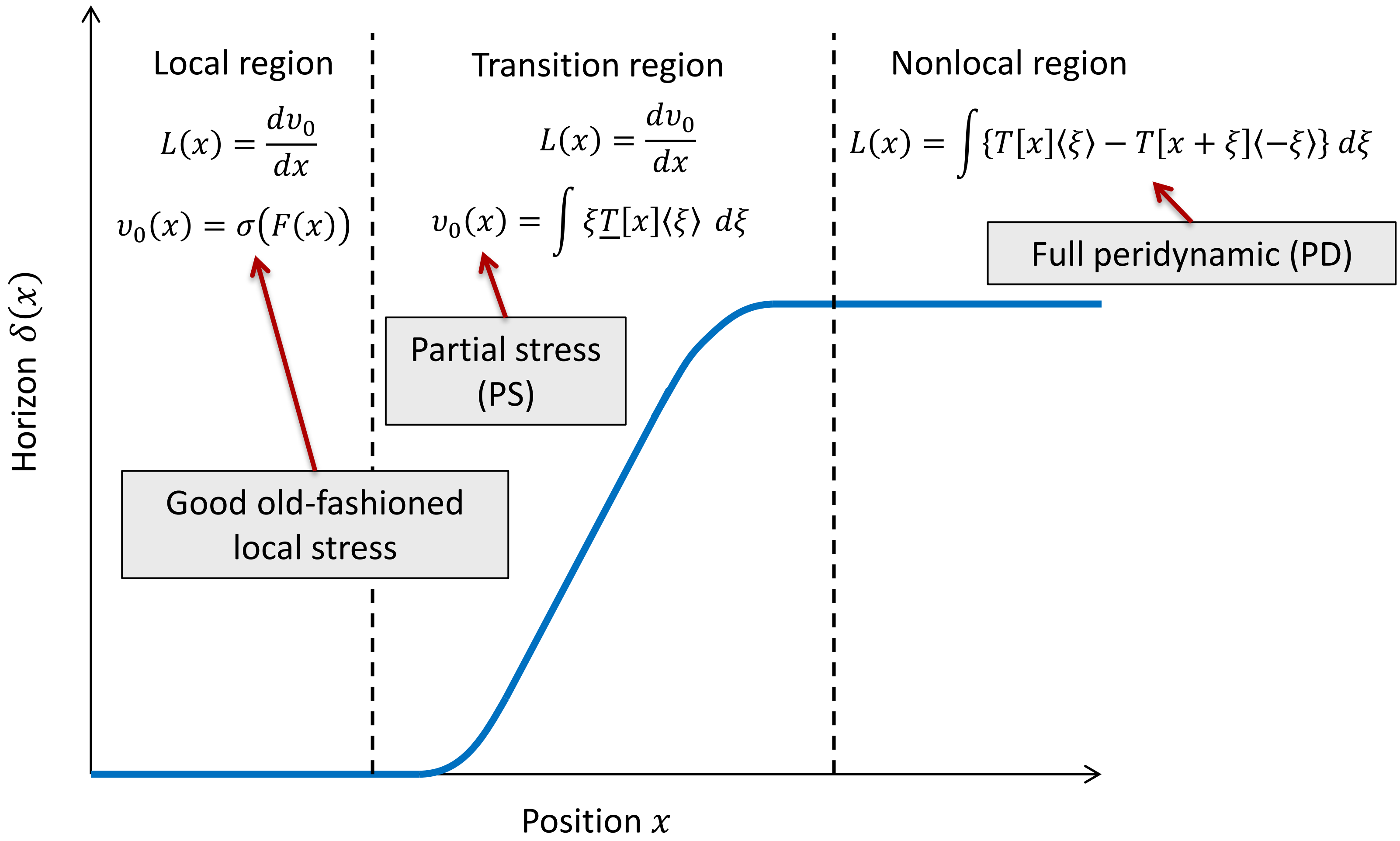
- Idea: within a coupling region in which δ is changing, compute the internal force density from

$$L(x) = \frac{dv_0}{dx}(x)\,, \qquad\qquad v_0(x) := \int_{-\infty}^{\infty} \xi T[x]\langle\xi\rangle \, d\xi$$

instead of the full PD nonlocal integral.

- Here, $T[x](x)$ is determined from whatever the deformation happens to be near x.
  - Z is no longer involved.
  - The material model has not changed from full PD, but the way of computing L has.

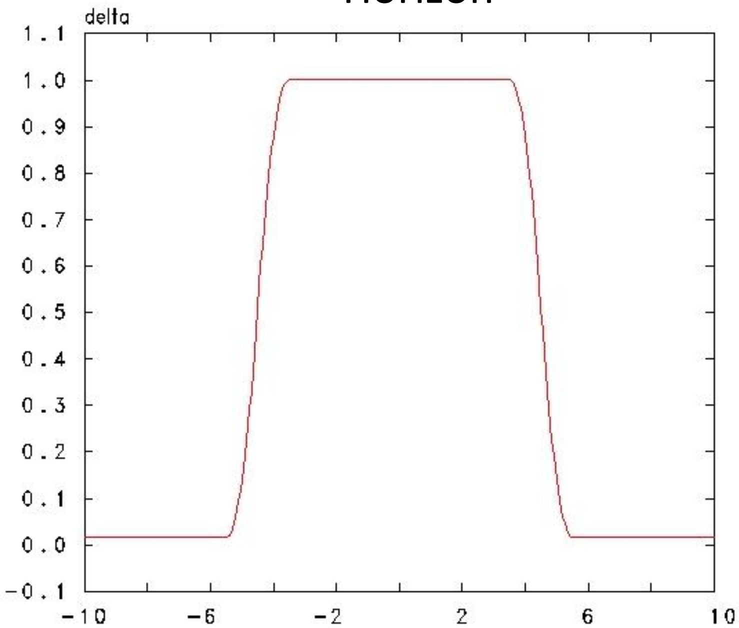
Local-nonlocal coupling idea



Continuum patch test results

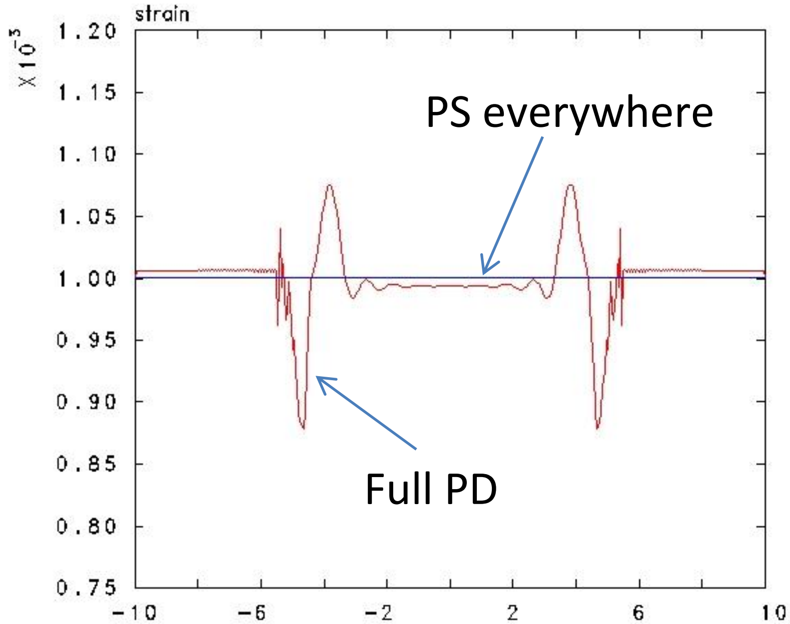
- Full PD shows artifacts, as expected.
- PS shows no artifacts, as promised.

Horizon

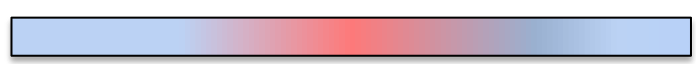


x

Strain



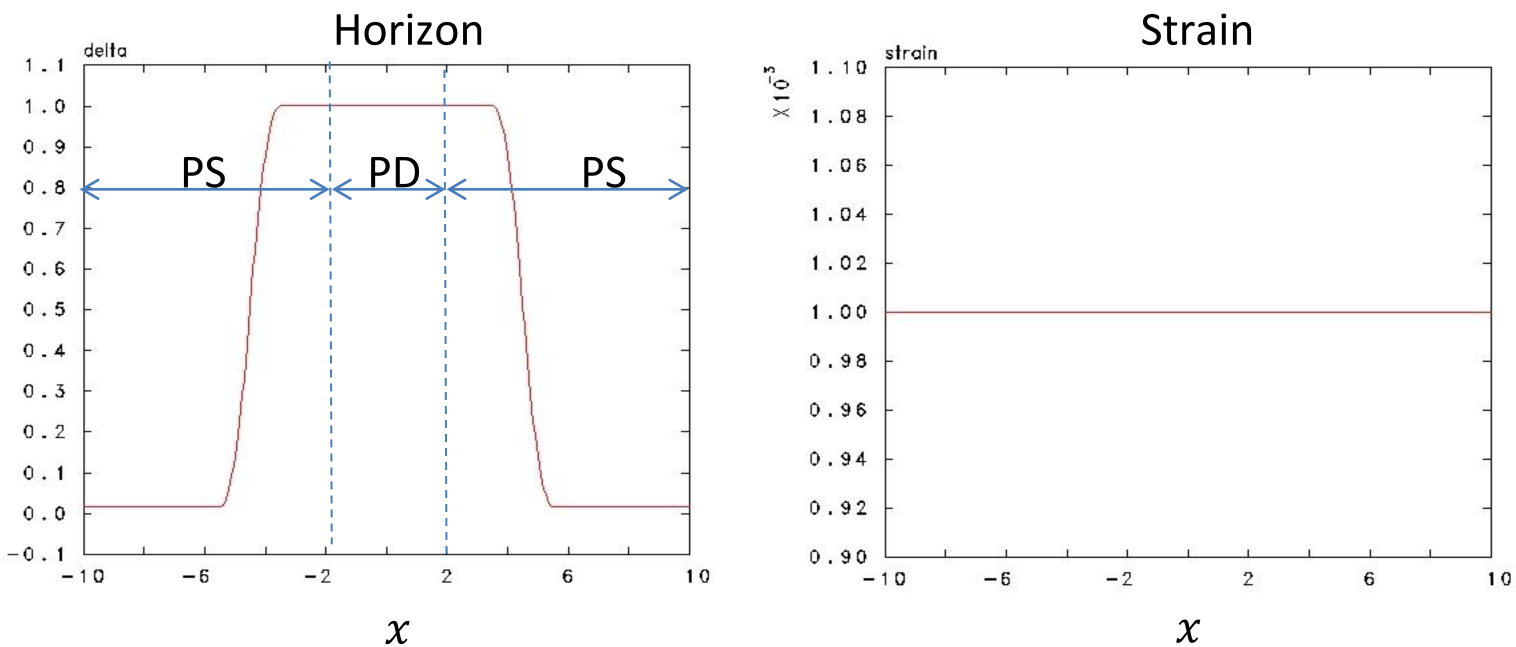
x

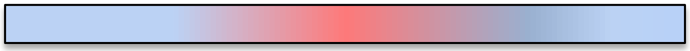


$u = 0$ $u = 0.02$

Continuum patch test with coupling

- No artifacts with PD-PS coupling (this was hoped for but not guaranteed).

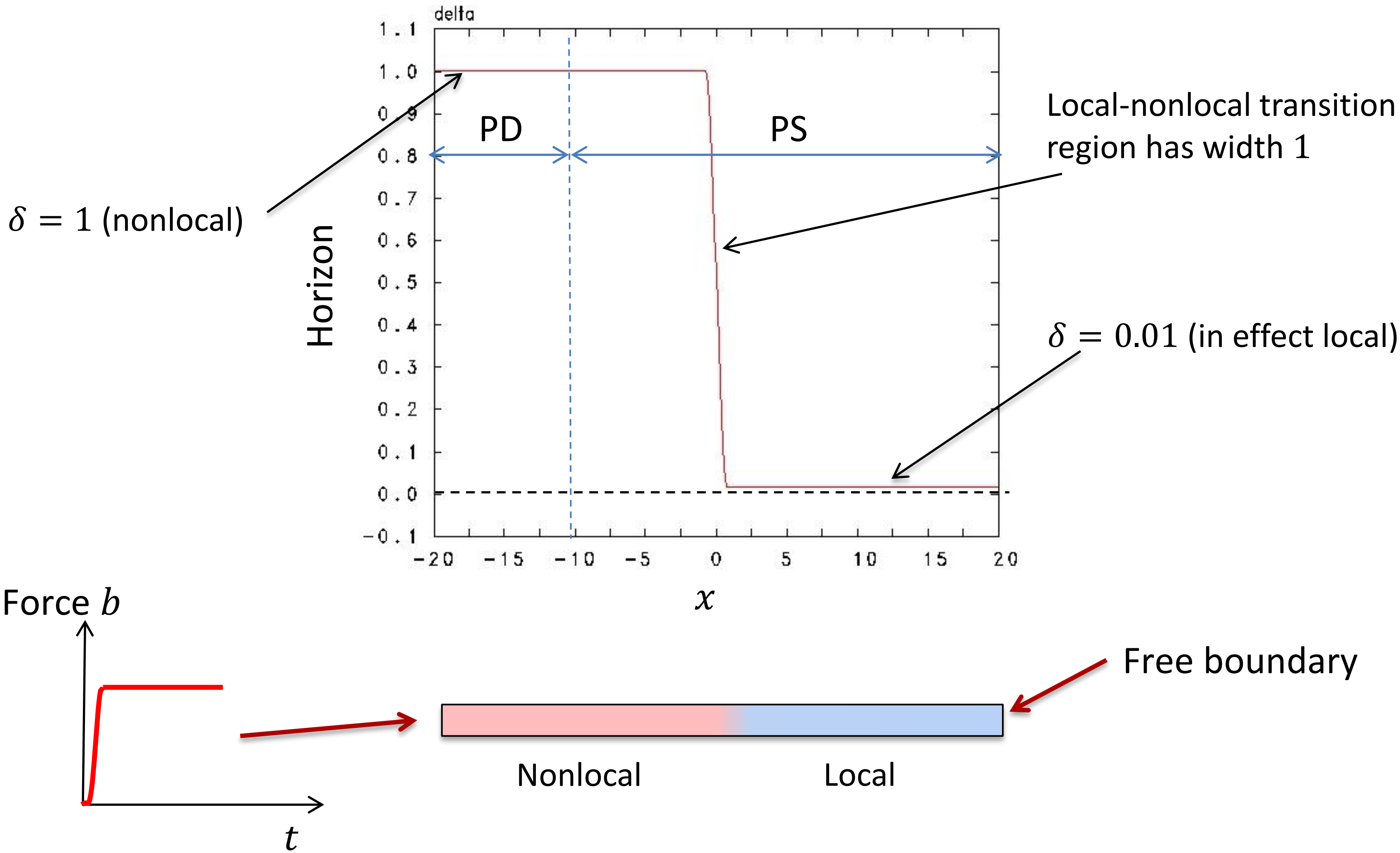




$u = 0$ $u = 0.02$

Pulse propagation test problem

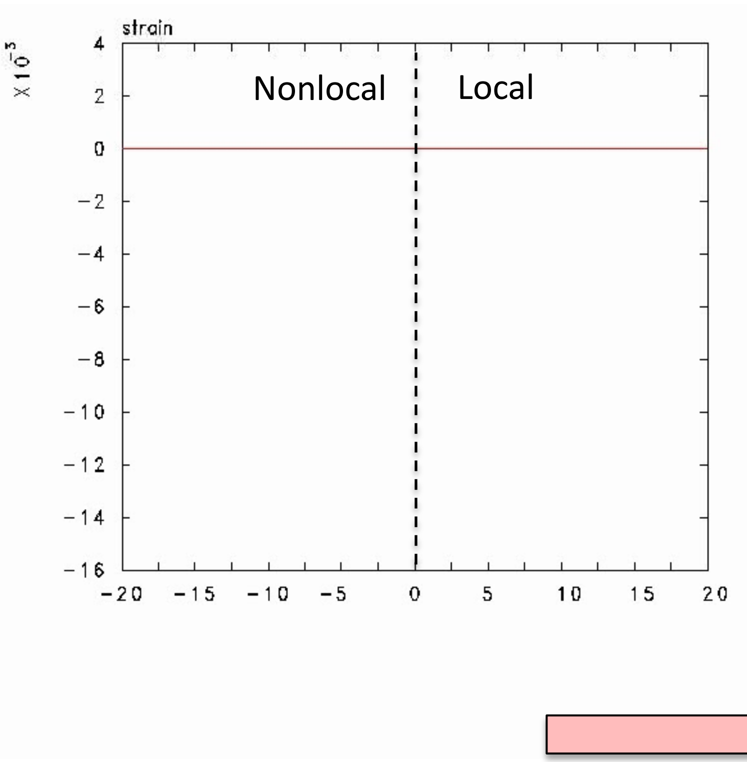
- Does our coupling method work for dynamics as well as statics with variable horizon?



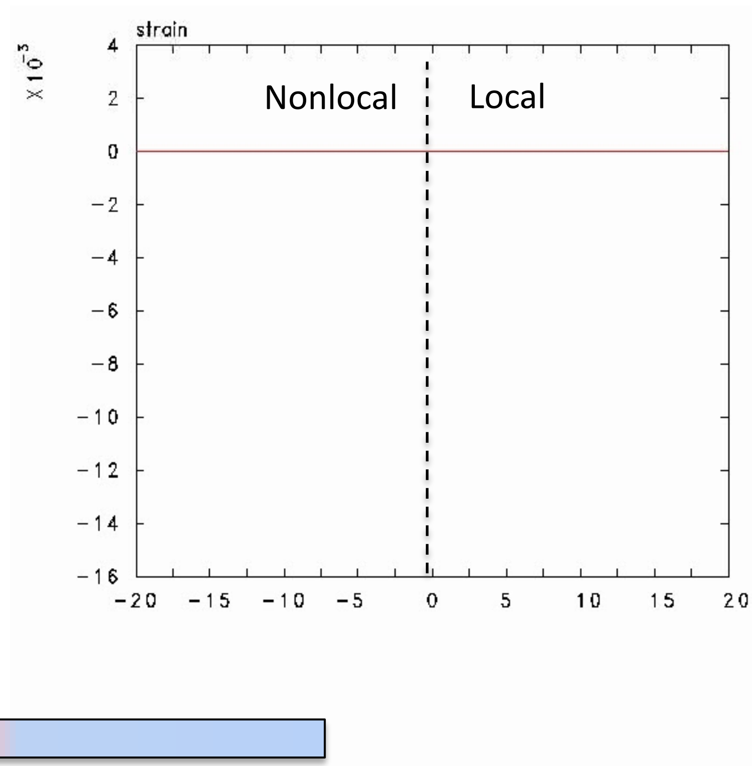
Pulse propagation test results

- Movies of strain field evolution

Full PD everywhere



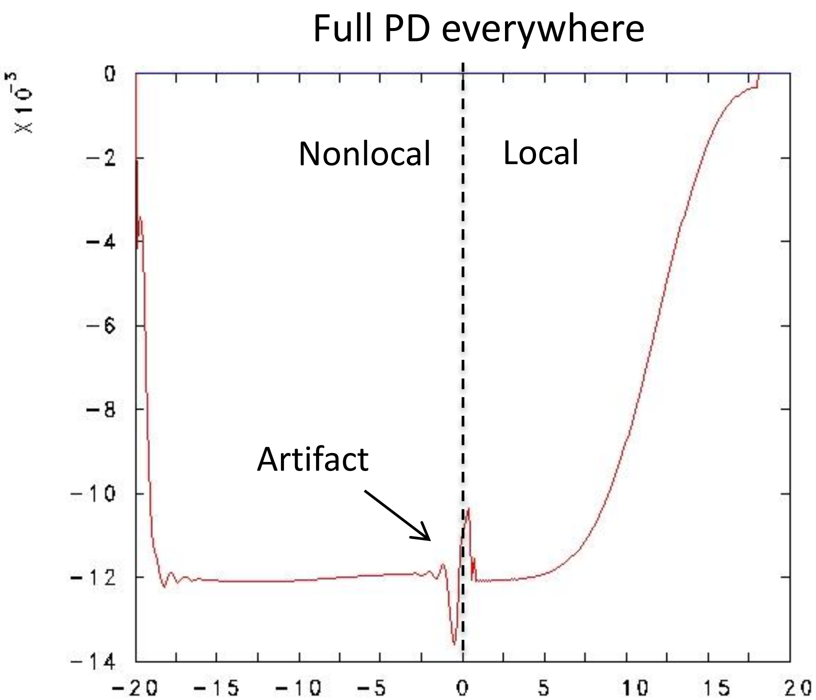
Coupled PD-PS

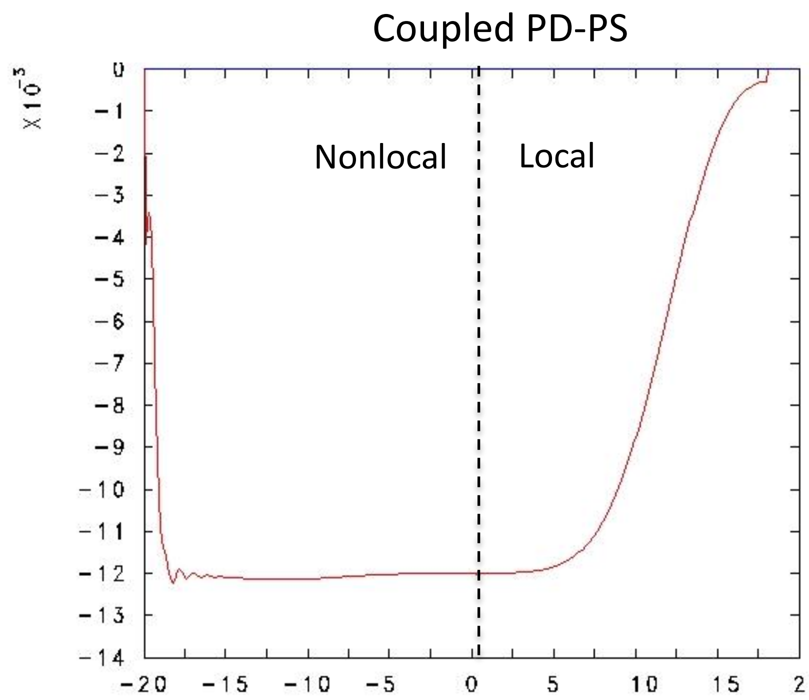


Nonlocal Local

Pulse propagation test results

- Strain field: no artifacts appear in the coupled model the local-nonlocal transition.





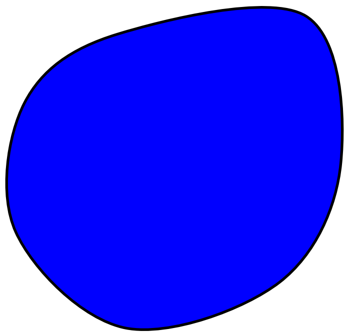
Discussion

- The partial stress approach may provide a means for local-nonlocal coupling within the continuum equations.
    - Uses the underlying peridynamic material model but modifies the way internal force density is computed.
    - Expected to work in 2D & 3D, linear & nonlinear.
- PS is inconsistent from an energy minimization point of view.
    - Not suitable for a full-blown theory of mechanics and thermodynamics (as full PD is).
    - Not yet clear what implications this may have in practice.
    - We still need to use full PD for crack progression.

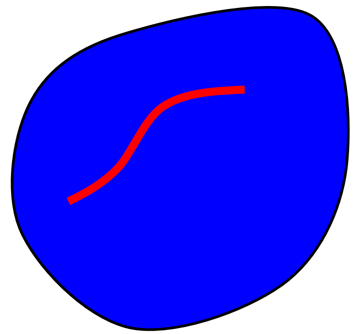
Extra slides

Purpose of peridynamics

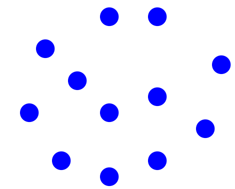
- To unify the mechanics of continuous and discontinuous media within a single, consistent set of equations.



Continuous body



Continuous body with a defect



Discrete particles

- Why do this? Develop a mathematical framework that help in modeling…
  - Discrete-to-continuum coupling
  - Cracking, including complex fracture patterns
  - Communication across length scales.

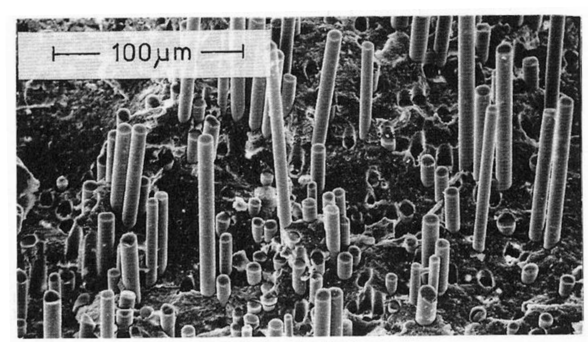


Figure 11.20 Pull-out: (a) schematic diagram; (b) fracture surface of 'Silceram' glass-ceramic reinforced with SiC fibres. (Courtesy H. S. Kim, P. S. Rogers and R. D. Rawlings.)

Peridynamic vs. local equations

State notation: $\underline{\text{State}}\langle\text{bond}\rangle = \text{vector}$

Relation	*Peridynamic theory*	*Standard theory*
Kinematics	$\underline{\mathbf{Y}}\langle\mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x})$	$\mathbf{F}(\mathbf{x}) = \dfrac{\partial \mathbf{y}}{\partial \mathbf{x}}(\mathbf{x})$
Linear momentum balance	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \int_{\mathcal{H}} \Big(\mathbf{t}(\mathbf{q},\mathbf{x}) - \mathbf{t}(\mathbf{x},\mathbf{q})\Big)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x})$	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \nabla\cdot\boldsymbol{\sigma}(\mathbf{x}) + \mathbf{b}(\mathbf{x})$
Constitutive model	$\mathbf{t}(\mathbf{q},\mathbf{x}) = \underline{\mathbf{T}}\langle\mathbf{q}-\mathbf{x}\rangle, \qquad \underline{\mathbf{T}} = \underline{\hat{\mathbf{T}}}(\underline{\mathbf{Y}})$	$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\mathbf{F})$
Angular momentum balance	$\int_{\mathcal{H}} \underline{\mathbf{Y}}\langle\mathbf{q}-\mathbf{x}\rangle \times \underline{\mathbf{T}}\langle\mathbf{q}-\mathbf{x}\rangle\, dV_{\mathbf{q}} = \mathbf{0}$	$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$
Elasticity	$\underline{\mathbf{T}} = W_{\underline{\mathbf{Y}}}$ (Fréchet derivative)	$\boldsymbol{\sigma} = W_{\mathbf{F}}$ (tensor gradient)
First law	$\dot{\varepsilon} = \underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} + q + r$	$\dot{\varepsilon} = \boldsymbol{\sigma}\cdot\dot{\mathbf{F}} + q + r$

$$\underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} := \int_{\mathcal{H}} \underline{\mathbf{T}}\langle\boldsymbol{\xi}\rangle \cdot \dot{\underline{\mathbf{Y}}}\langle\boldsymbol{\xi}\rangle\, dV_{\boldsymbol{\xi}}$$