Intro to Mesh Generation

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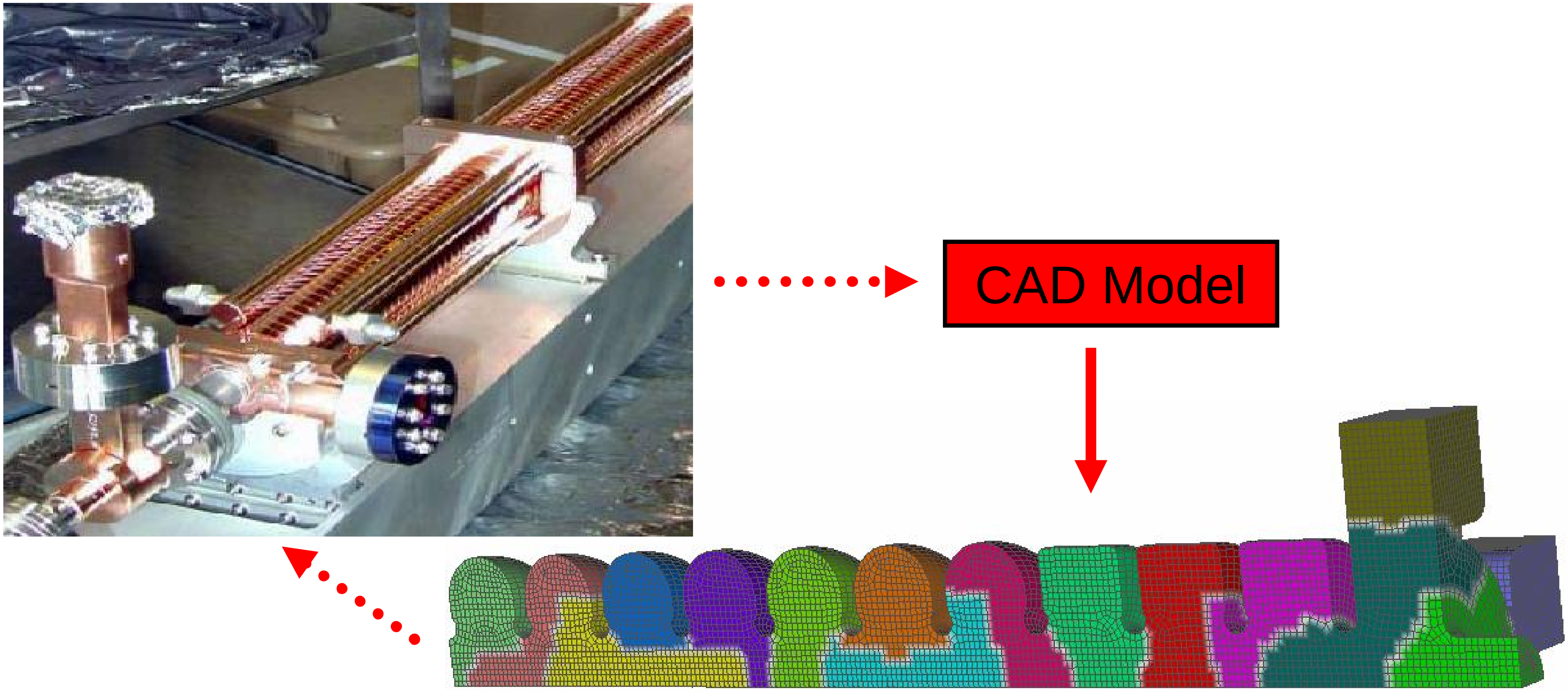
April 20, 2005

Overview

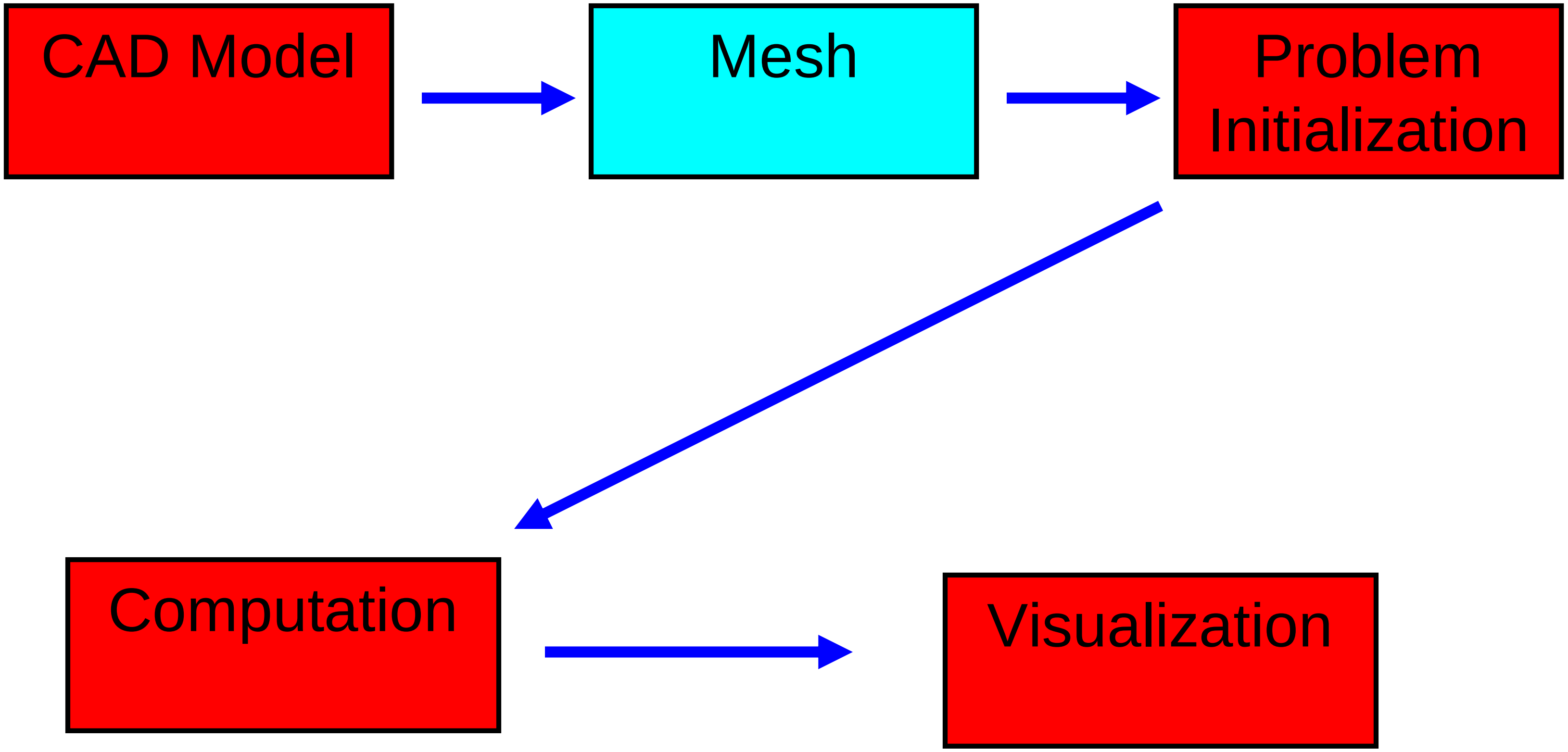
- Introduction to Mesh Generation
- Mesh Quality
- Serial Meshing Methods
  - Quadtree/Octree
  - Advancing Front
  - Delaunay
- Parallel Mesh Generation
  - Why Parallel?
  - Categorization Parallel methods
  - Subdomains, interfaces, separators
- CSAR Mesh Repair in Rocket Simulation

Introduction to Meshing

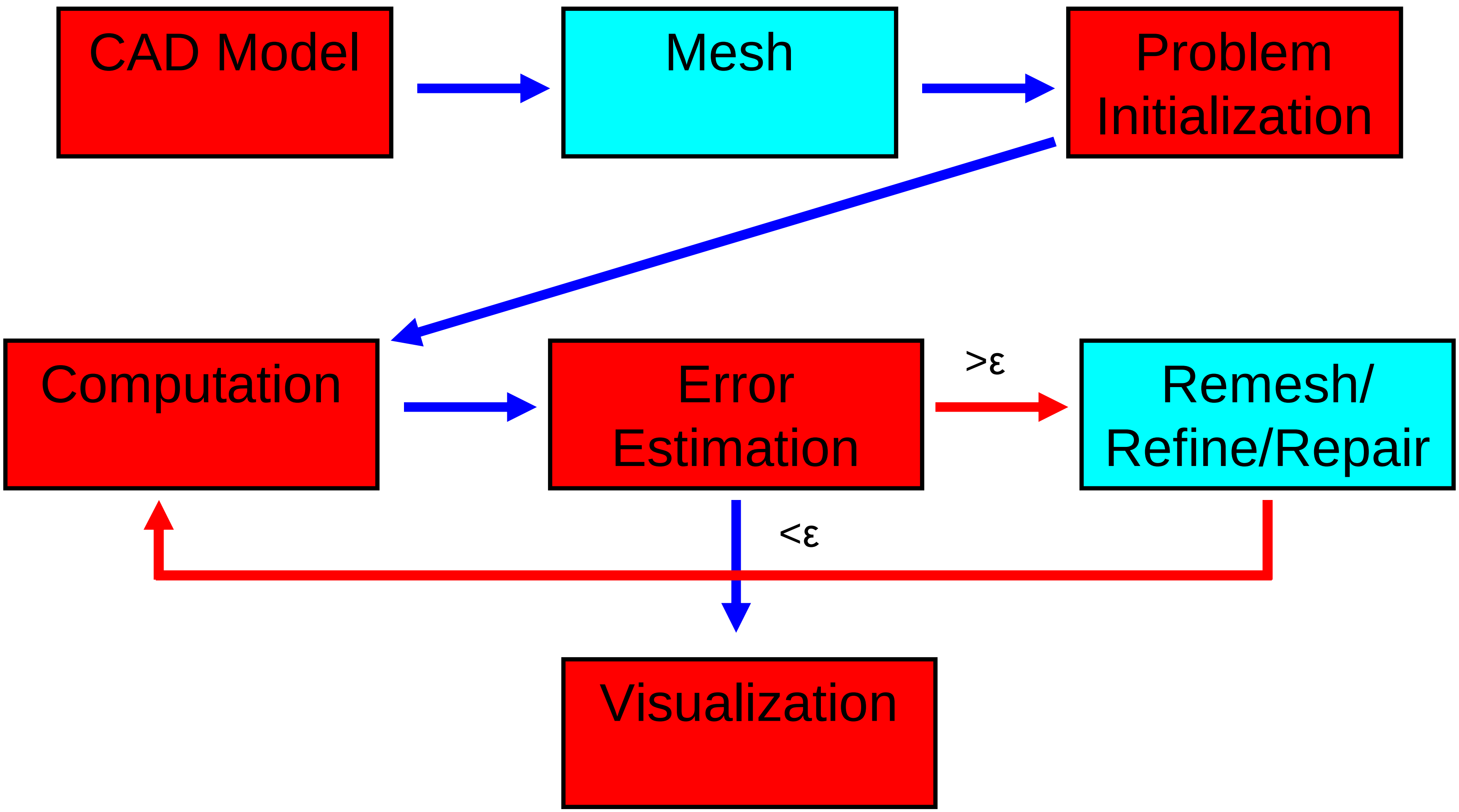


- CAD (Continuous Model)
- Mesh (Discrete Model)
  - Domain on which to compute

Simulation Process

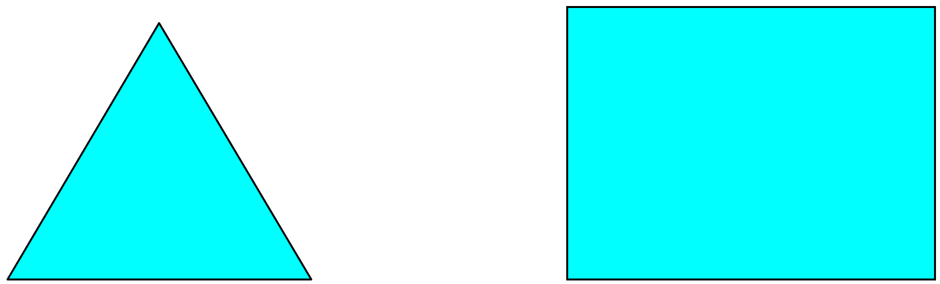


Adaptive Simulation Process

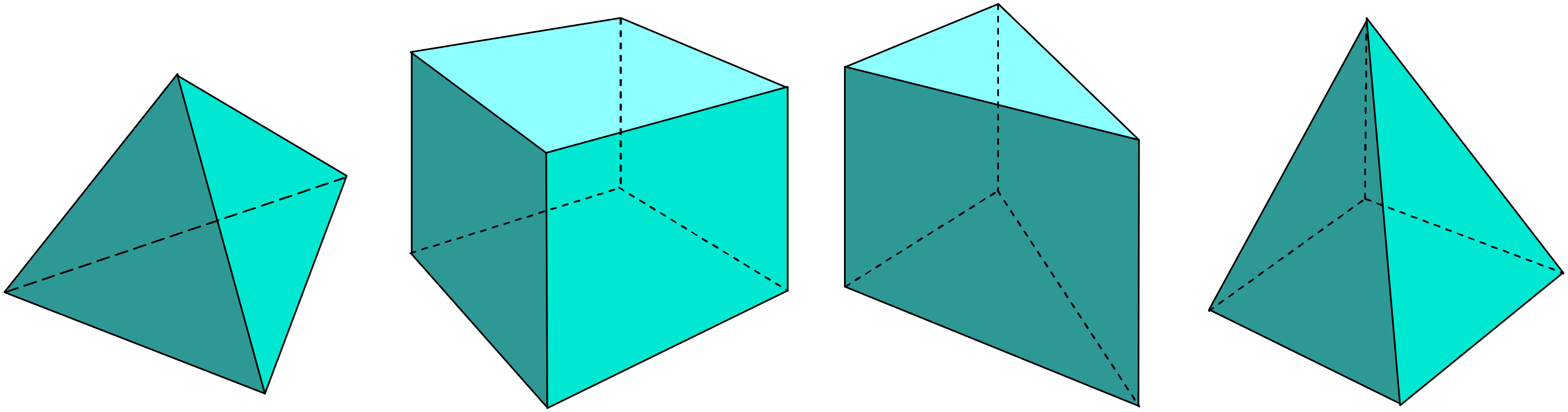


Types of Meshes: Typical Element Types

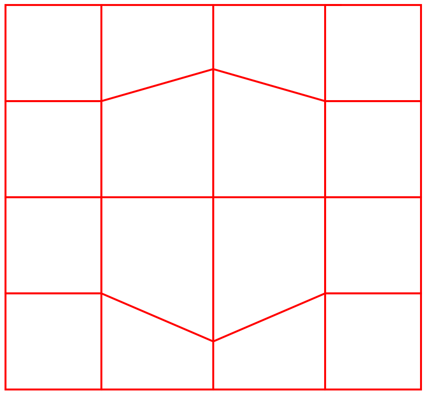
- 2D
  - Triangles, Quadrilaterals



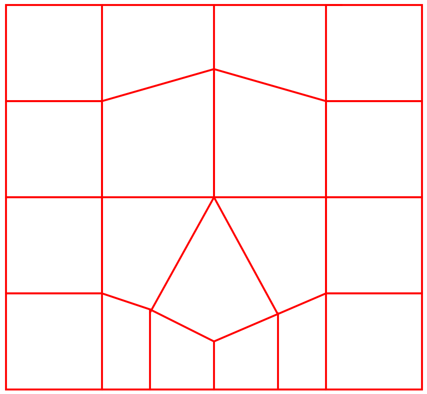
- 3D
  - Tetrahedra, Hexahedra, Prisms, Pyramids



Types of Meshes: Regular vs Irregular



Regular



Irregular

- Regular (Structured)
  - Interior nodes attached to same number of elements
- Irregular (Unstructured)
  - Interior nodes attached to variable number of elements

Mesh Quality

- Poor quality elements often yield poor solutions
- Usually regular tetrahedron (4 equilateral faces) is prototypic good element
- How to quantify “Good” element
  - Dihedral angles
  - Volume
  - Skew
  - Algebraic means
  - Etc.

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  - Quadtree/Octree
  - Advancing Front
  - Delaunay
- Parallel Mesh Generation
  - Why Parallel?
  - Categorization Parallel methods
  - Subdomains, interfaces, separators
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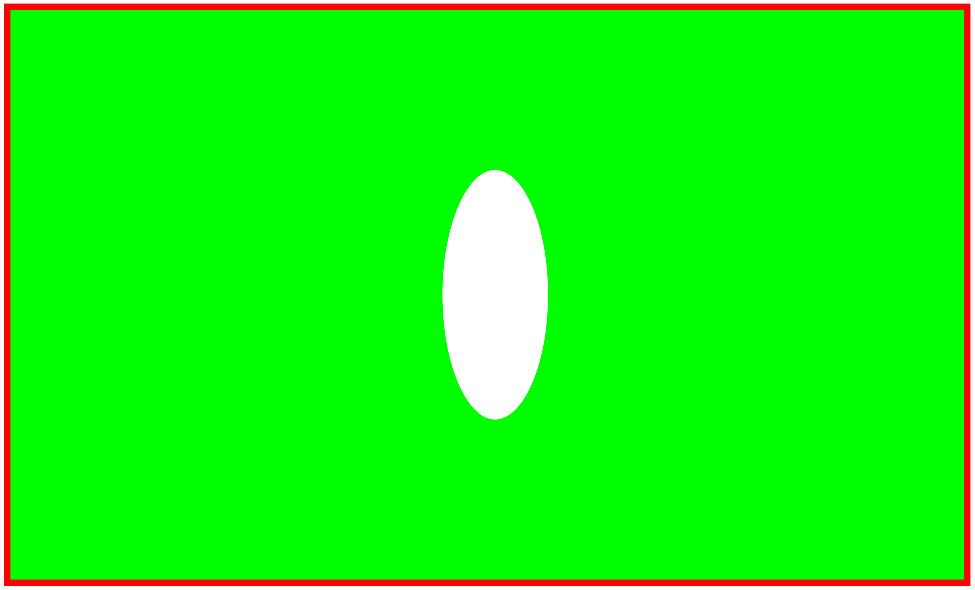
Serial Meshing Methods

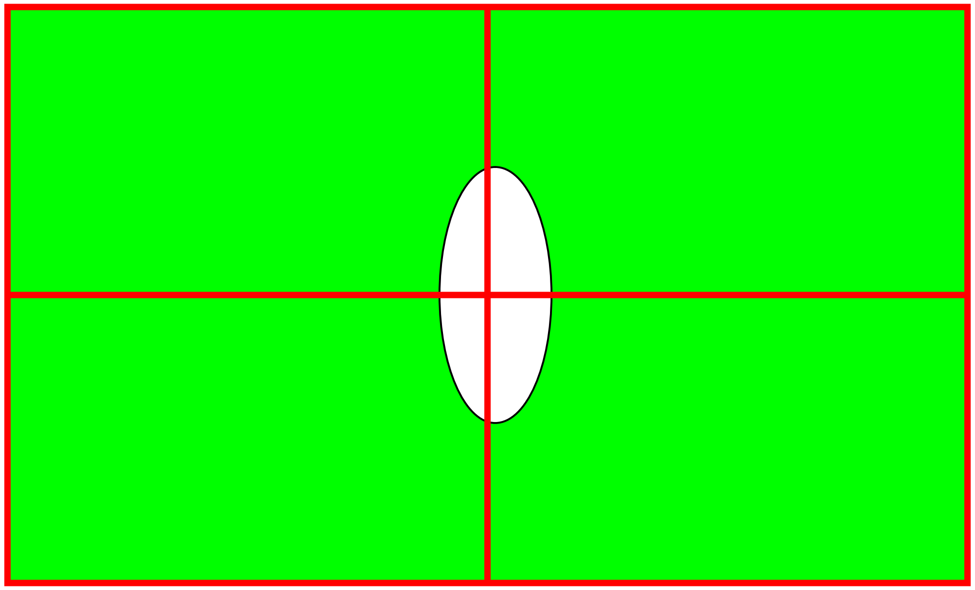
- Going to present 2D versions of methods but 3D equivalents are similar
- Focus on Triangle methods but there are numerous interesting Quad methods

Quadtree/Octree



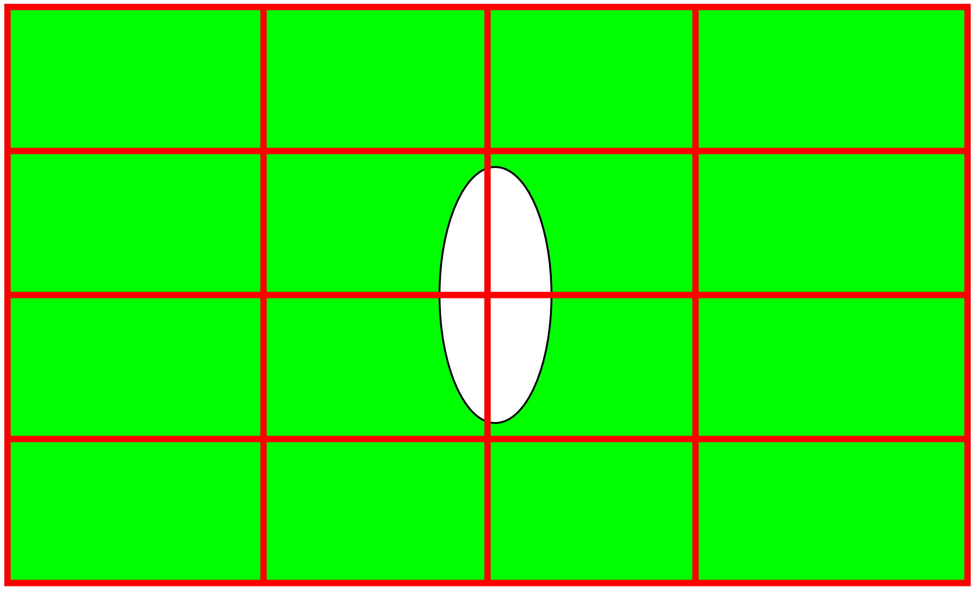
- Setup Bounding Box

Quadtree/Octree



- Recursively Build Quadtree to resolve geometry

Quadtree/Octree



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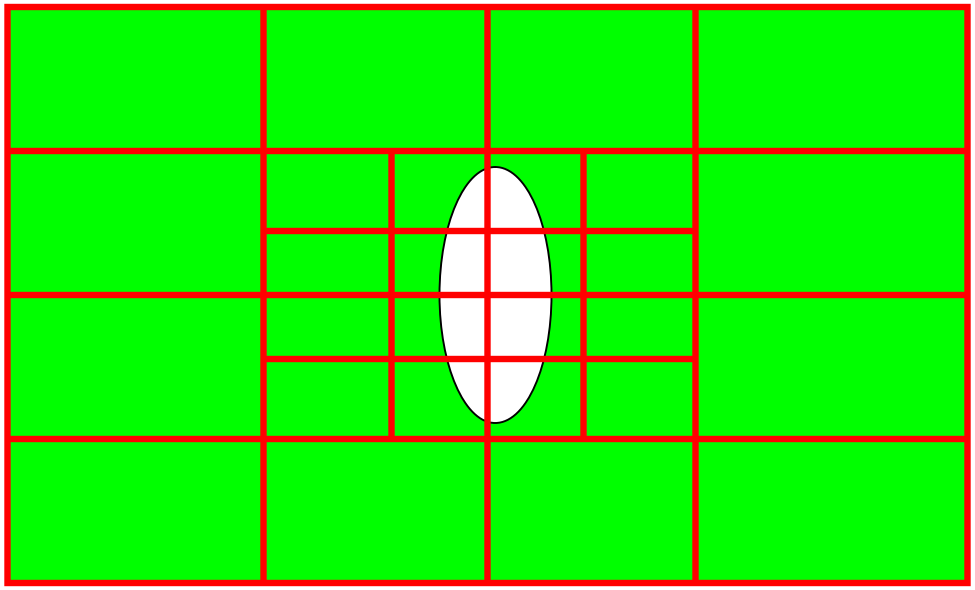
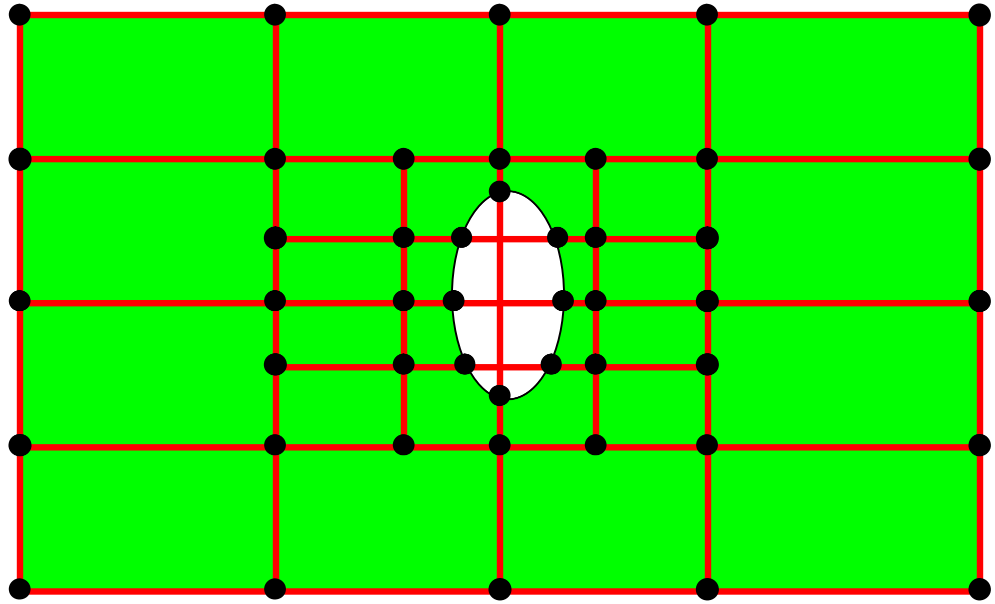
Quadtree/Octree



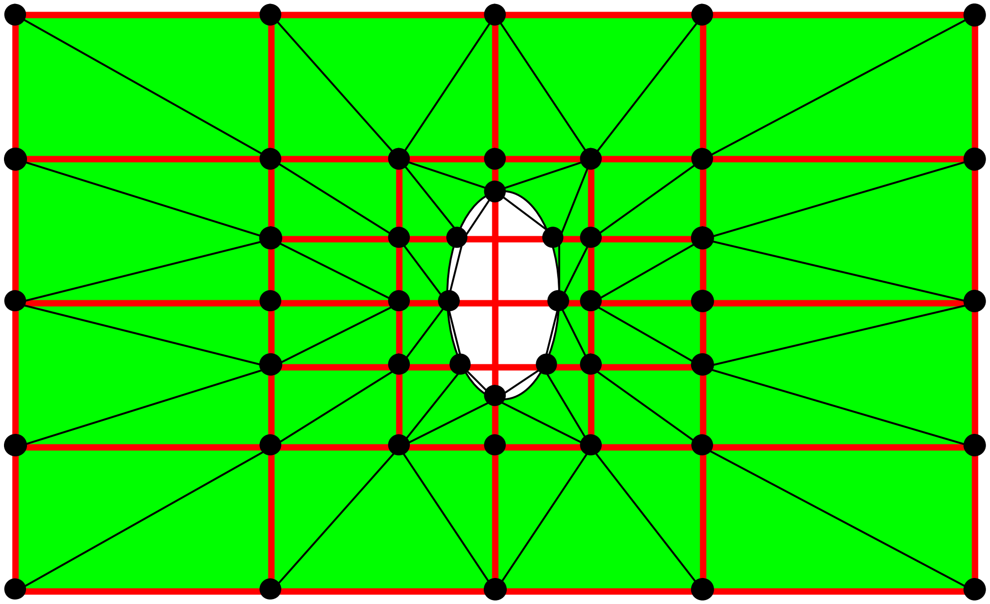
- Recursively Build Quadtree to resolve geometry

Quadtree/Octree



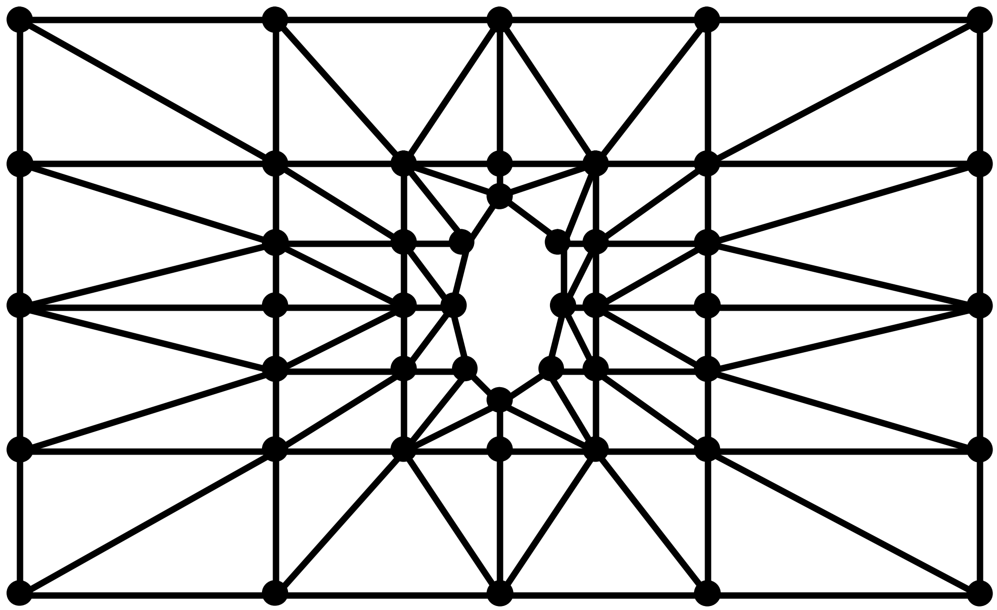
- Add nodes to:
  - Intersection of 2 quadtree lines
  - Intersection of boundary and quadtree line
- Remove nodes not outside boundary

Quadtree/Octree



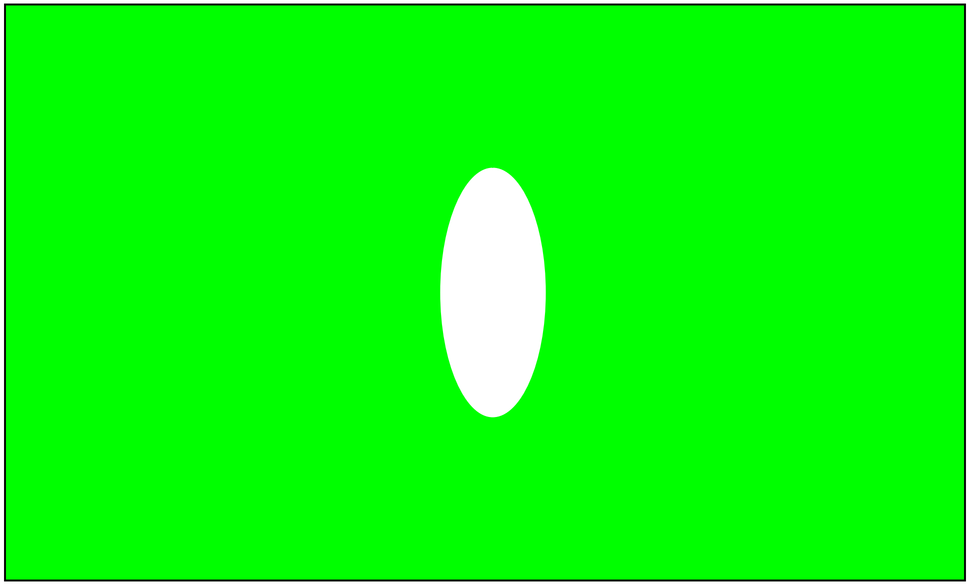
- Mesh Structure using nodes with triangles

Quadtree/Octree

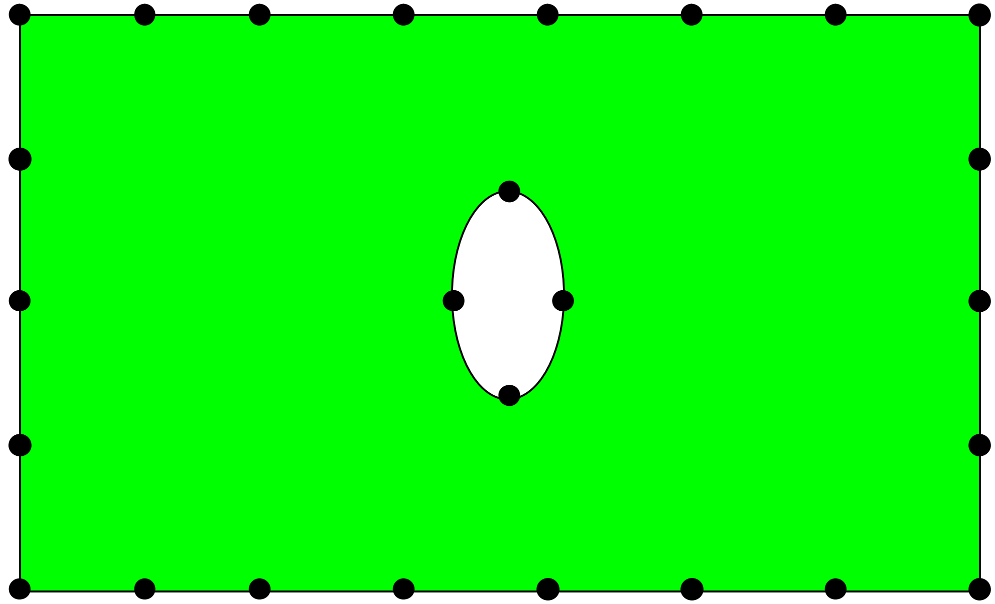


- Final Mesh

Advancing Front

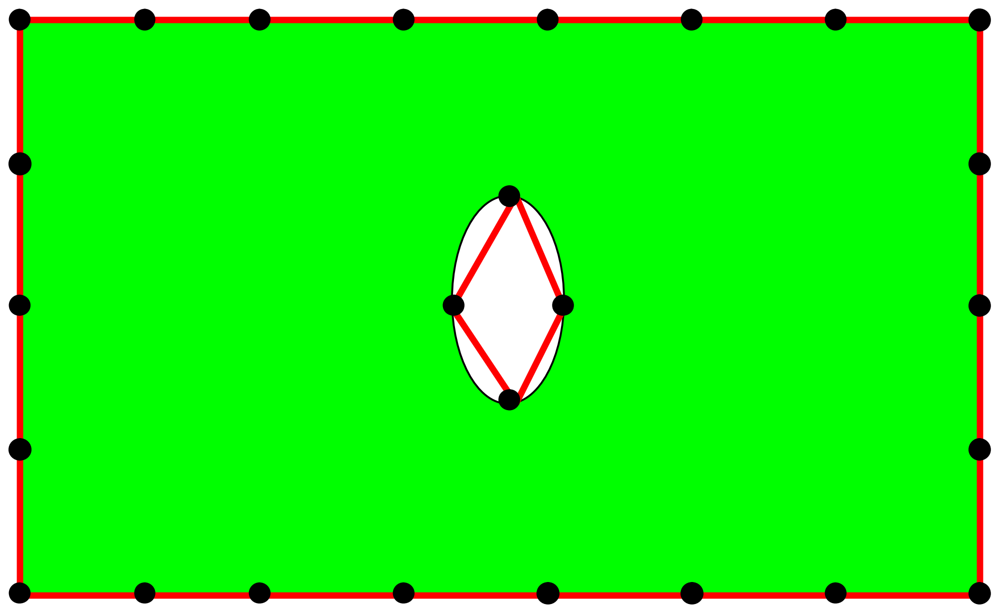


Advancing Front



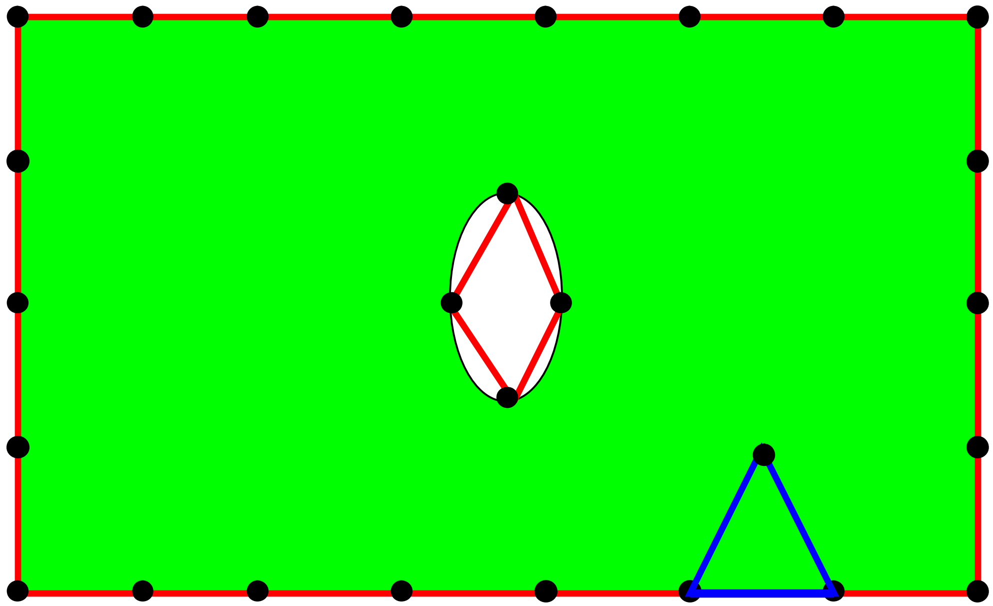
- Place nodes around boundary.

Advancing Front



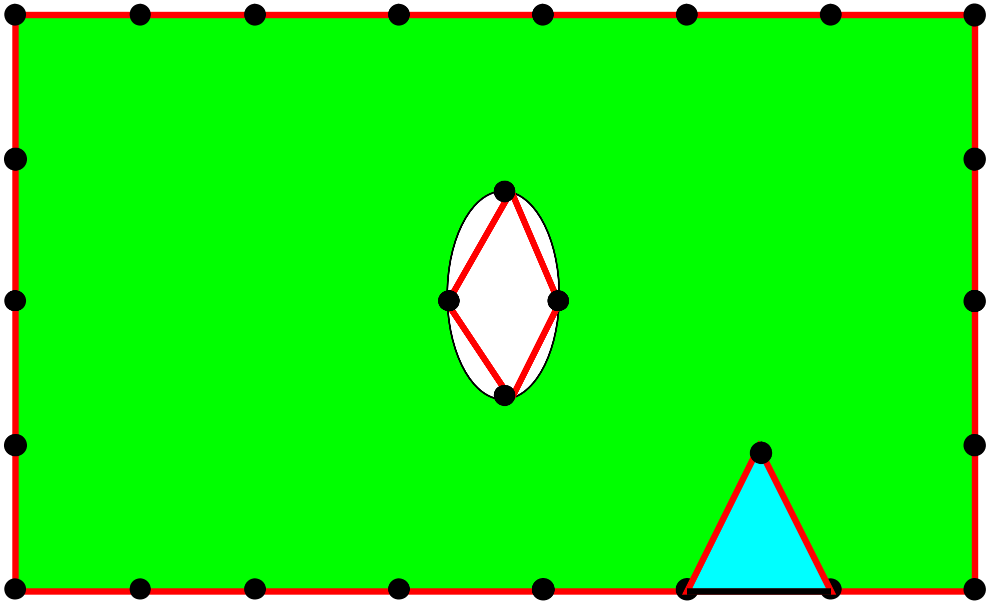
- Front initially set to be boundary.

Advancing Front



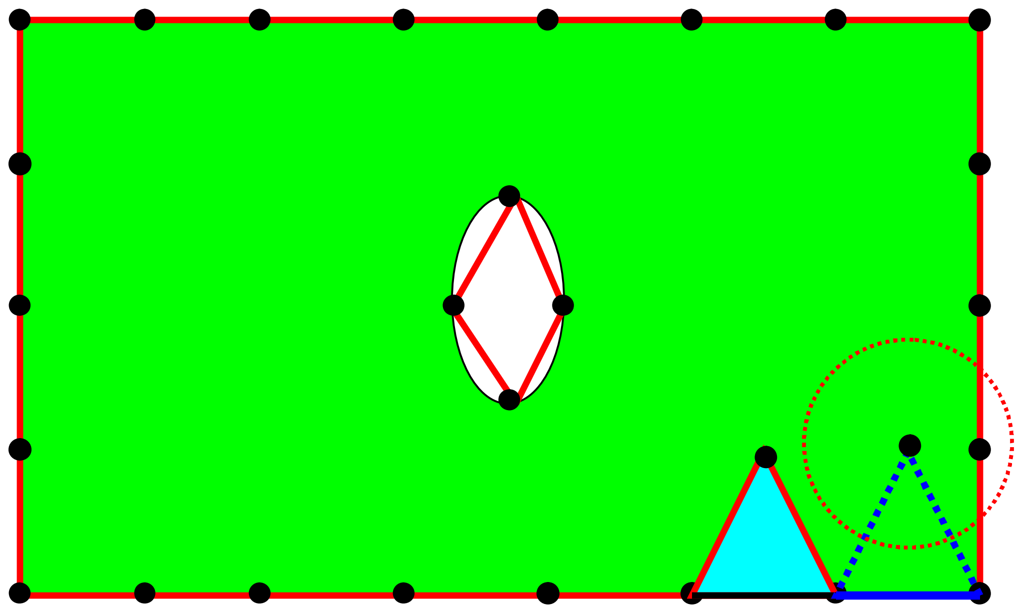
- Loop through all edges on front.
  - Find vertex which is optimal for each edge

Advancing Front



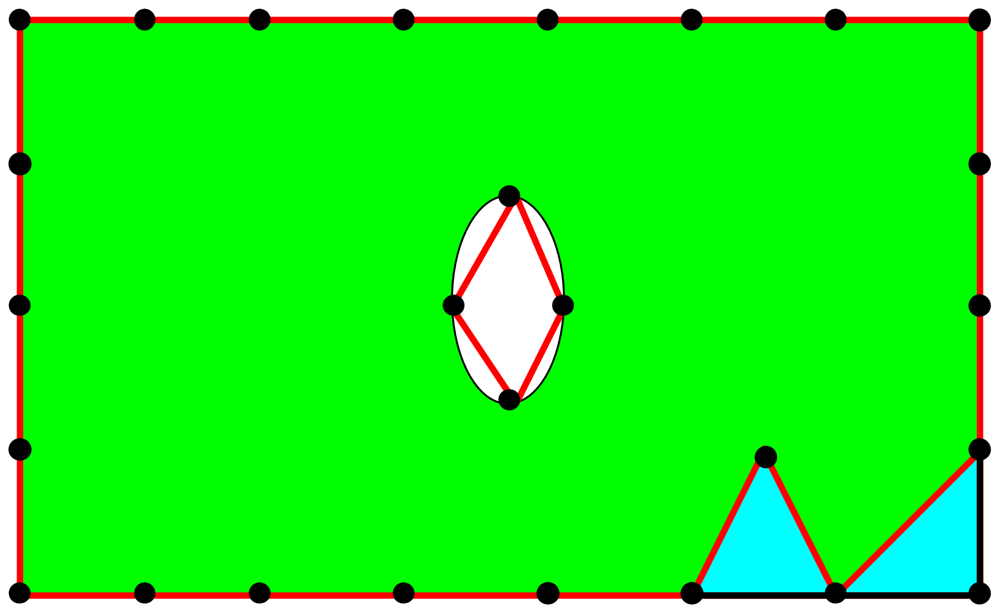
- Create triangle
- Remove edge from front
- Add new edges to front

Advancing Front



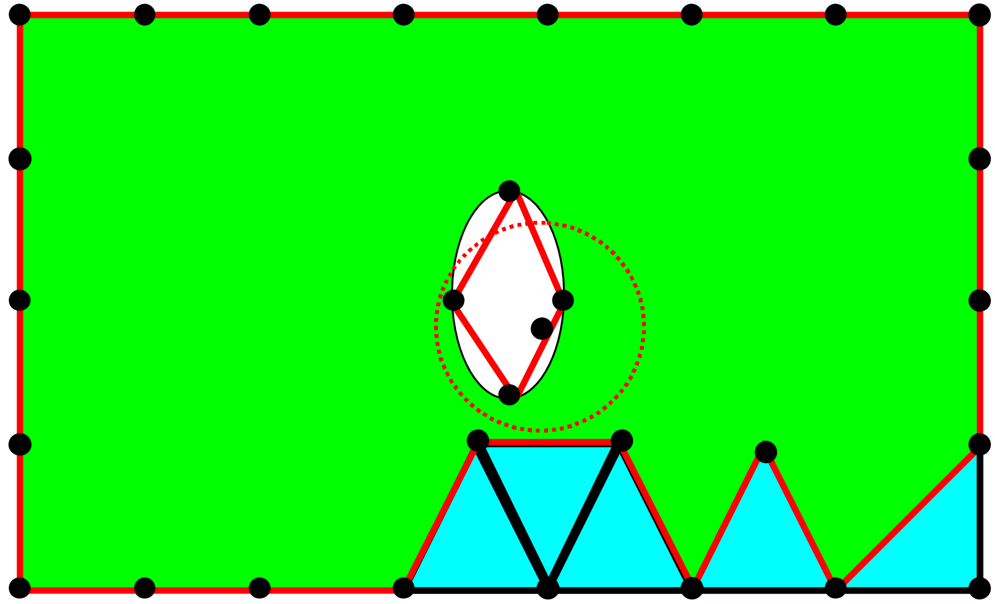
- Check radius around optimal node for nodes currently on front

Advancing Front



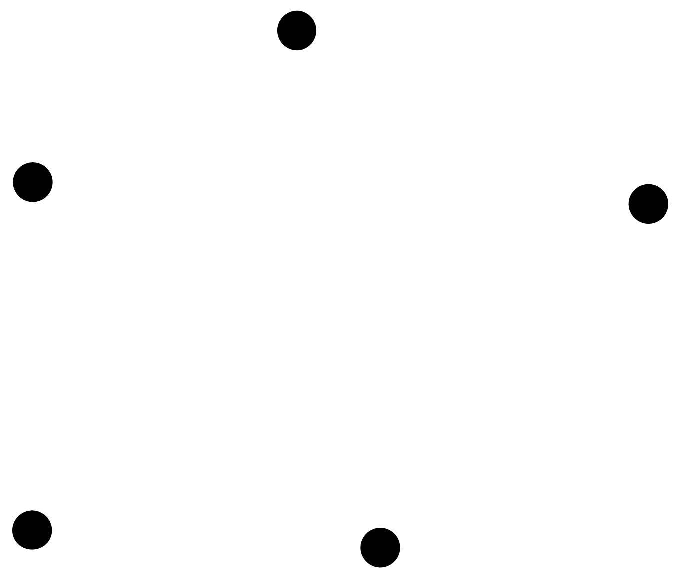
- If frontal node is found in radius, use instead

Advancing Front

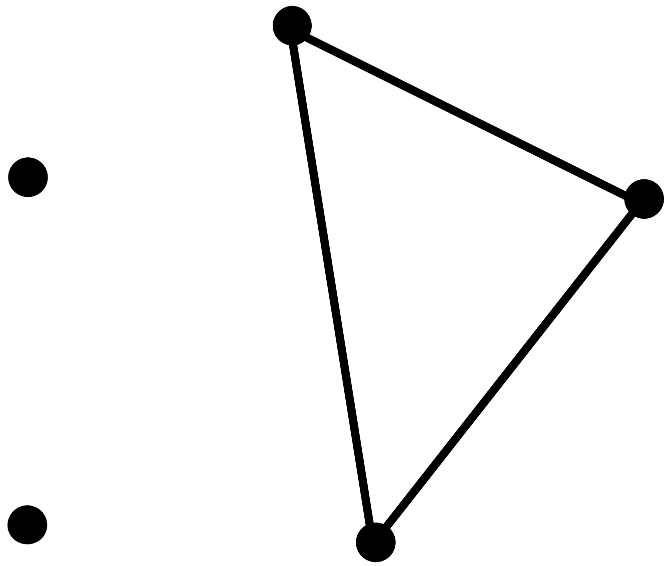


- If choice between multiple nodes, chose best quality element
- Continue until finished

Delaunay

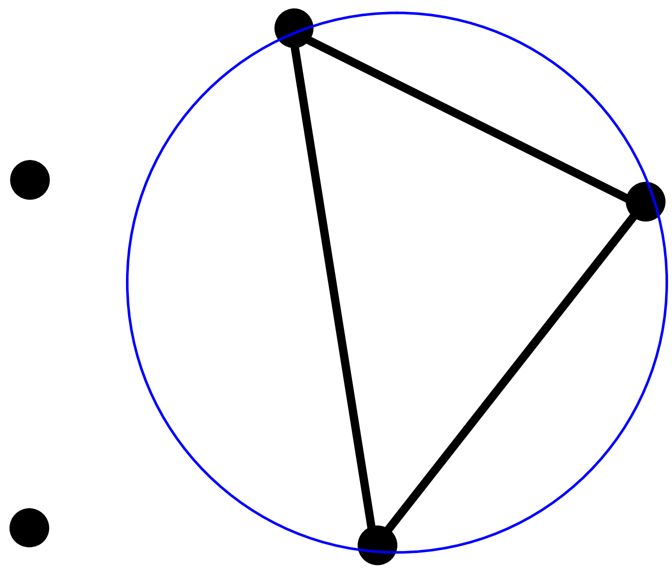


Delaunay



Empty Circle (Sphere) Property:
No other vertex is contained within the circumcircle of any triangle

Delaunay



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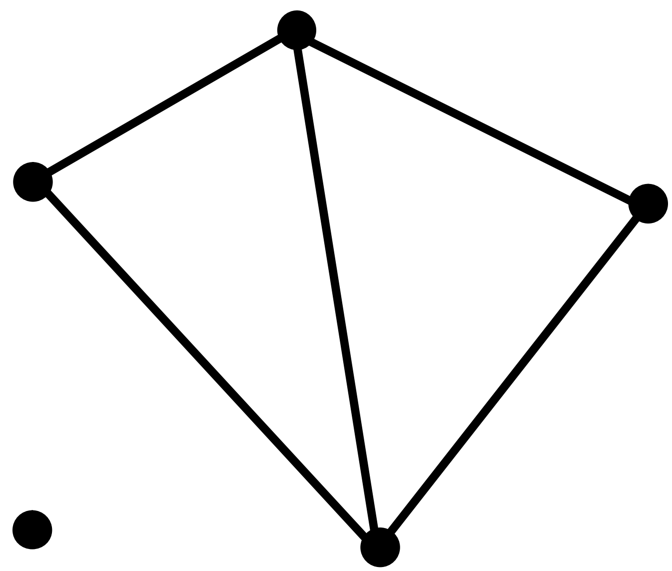
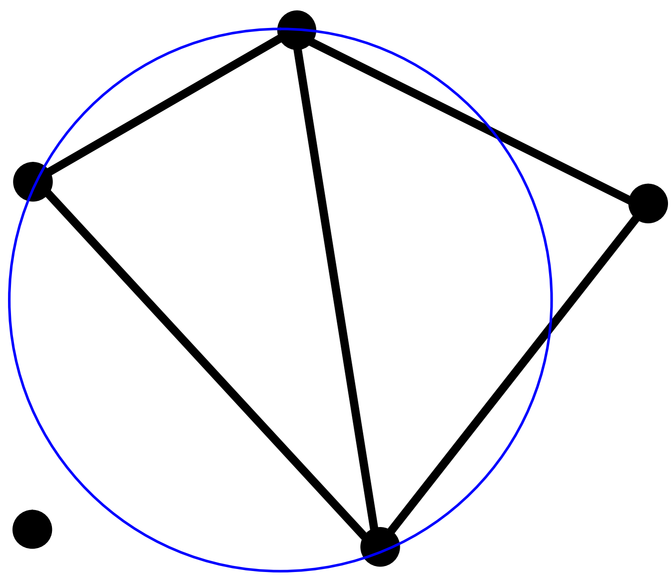


Delaunay



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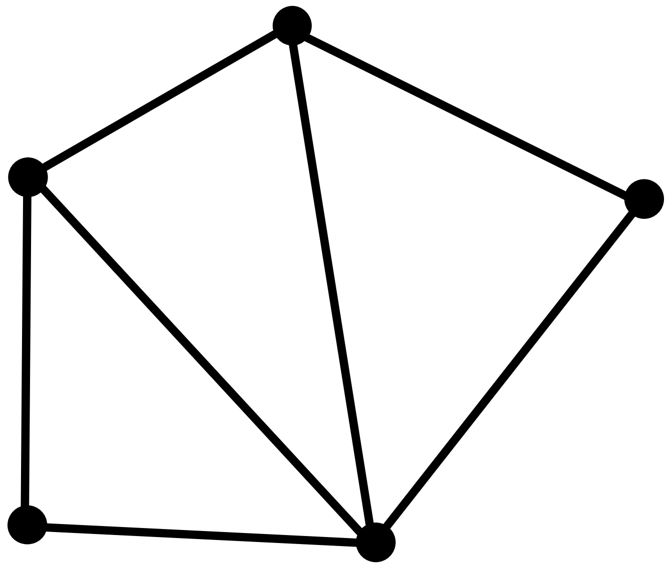
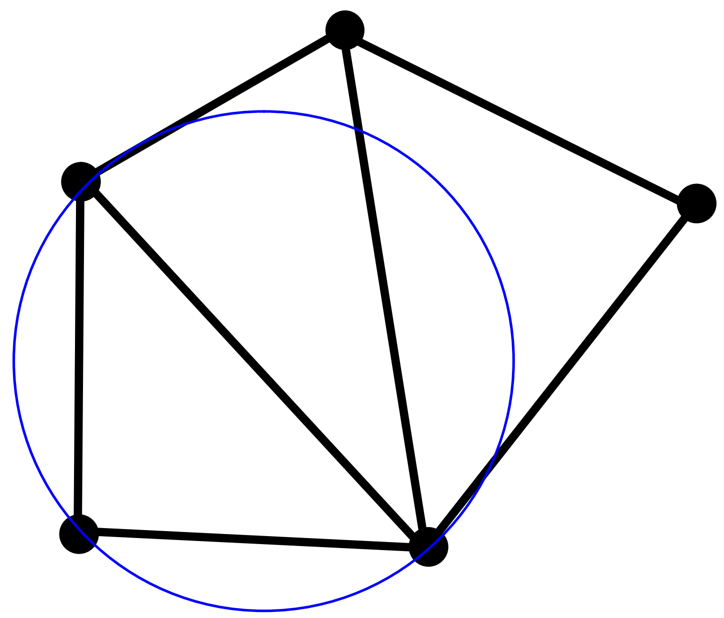


Delaunay



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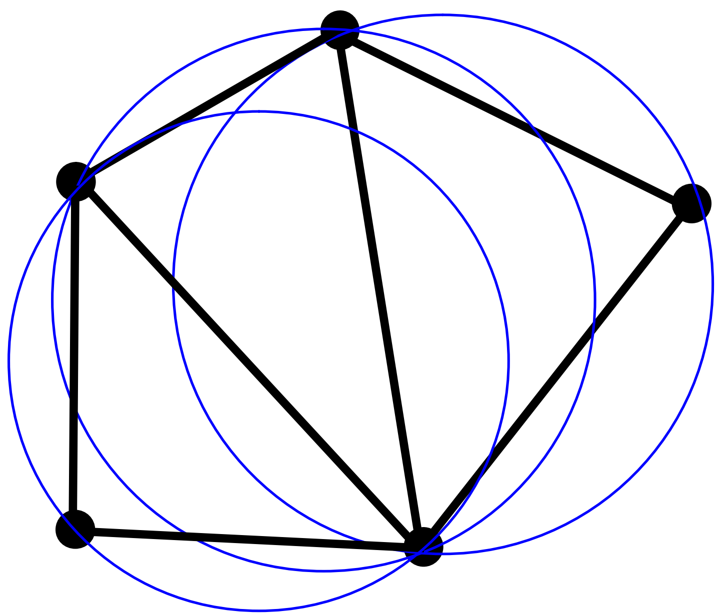

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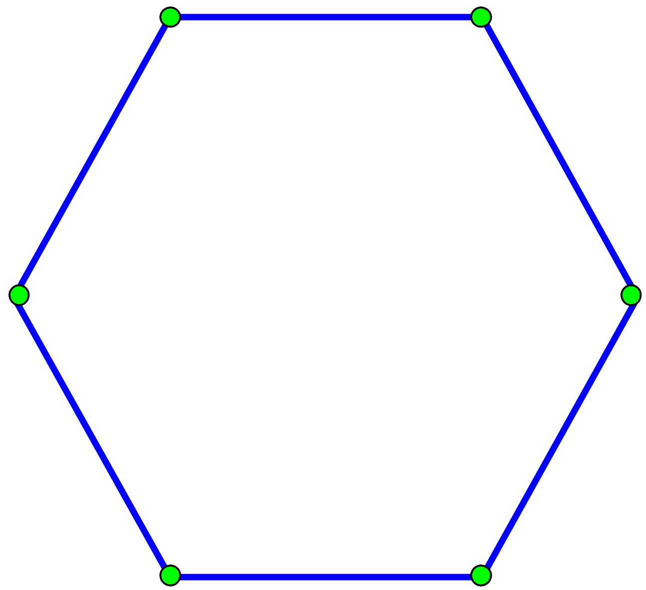
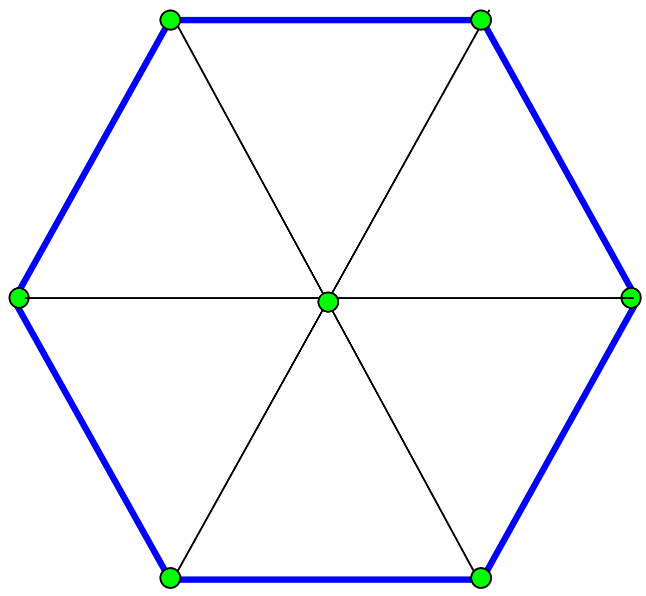


Valid Delaunay Triangulation



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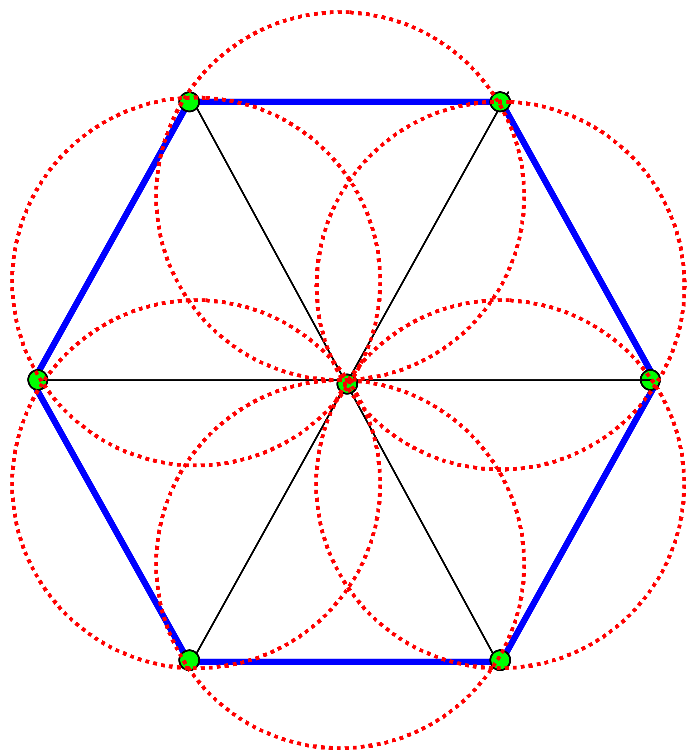
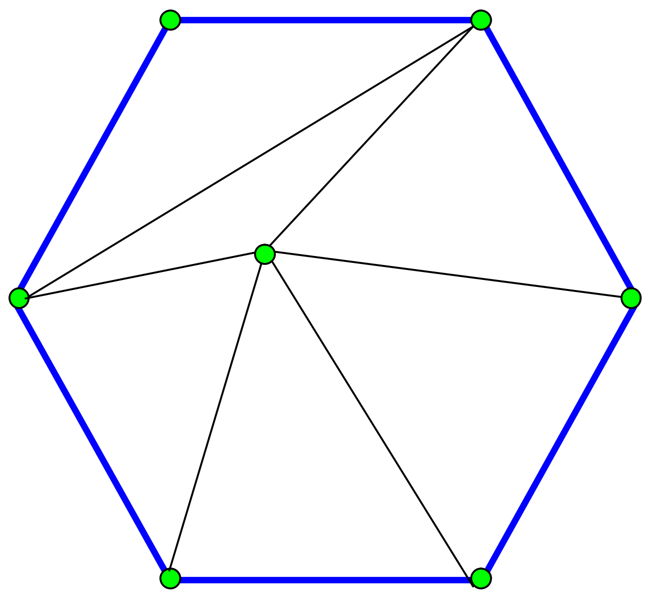


Non-Delaunay Triangulation



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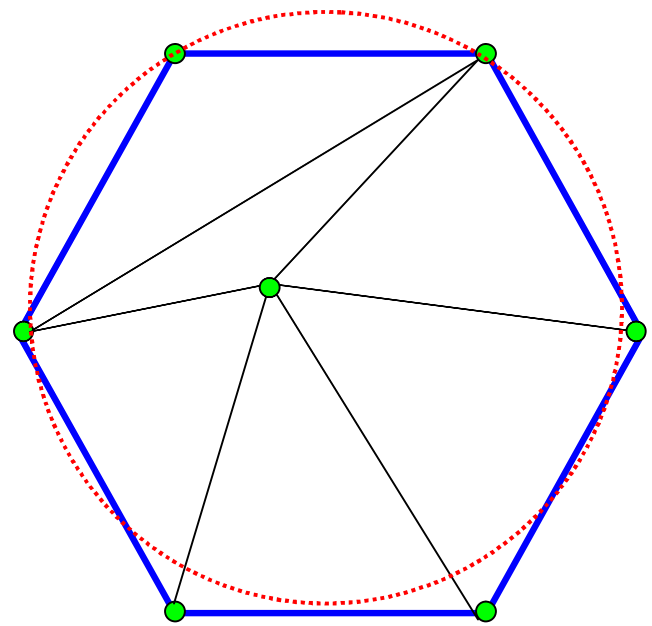
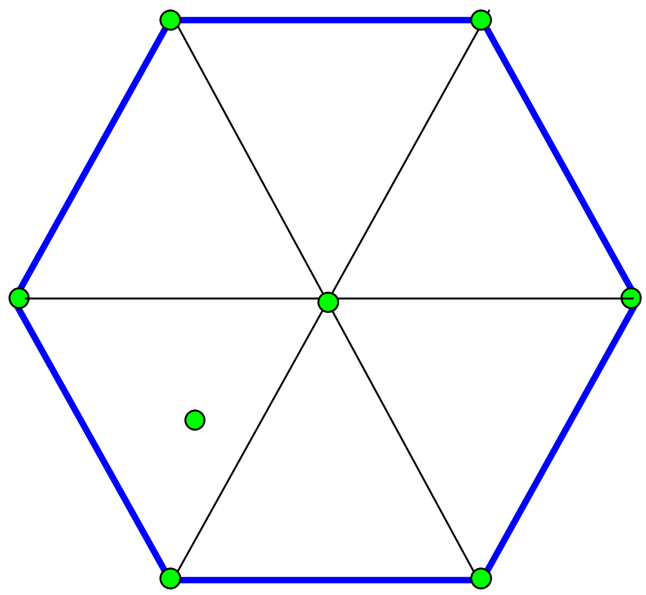
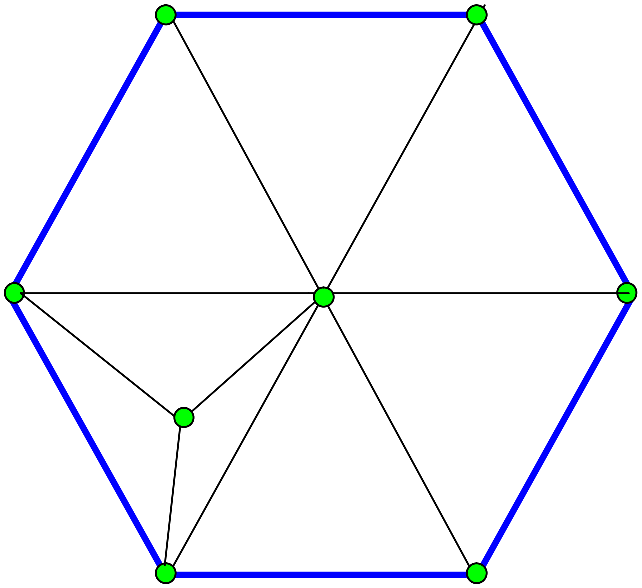


Delaunay – Node Insertion

Want to insert one node

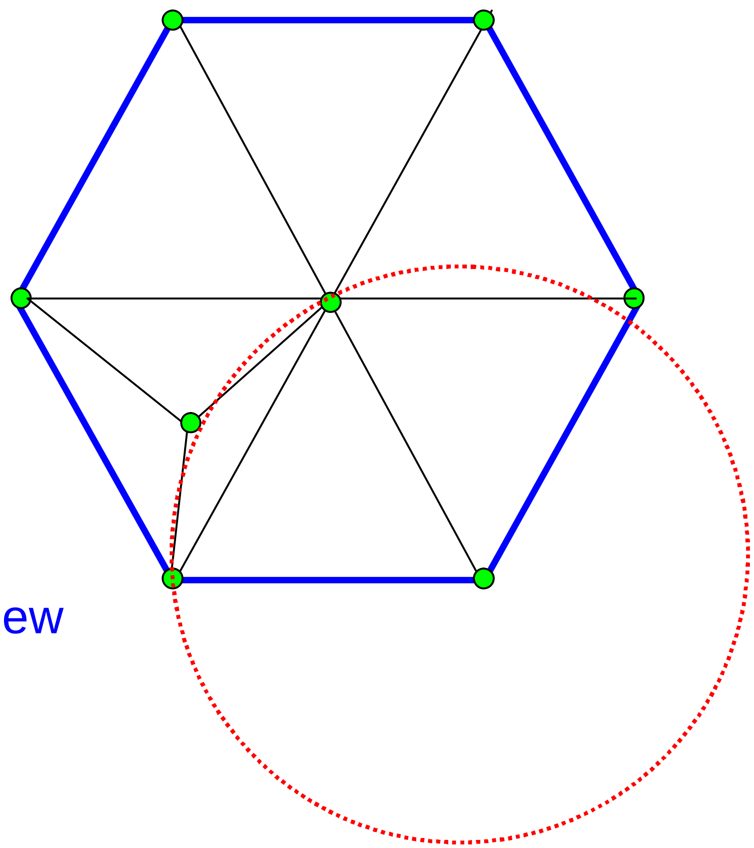


Delaunay – Node Insertion (Lawson)



Lawson Algorithm
1. Subdivide triangle that contains new point

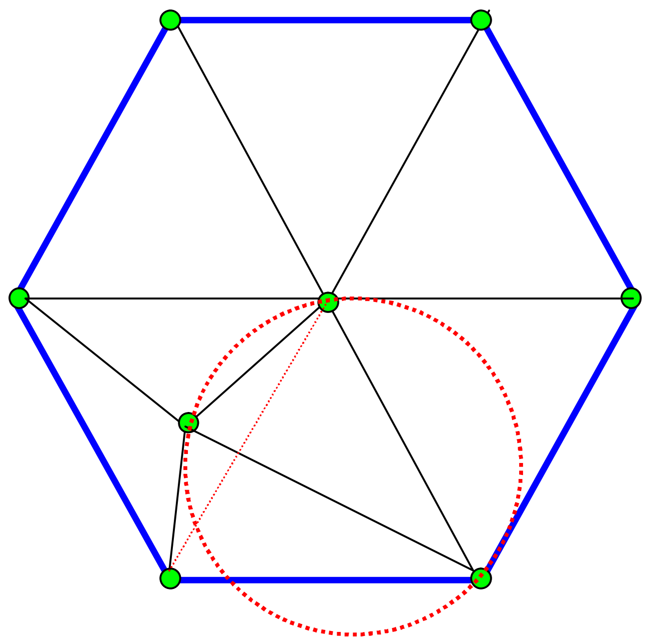
Delaunay – Node Insertion (Lawson)



Lawson Algorithm

2. Empty circle check for new and surrounding triangles

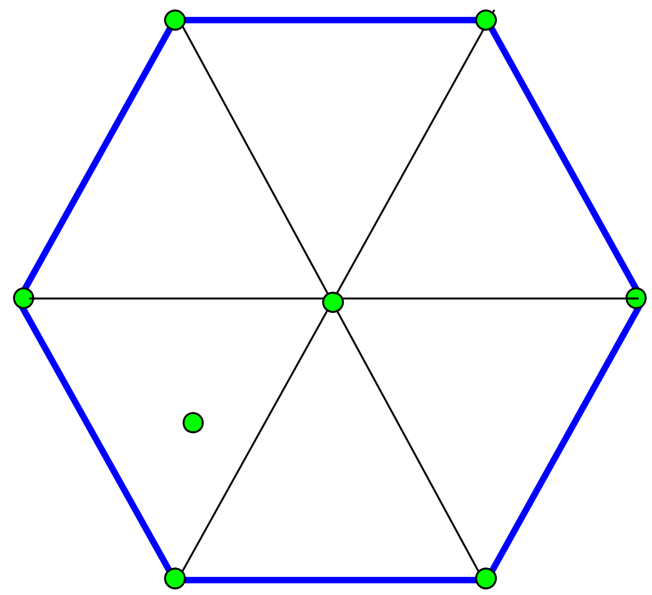
Delaunay – Node Insertion (Lawson)



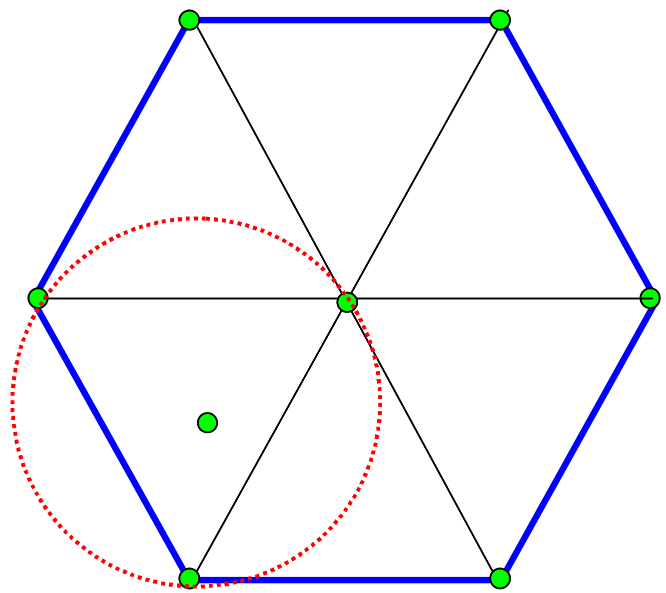
Lawson Algorithm
3. Move diagonal if necessary and recheck

Delaunay – Node Insertion

Want to insert one node



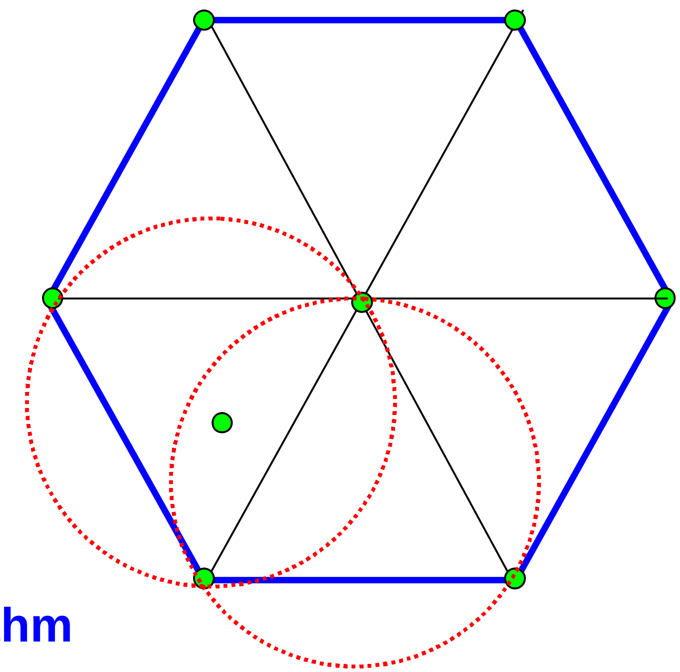
Delaunay – Node Insertion (Bowyer-Watson)



Bowyer-Watson Algorithm

1. Find all triangles whose circumcircle contains the new node.

Delaunay – Node Insertion (Bowyer-Watson)



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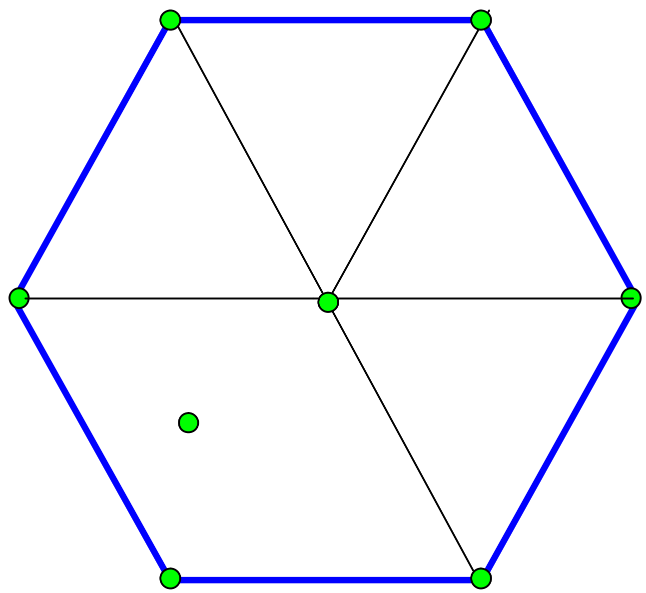
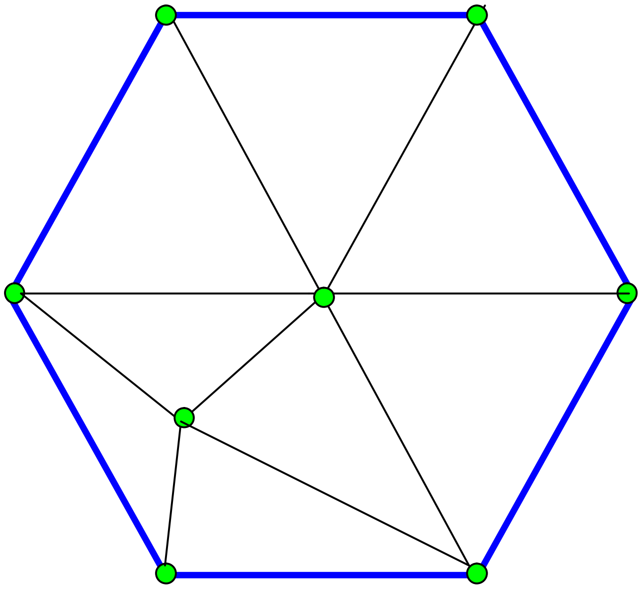


Bowyer-Watson Algorithm
2. Remove edges interior to these triangles

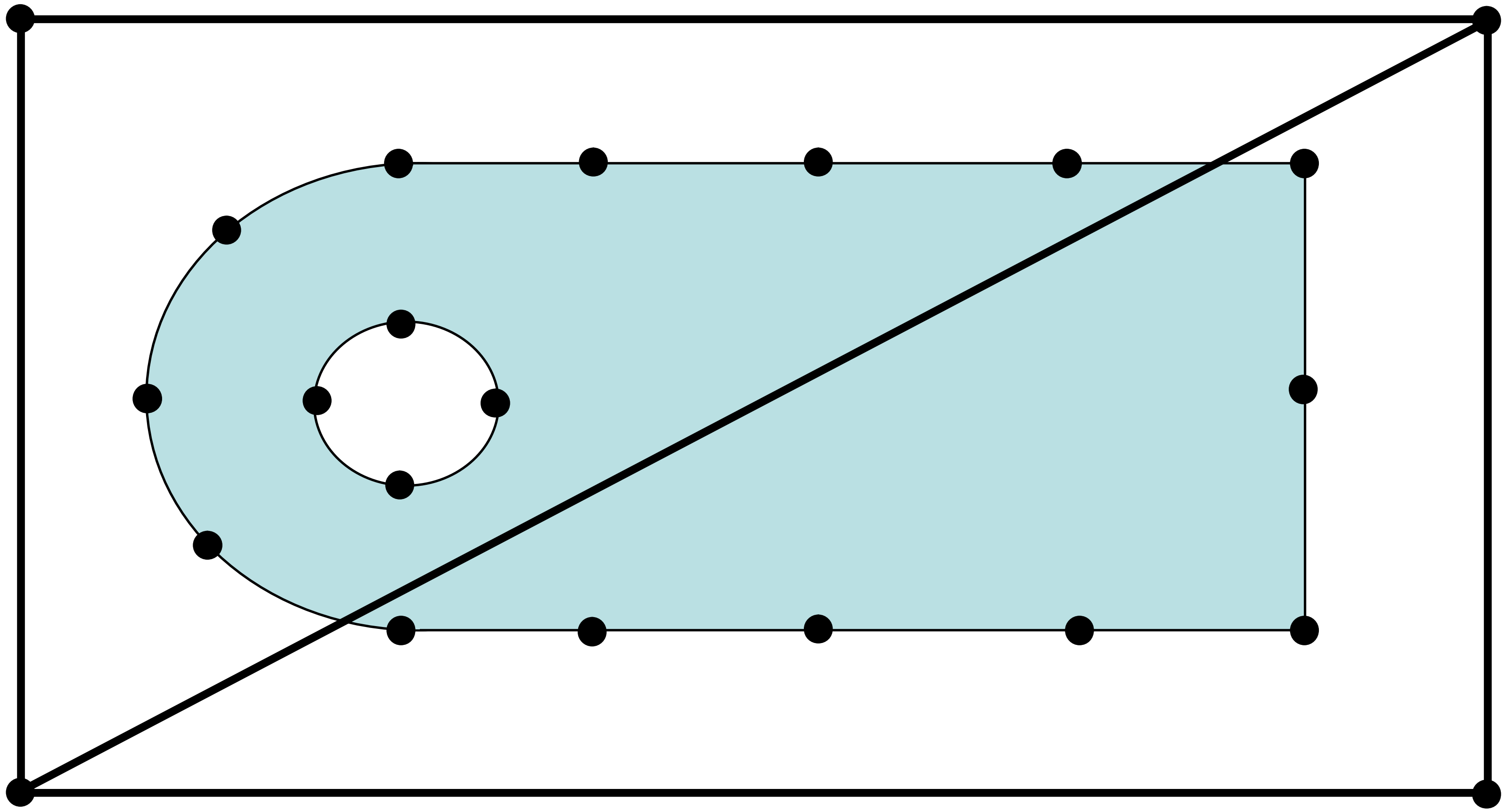
Delaunay – Node Insertion (Bowyer-Watson)



Bowyer-Watson Algorithm

3. Connect nodes of this empty space to new node.

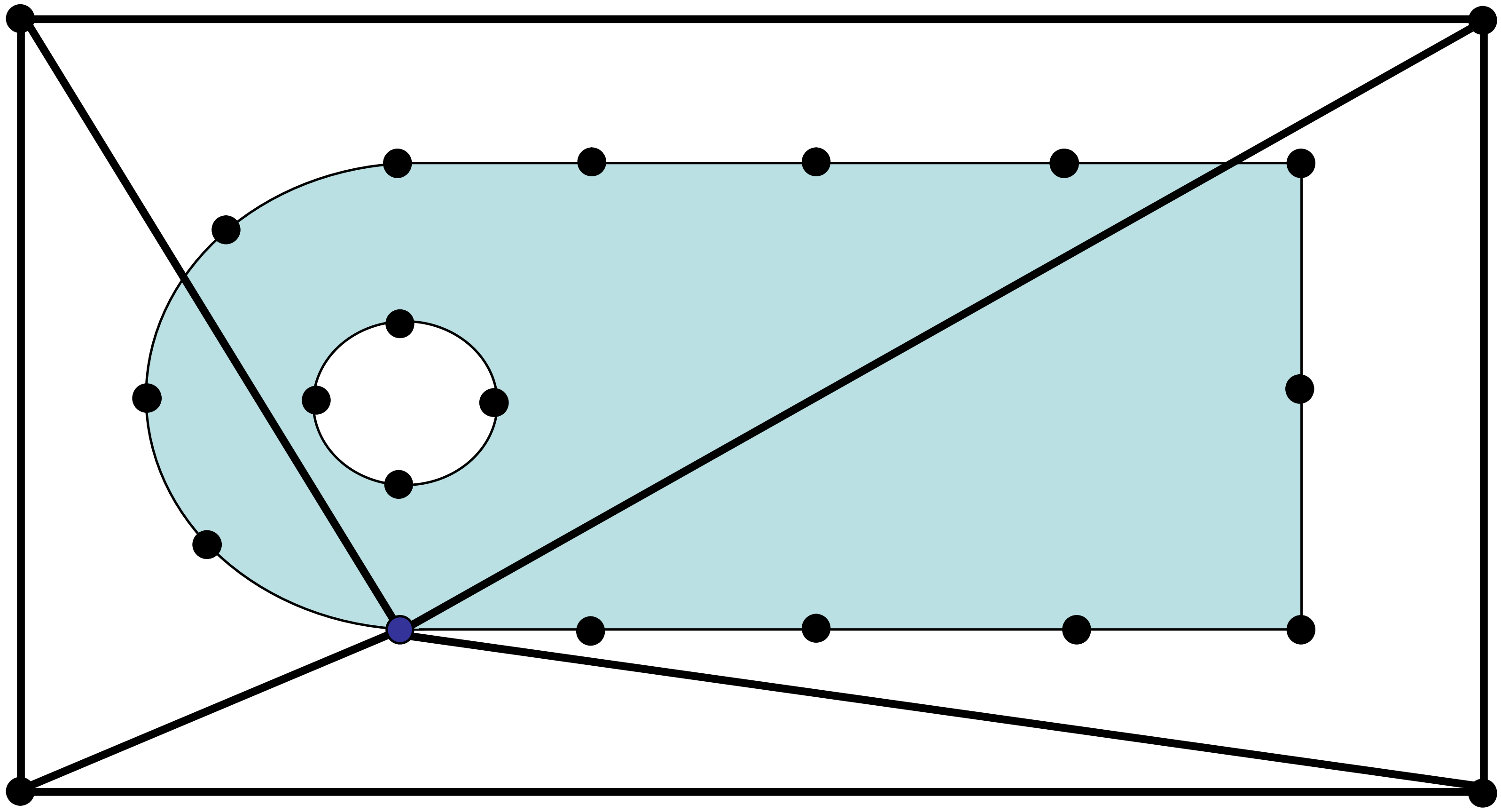
Delaunay



•Begin with Bounding Triangles

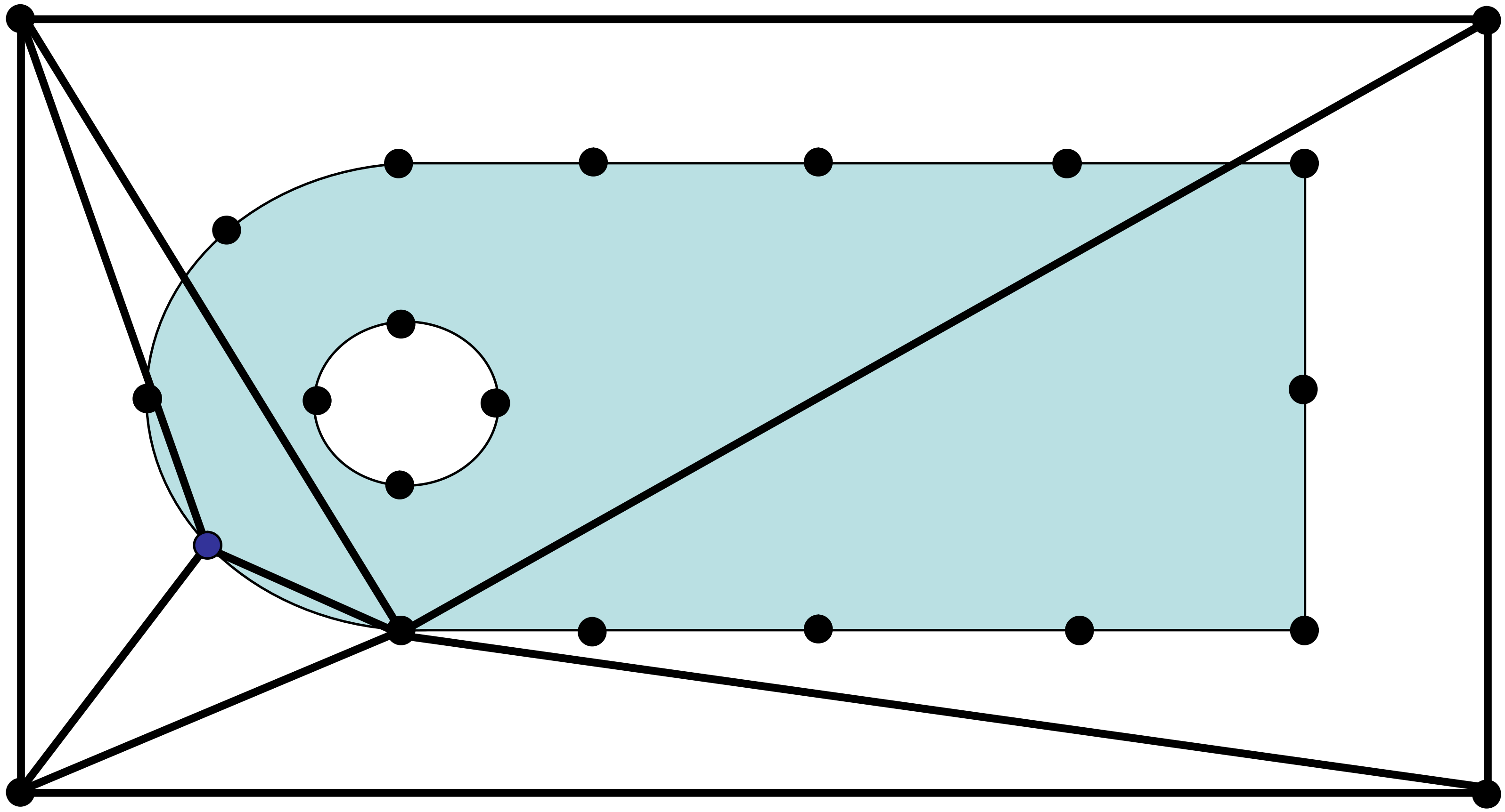
* From S. Owen

Delaunay



•Insert boundary nodes using Delaunay method (Lawson or Bowyer-Watson)

Delaunay



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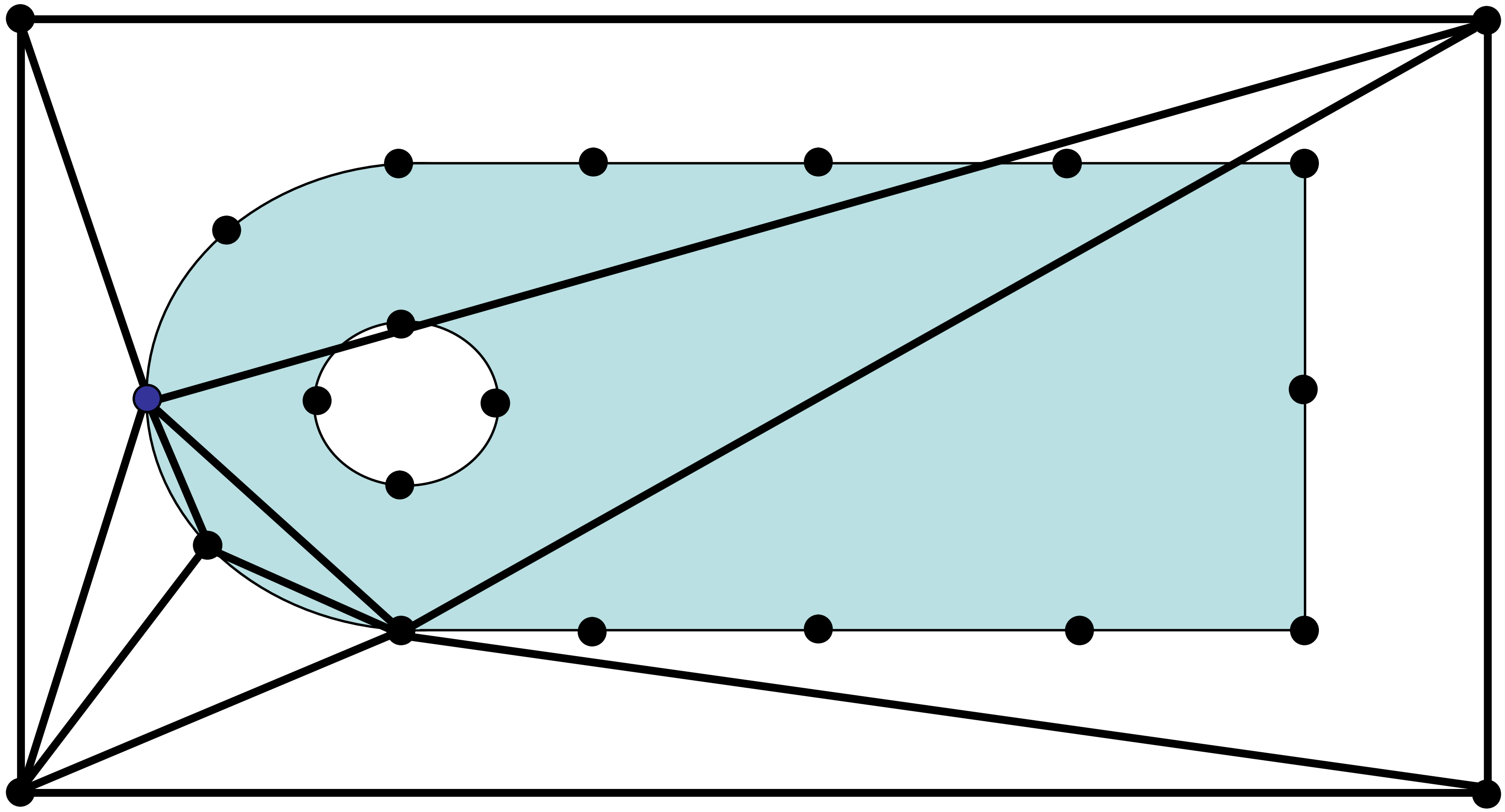

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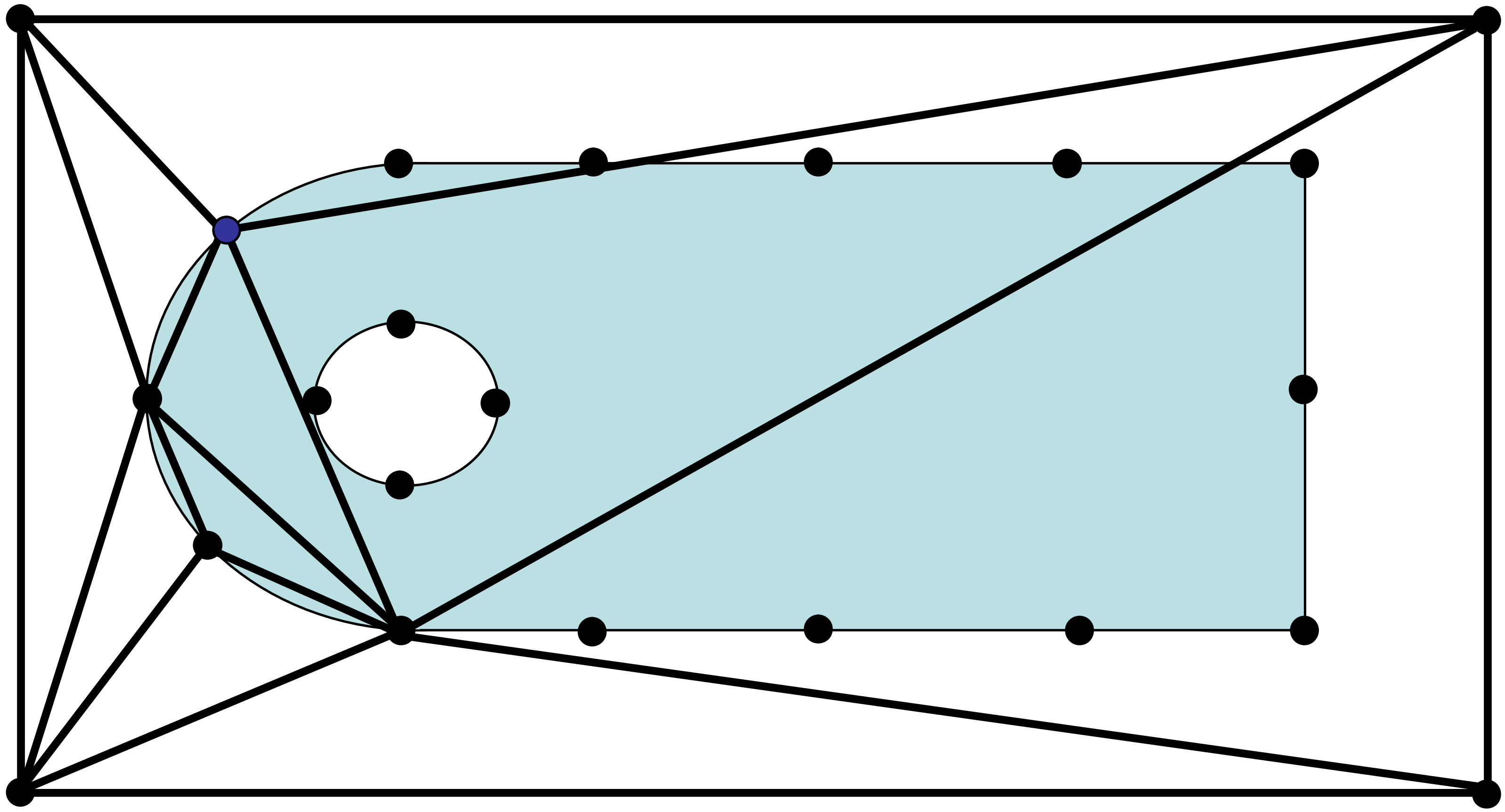

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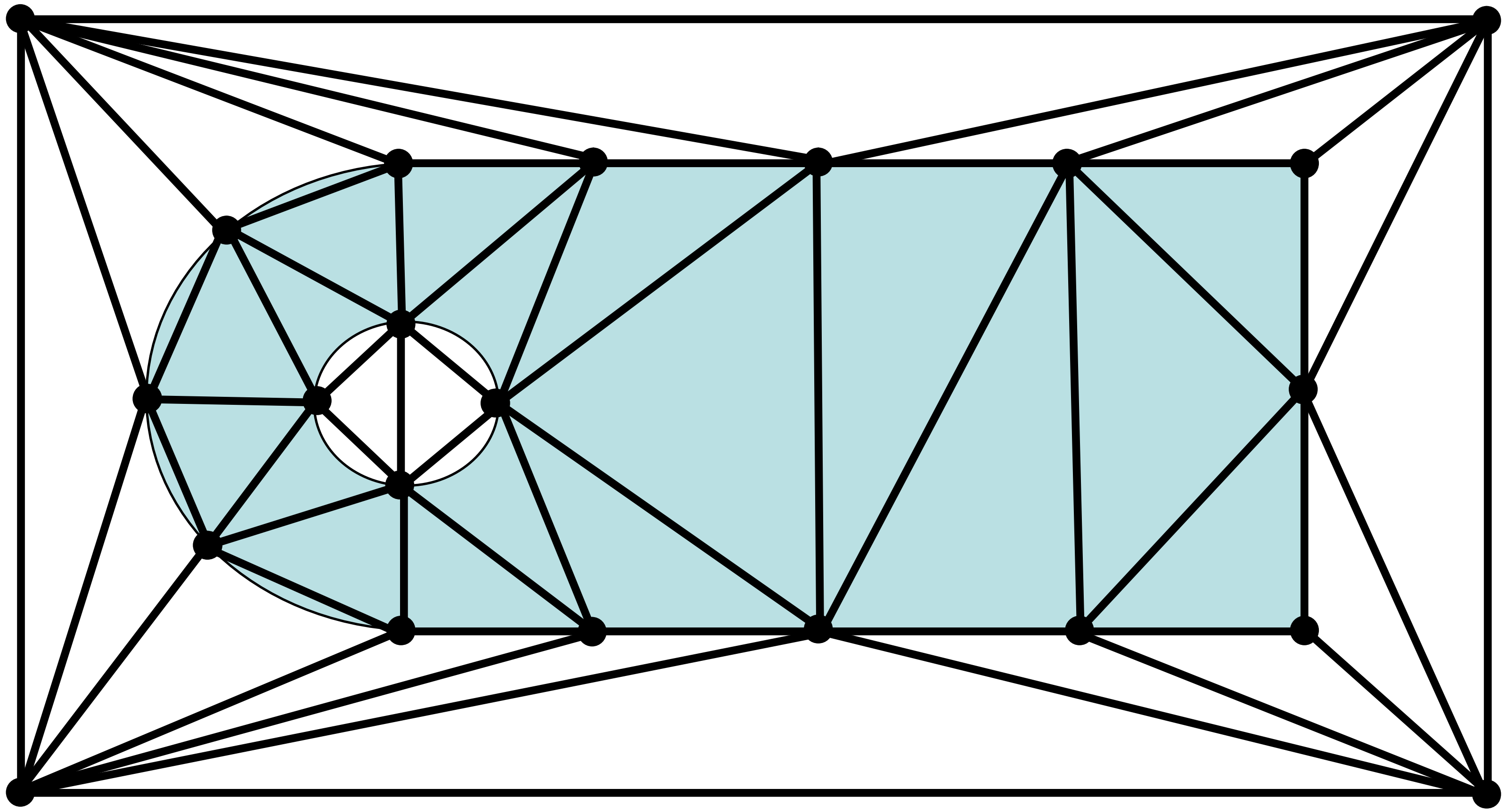

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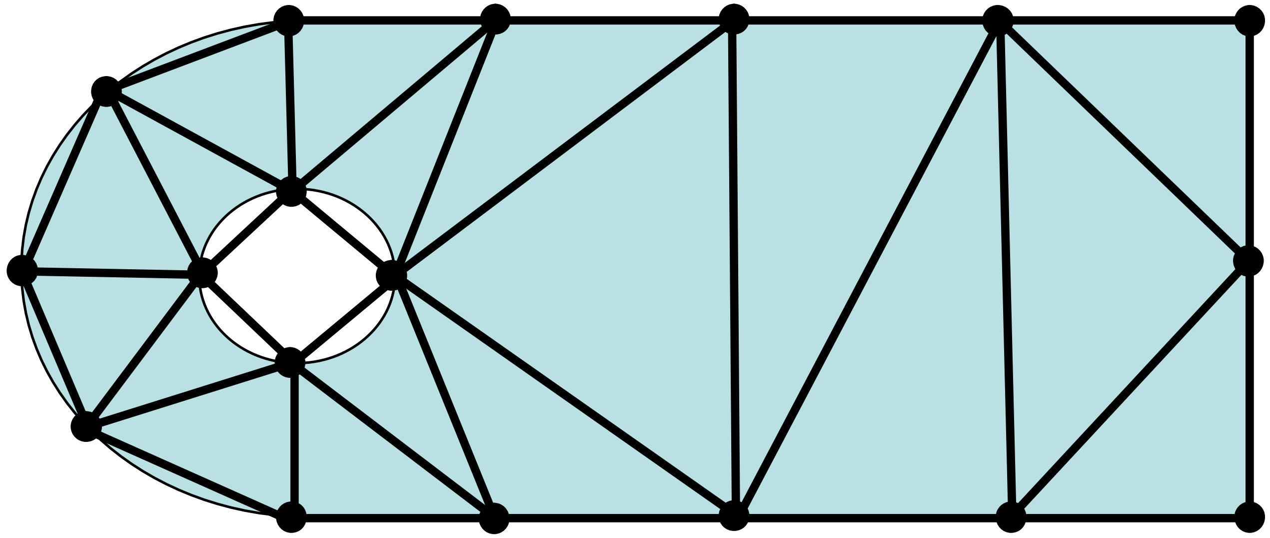
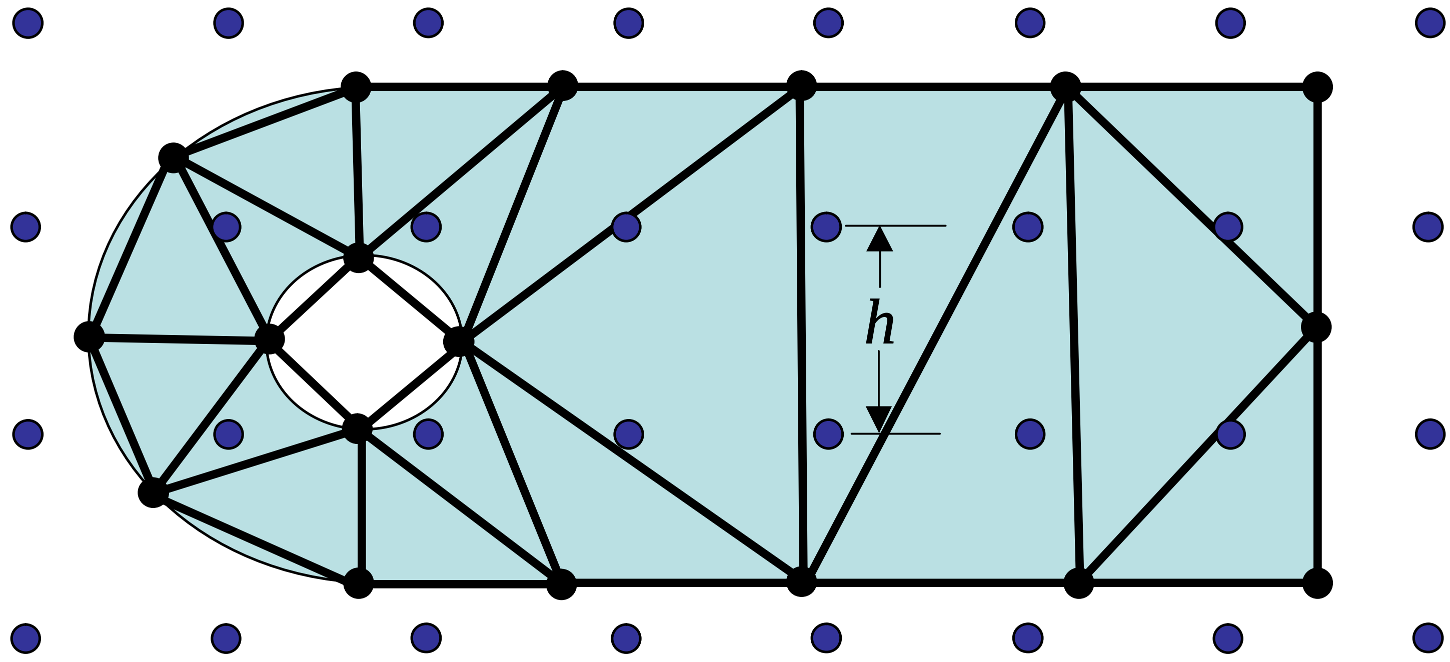


•Delete outside triangles

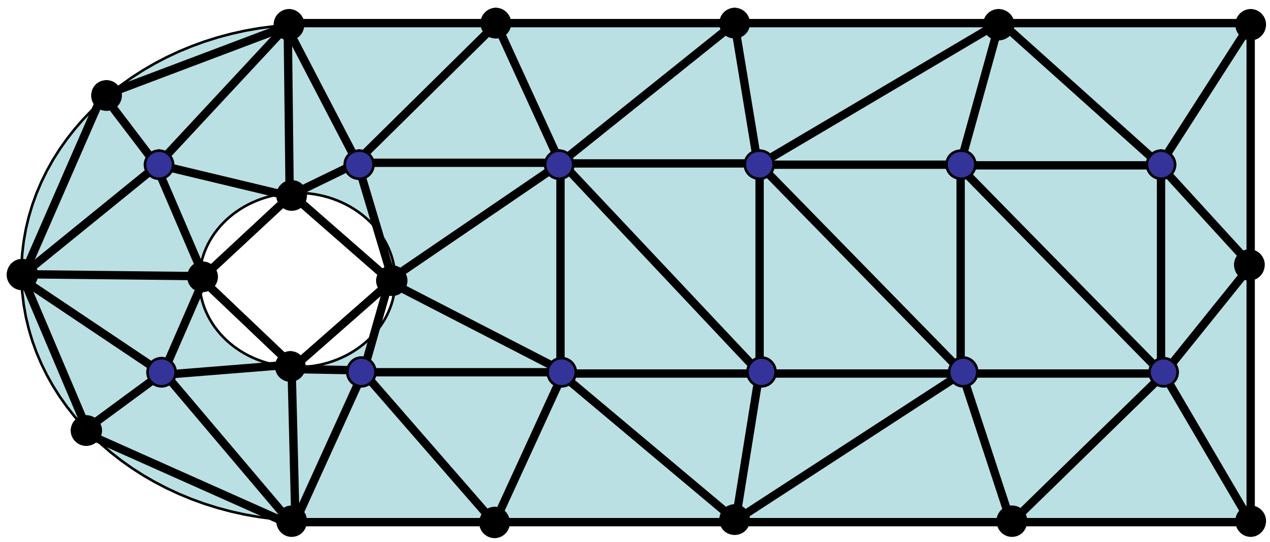
Delaunay – Interior Nodes



Grid Based

•Nodes introduced based on a regular lattice

Delaunay – Interior Nodes



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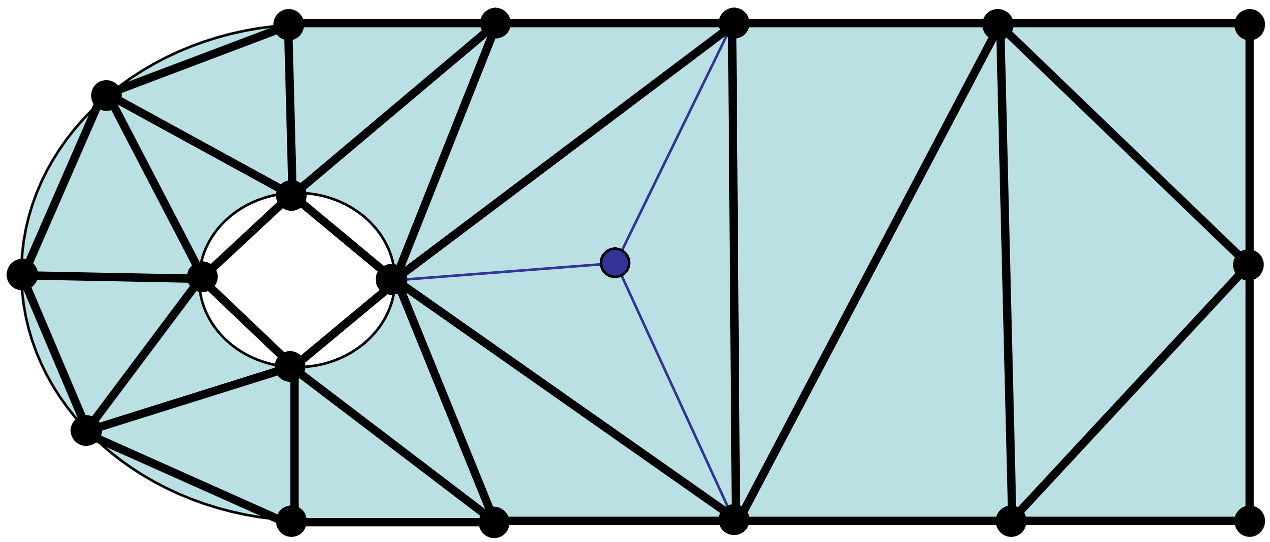
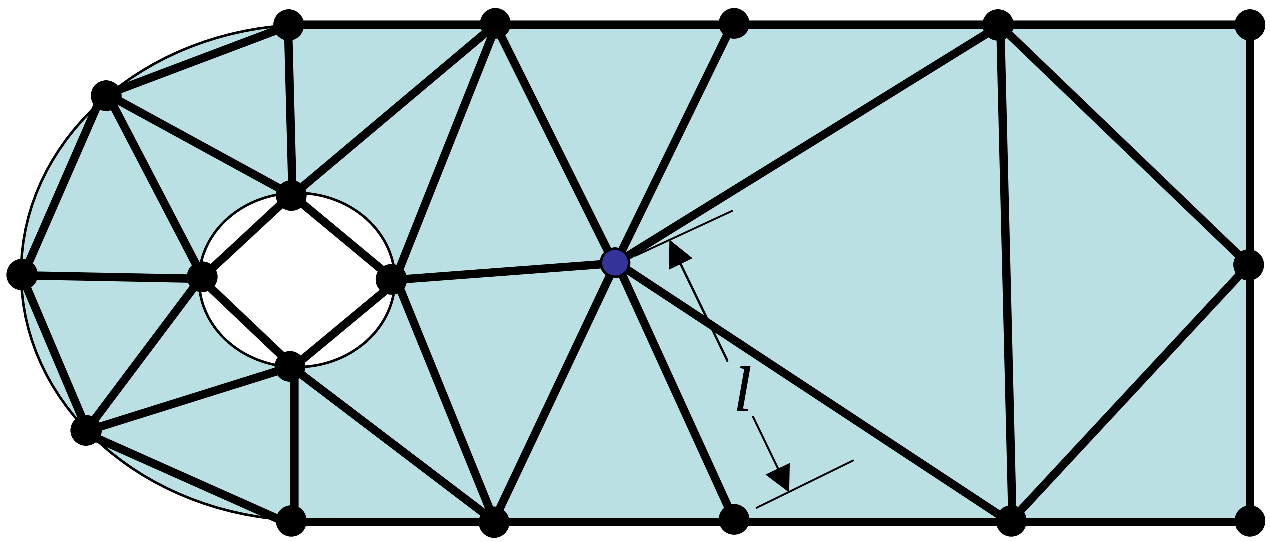


Centroid

- Nodes introduced at triangle centroids
- Continues until edge length, $l \approx h$

Delaunay – Interior Nodes



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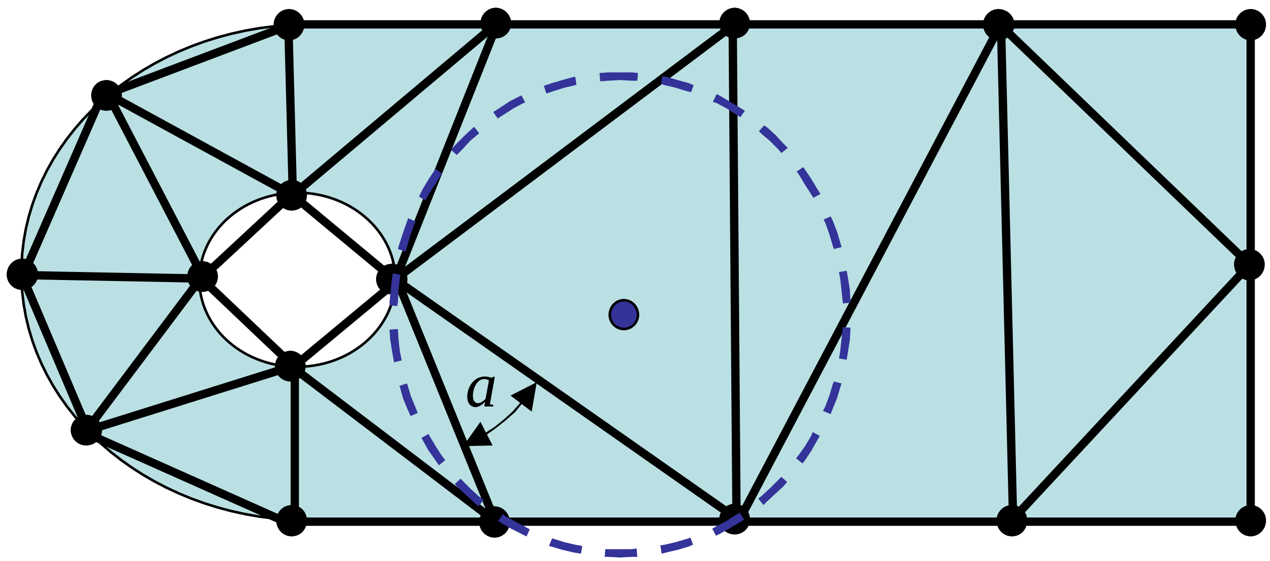
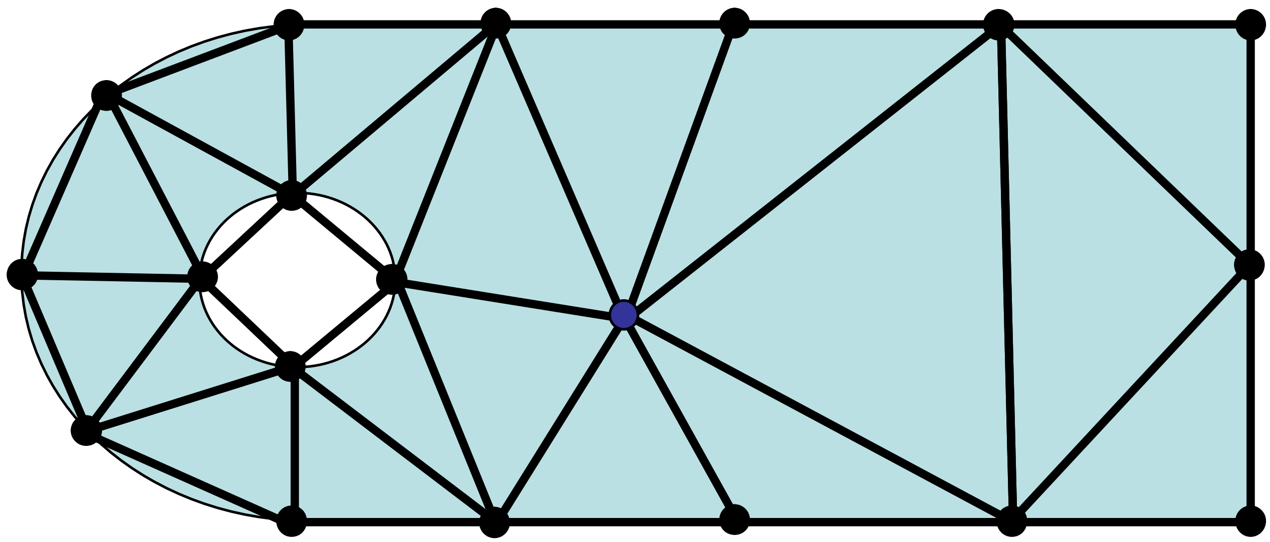


Circumcenter

- Nodes introduced at triangle circumcenters
- Order of insertion based on minimum angle of any triangle
- Continues until minimum angle > predefined minimum $(a \approx 30^{0})$

(Chew,Ruppert,Shewchuk)

Delaunay – Interior Nodes



Circumcenter ("Guaranteed Quality")

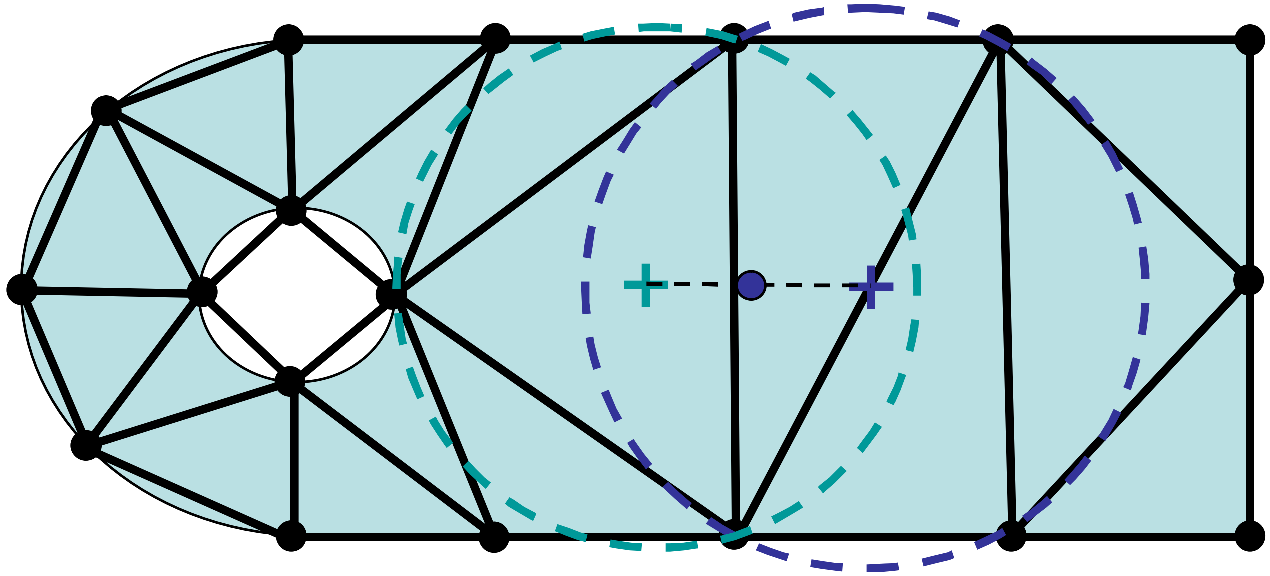
•Nodes introduced at triangle circumcenters

•Order of insertion based on minimum angle of any triangle

•Continues until minimum angle > predefined minimum ($a \approx 30^{\mathbf{0}}$)

(Chew,Ruppert,Shewchuk)

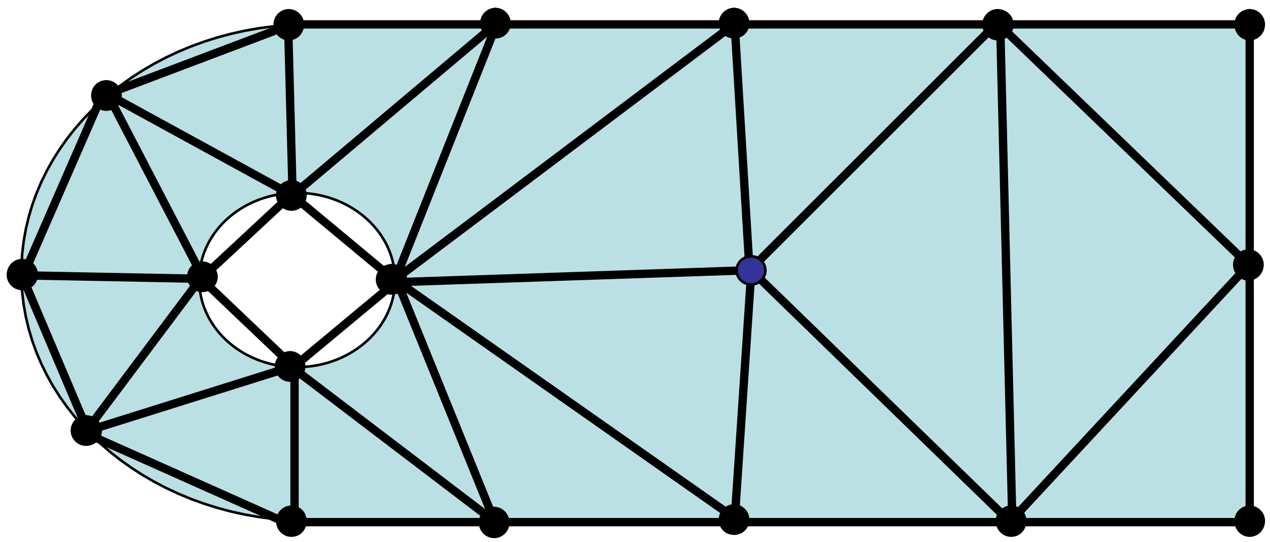
Delaunay – Interior Nodes



Voronoi-Segment

•Nodes introduced at midpoint of segment connecting the circumcircle centers of two adjacent triangles

Delaunay – Interior Nodes



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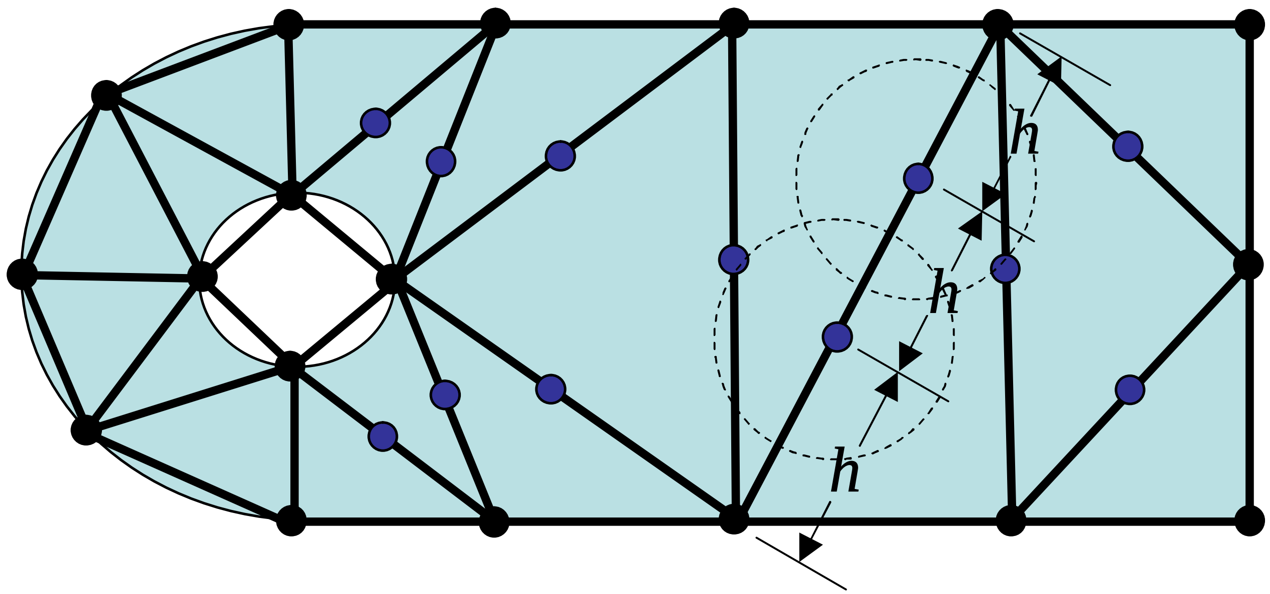
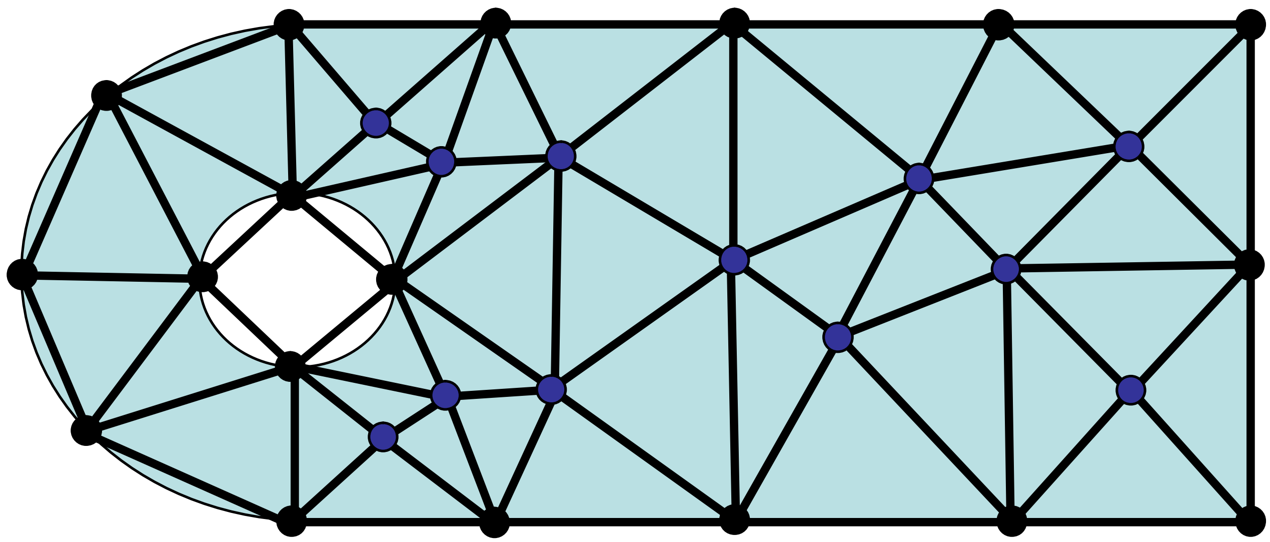


Edges

•Nodes introduced at along existing edges at $l=h$

•Check to ensure nodes on nearby edges are not too close

Delaunay – Interior Nodes

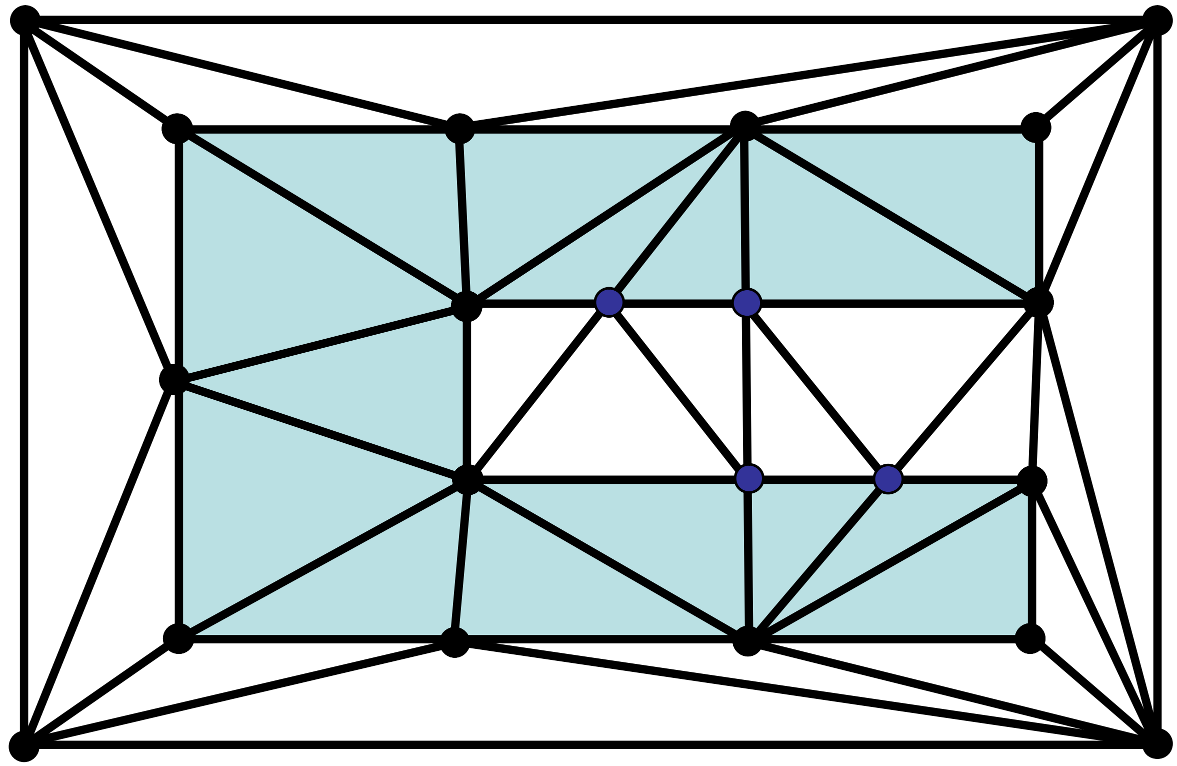


Edges

•Nodes introduced at along existing edges at $l=h$

•Check to ensure nodes on nearby edges are not too close

Delaunay – Constrained Boundaries

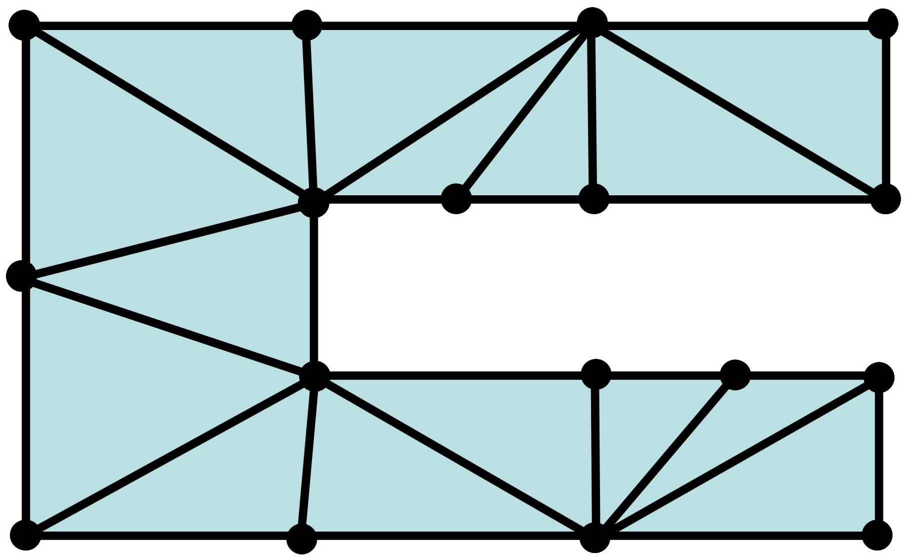


Boundary Intersection

•Nodes and edges introduced where Delaunay edges intersect boundary

* From S. Owen

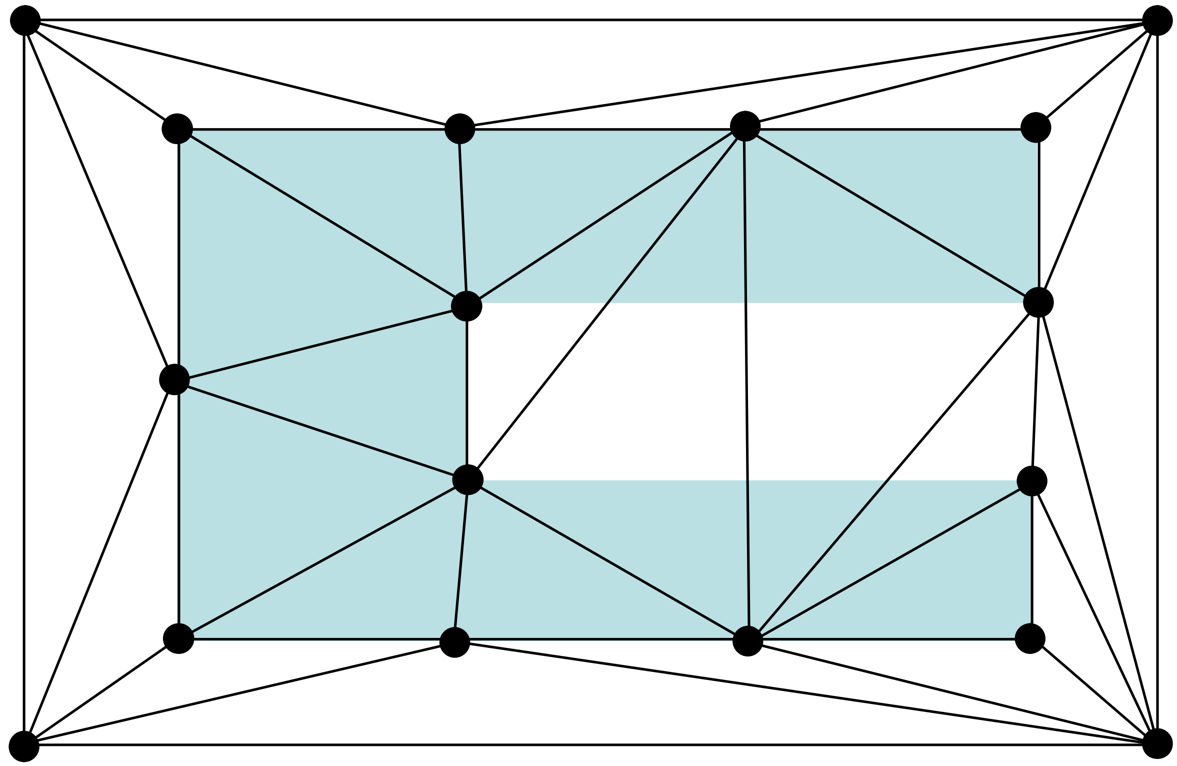
Delaunay – Constrained Boundaries



Boundary Intersection

•Nodes and edges introduced where Delaunay edges intersect boundary

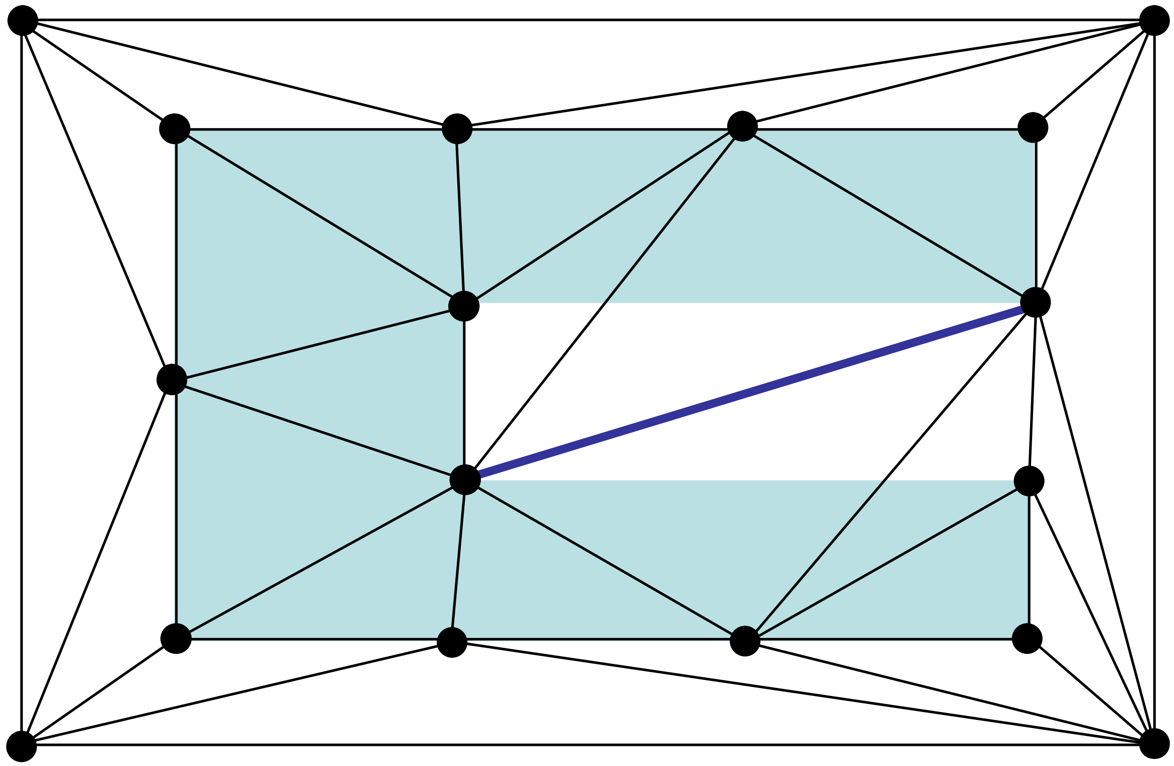
Delaunay – Constrained Boundary



Local Swapping

•Edges swapped between adjacent pairs of triangles until boundary is maintained

Delaunay



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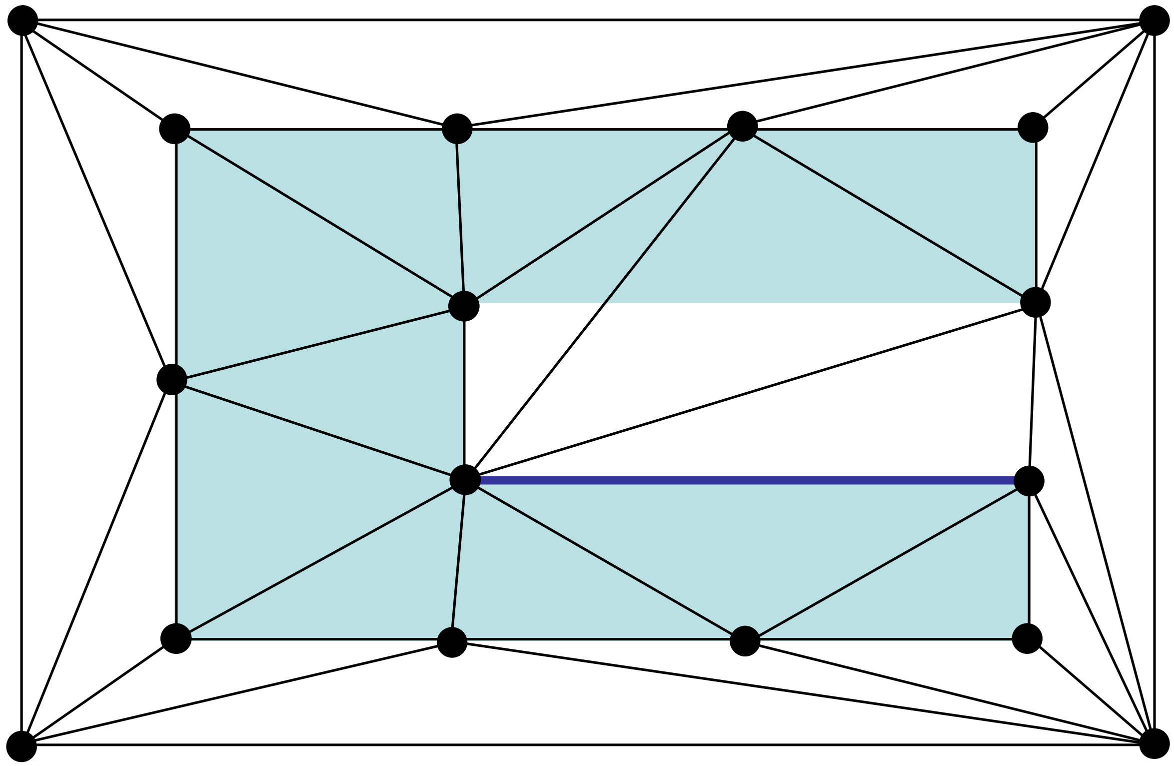
Delaunay – Constrained Boundary



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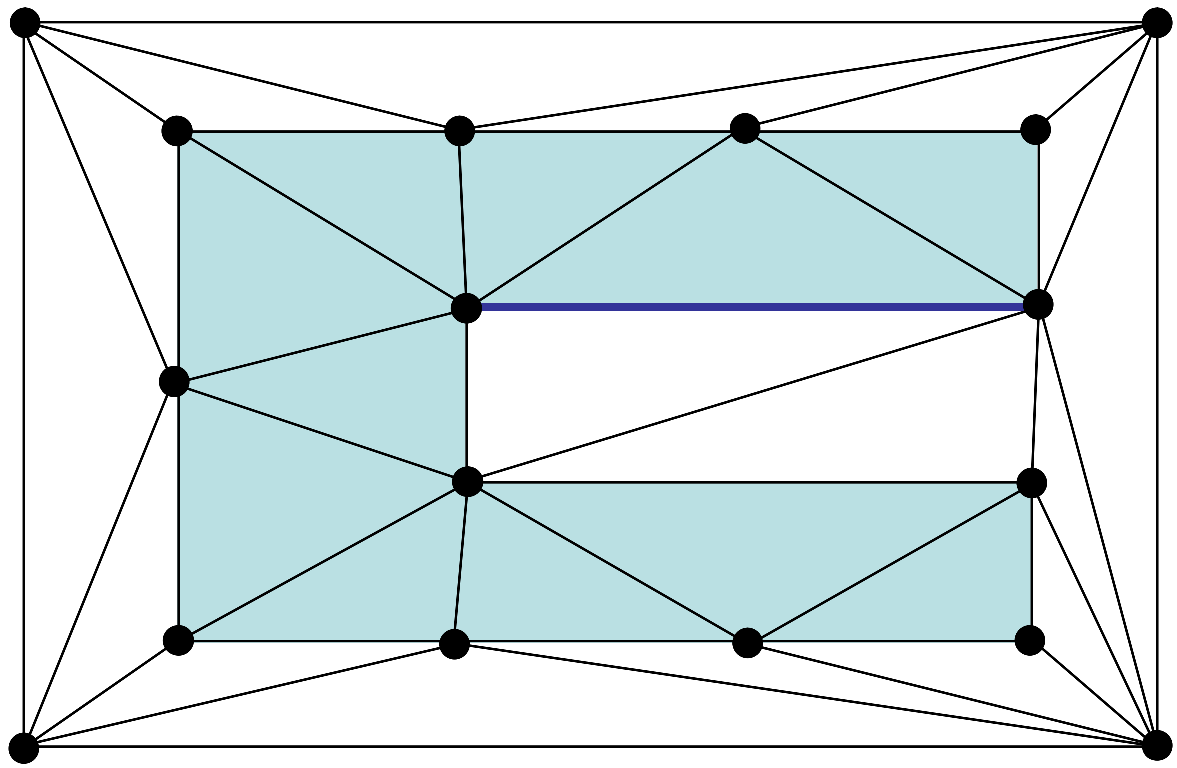
Delaunay – Constrained Boundary



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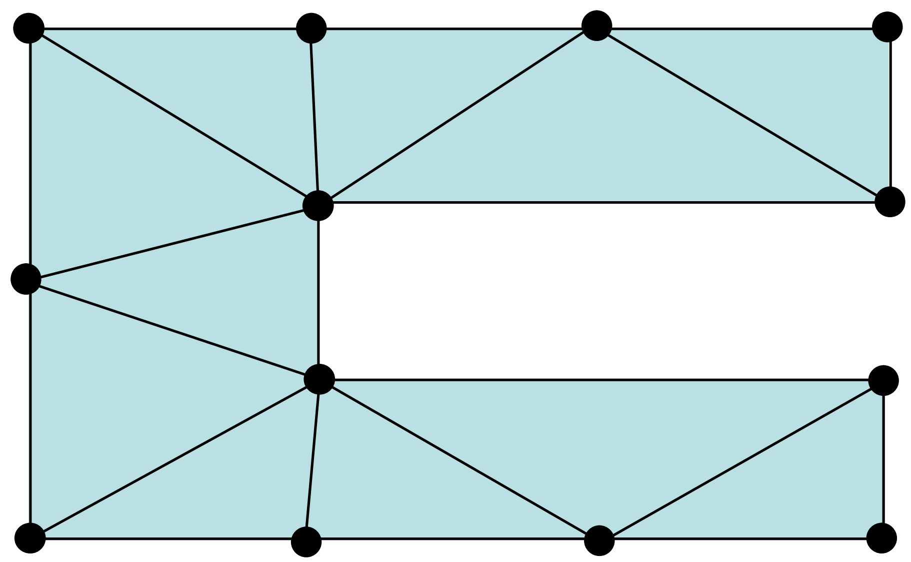
Delaunay – Constrained Boundary



Local Swapping

•Edges swapped between adjacent pairs of triangles until boundary is maintained

Overview

- Introduction to Mesh Generation
- Mesh Quality
- Serial Meshing Methods
  - Quadtree/Octree
  - Advancing Front
  - Delaunay
- Parallel Mesh Generation
  - Why Parallel?
  - Categorization Parallel methods
  - Subdomains, interfaces, separators
- CSAR Mesh Repair in Rocket Simulation

Parallel Mesh Generation

- Why Parallel?
  - Meshes require too much memory to generate serially
  - Mesh generation becomes computational bottleneck in simulation
  - Already have parallel simulation and need to remesh/repair/refine

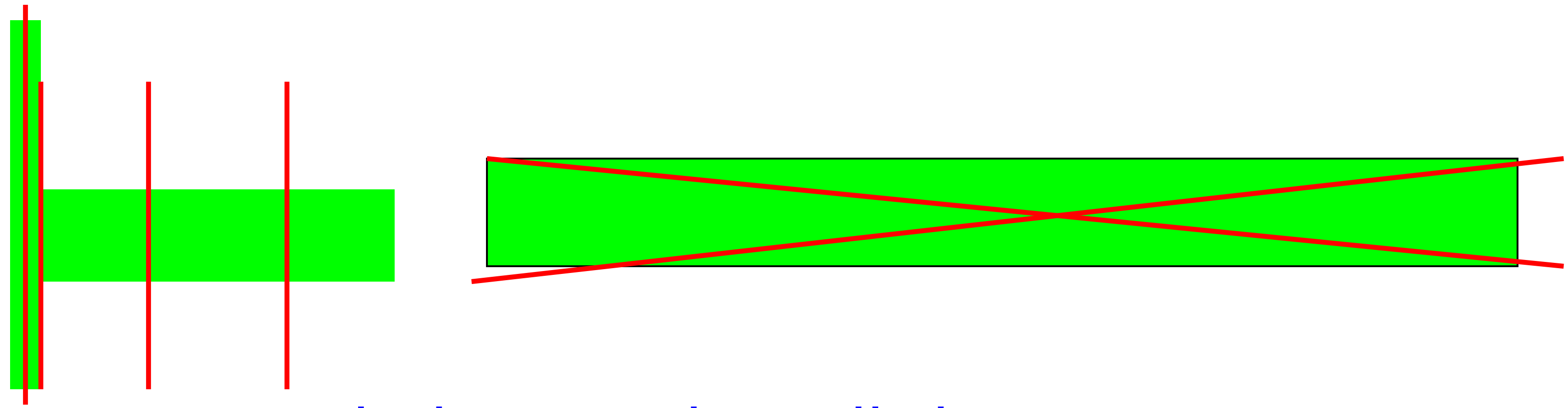
Categorization of Parallel Mesh Generation

- Nikos Chrisochoides in [1] advocated the use of “off-the-shelf” serial mesh generators to develop parallel mesh generator.
- Using this idea parallel mesh generators can be categorized by:
  - Underlying sequential mesh generation algorithm
  - Parallel Coupling

Categorization of Parallel Mesh Generation

- Underlying sequential mesh generation algorithm
  - Octree
  - Delaunay
  - Etc.
- Parallel Coupling
  - Process interface meshed before subproblems meshed
  - Subproblems meshed and then process interface meshed
  - Process interface and subproblems simultaneously meshed

Interface/Artificial Boundary



- Process Boundaries must be well chosen
  - Load must be balanced
  - Process boundaries should be well spaced
  - Process boundaries should not form small angle with other process boundaries or physical boundaries
- Usually not a problem if mesh partitioner is reasonable
- Constrained optimization
- Changing domains can pose a problem

Overview

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Mesh Repair in Rocket Simulation

- Independent Study with Professor Heath and Damrong Guoy
- Want to improve mesh quality of adaptively refined mesh in rocket simulation
- Center for the Simulation of Advanced Rockets (CSAR)
- Terry Wilmarth and Phil Alexander also working on aspects of this project

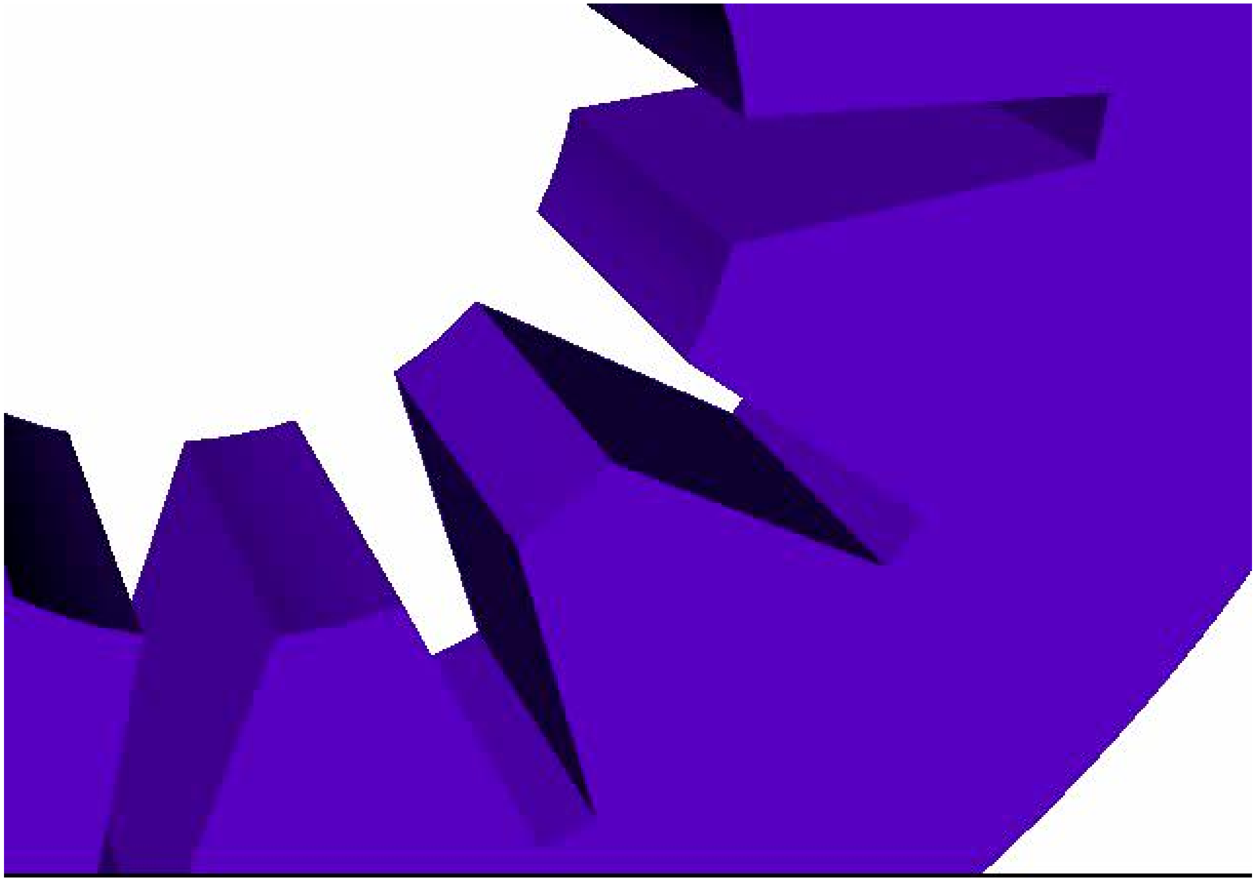
Evolving Geometry of Rocket

- Shrinking solid propellant
- Expanding gas flow
- Deforming due to high pressure
- Crack propagation

Courtesy of Damrong Guoy, CSAR

Evolving Geometry

- http://www.cse.uiuc.edu/~jiao/Rocprop/movies/starslice_entropy.mpg
- http://www.cse.uiuc.edu/~jiao/Rocprop/results.html



Courtesy of Jim Jiao (via Damrong Guoy), CSAR

Poor Quality Elements

- Elements are distorted as a result of the changing geometry
- Elements in expanding region are stretched
- Elements in compressed region are flattened

Solving Mesh Distortion problem

- Mesh Smoothing
  - Moderate change in geometry
- Local mesh repair
  - Significant distortion in local region
- Global remeshing
  - Severe deformity beyond repair

Courtesy of Damrong Guoy, CSAR

Local Mesh Repair

- Repair local distortion
- Preserve large part of the mesh
- Locally refine and coarsen the mesh
- Many basic operations

Courtesy of Damrong Guoy, CSAR

Local Mesh Repair

- Basic operations
  - Vertex relocation
  - Vertex insertion
  - Edge contraction
  - Connectivity flip

Courtesy of Damrong Guoy, CSAR

Local Mesh Repair

- Basic operations
  - Vertex relocation
  - Vertex insertion
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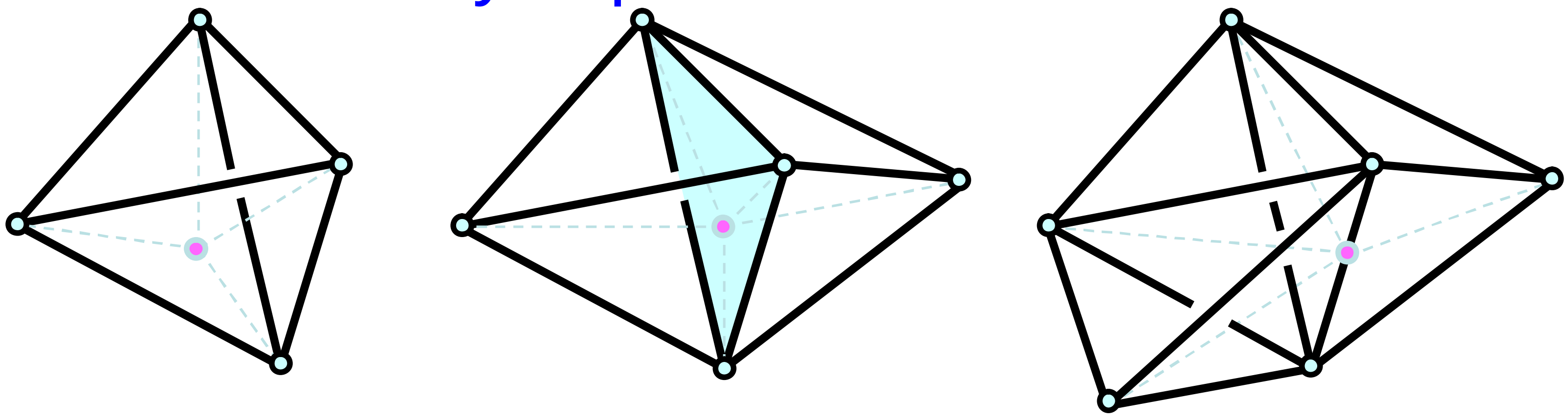

Courtesy of Damrong Guoy, CSAR

Local Mesh Repair

- Basic operations
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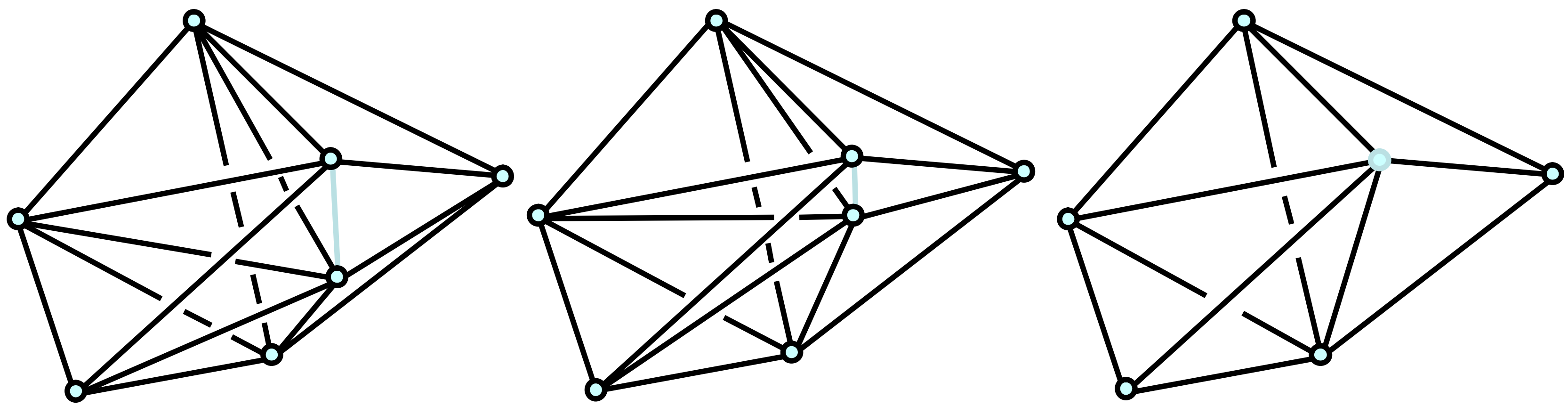

Courtesy of Damrong Guoy, CSAR

Local Mesh Repair

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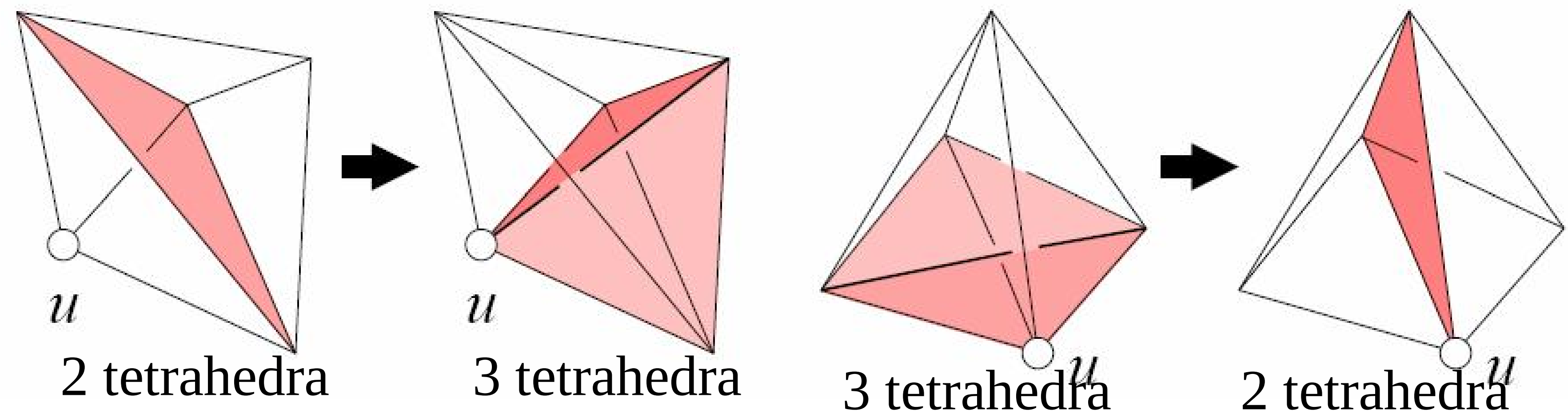
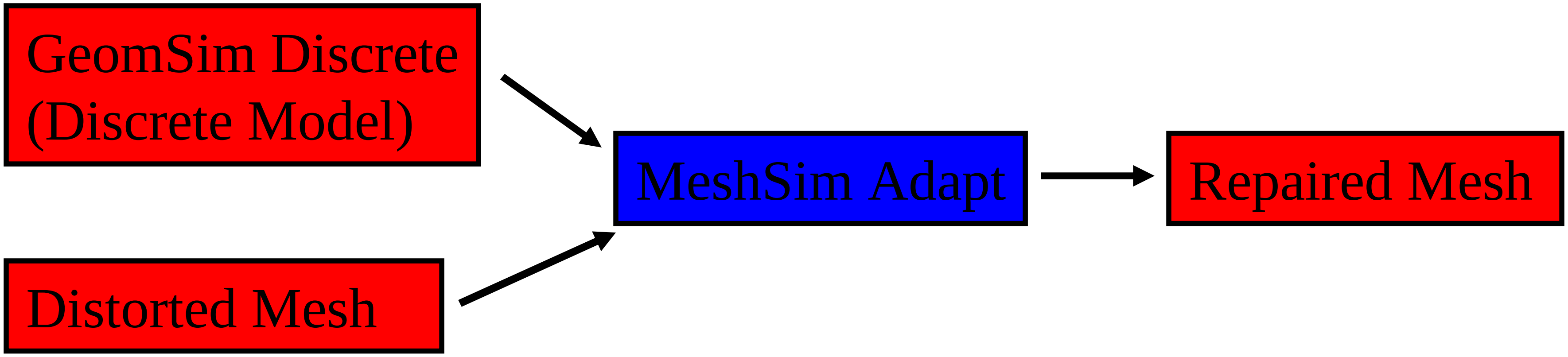


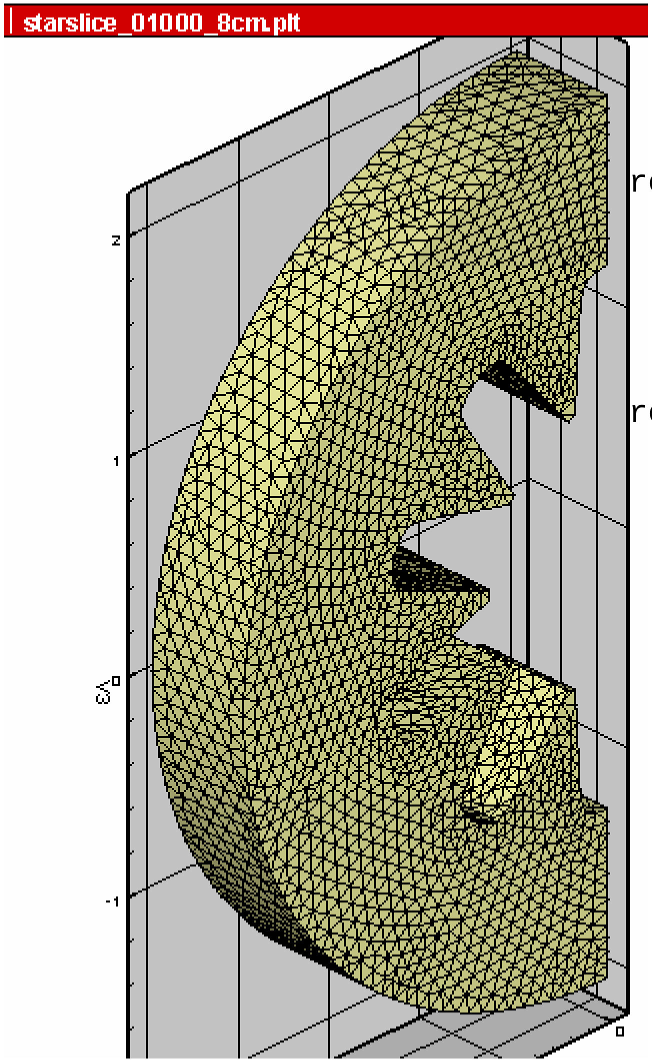
Courtesy of Damrong Guoy, CSAR

Simmetrix

- Using Simmetrix software (M. Shephard) for mesh repair
  - Linux,Mac OS X, Windows
  - Serial and parallel (?)
  - Geometric and discrete model support



Damrong's Global Remeshing Result




```
reportNumMeshEntity() num entity in mesh:-
    13979 vertices
    85533 edges
   139095 faces
    67540 regions

reportMeshQualityStatistics() supported metrics:-
  1. aspect ratio    = longest edge by shortest altitude
  2. smallest dihedral angle
  3. largest dihedral angles
  4. volume skewness = ((optimal size) - (size)) / (optimal size)
  5. rbyR            = unitized ratio of inradius to circumradius
  6. volume
                        _______minimum _______average _______maximum
  aspect ratio                    1.32           2.93          22.09
  small dih angle                 2.97          41.57          68.88
  large dih angle                72.59         105.15         155.65
  volume skewness          0.006635360    0.520993266    0.992803313
  rbyR                     0.103372444    0.670502505    0.996991125
  volume                   0.000000625    0.000034702    0.000242023
```

Courtesy of Damrong Guoy, CSAR

Before Mesh Repair

reportNumMeshEntity() num entity in mesh:-
143389 vertices
935693 edges
1560104 faces
767799 regions

reportMeshQualityStatistics() supported metrics:-
1. aspect ratio = longest edge by shortest altitude
2. smallest dihedral angle (degree)
3. largest dihedral angles (degree)
4. volume skewness = ((optimal size) - (size)) / (optimal size)
5. rbyR = unitized ratio of inradius to circumradius
6. volume

	minimum	average	maximum
aspect ratio	1.24	4.56	169.83
small dih angle	0.71	34.87	69.77
large dih angle	71.08	116.54	**178.36**
volume skewness	0.000208097	0.707317863	0.999999960
rbyR	0.000517771	0.490753236	0.999882321
volume	0.000000044	0.000003054	0.000028641

After Mesh Repair

reportNumMeshEntity() num entity in mesh:-

39211 vertices
219771 edges
336631 faces
156070 regions

reportMeshQualityStatistics() supported metrics:-

1. aspect ratio = longest edge by shortest altitude
2. smallest dihedral angle (degree)
3. largest dihedral angles (degree)
4. volume skewness = ((optimal size) - (size)) / (optimal size)
5. rbyR = unitized ratio of inradius to circumradius
6. volume

	minimum	average	maximum
aspect ratio	1.29	3.28	29.84
small dih angle	2.50	38.90	69.07
large dih angle	72.34	108.67	**173.16**
volume skewness	0.004655401	0.550685298	0.999879335
rbyR	0.014748121	0.631093733	0.997732522
volume	0.000000214	0.000015025	0.000096563

Future Work (near future)

- Better improvement of mesh quality
  - Learn how to use Symmetrix better
  - More iterative mesh-repairing strategy
- Parallel mesh-repair

Acknowledgements

[1] L. Paul Chew, Nikos Chrisochoides, and Florian Sukup. "Parallel Constrained Delaunay Meshing," *In the proceedings of 1997 ASME/ASCE/SES summer meeting, Special Symposium on Trends in Unstructured Mesh Generation,* pp 89-96, June 29 - July 2, 1997, Northwestern University, Evanston, IL.

[2] Nikos Chrisochoides. "A Survey of Parallel Mesh Generation Methods," BrownSC-2005-09.

[3] Damrong Guoy. "Tools and Techniques for Mesh Repair in Rocket Simulation" CSAR seminar. March 30, 2005.

[4] Steve Owen. "An Introduction to Unstructured Mesh Generation." Mesh Generation and Simulation: A Short Course. USNCCM'03 http://www.andrew.cmu.edu/user/sowen/usnccm03/short_course.html