Self organized spatial-temporal structure within the fractured Vadose Zone: Influence of fracture intersections

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[1] Under conditions of unsaturated flow, others have shown experimentally that fracture intersections can direct flow to a single exiting fracture. In addition, they have been found to gather water from above to release as a pulse below. We formulate a simple model where these two behaviors are embedded within a network. With slow steady inflow distributed randomly along the top of the network, the system self organizes to form avalanches of water that can penetrate to great depths. When all intersections split their outflow, flow diverges with depth and develops into a self-organized dynamical state where the distribution of avalanche sizes follows a power-law over many decades. As the fraction of intersections that direct outflow singly is increased, spatial structure passes from divergent through braided to a fully convergent, hierarchical flow regime where avalanche size is minimized along one-dimensional slender pathways. *INDEX TERMS:* 1829 Hydrology: Groundwater hydrology; 1848 Hydrology: Networks; 1869 Hydrology: Stochastic processes; 1875 Hydrology: Unsaturated zone; 3220 Mathematical Geophysics: Nonlinear dynamics. **Citation:** Glass, R. J., and R. A. LaViolette (2004), Self organized spatial-temporal structure within the fractured Vadose Zone: Influence of fracture intersections, *Geophys. Res. Lett.*, *31*, L15501, doi:10.1029/2004GL019511.

1. Introduction

[2] Recent results from both laboratory and field experiments in fractured rock suggest that fracture intersections can force the convergence of pathway; flow from an intersection need not be divided amongst all exiting fractures but can be directed into only one [*Glass et al.*, 2002b; *LaViolette et al.*, 2003]. Laboratory experiments have also shown that fracture intersections can act as hysteretic gates that integrate flow from above until a threshold volume is exceeded, and then release all or a portion of the volume as a pulse below [*Glass et al.*, 2002a; *Wood et al.*, 2002]. The implications of these behaviors at intersections are of great importance to applied hydrological problems in the fractured Vadose Zone where one must characterize the pristine or contaminated formation with limited sampling, predict travel times for water and contaminants, and interpret the results of each to answer specific questions of water quantity and quality. For these situations, understanding the possible spatial-temporal structure for flow and transport is required.

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[3] Here we consider the influence of fracture intersections alone on generating spatial-temporal structure within the fractured Vadose Zone. We model the intersections linked together in a stylized network where each intersection is treated as an integrator and outflow from the intersection can either be split between exiting fractures equally or pass to only one. When all intersections split their outflow, our model is similar to the directed "Sand-Pile", or DSP, that exhibits "Self Organized Criticality" [*Dhar and Ramaswamy*, 1989; *Jensen*, 1998; *Dhar*, 1999]. As the fraction of single outflow increases, channels form due to convergence within the network; spatial structure with depth transitions from divergent to braided to the fully convergent hierarchal end member when only single outflow is allowed. For steady inflow randomly applied to the top of the network, water moves through these defining structures as avalanches that can penetrate to great depths. From divergent to convergent regimes, avalanche size distribution transitions from a smooth power-law to a single value where every avalanche spans the entire system but transmits the minimum volume of water. These results suggest that alone, the individual local action of fracture intersections can create self organized spatial-temporal structure within the fractured Vadose Zone with a wide range of possibility.

2. Model Formulation and Simulations

[4] Let us consider the fracture network as composed of an array of intersections connected by fractures. Water moves down the plane of an individual fracture either as fingers, droplets or in films [e.g., *Glass et al.*, 1995]. If the intersection acts as a capillary barrier, water reaching the adjoining intersection will pool above such as has been seen in experiment [*Wood et al.*, 2002]. As the pool height increases, it will reach a level where the pressure at the intersection is high enough for water to enter the intersection. At this height, the barrier is breached and the pool decreases to a minimum as it passes water below to one or all of the exiting fractures. While the rise or integration phase of the process may be slow, the discharge phase is rapid, as will be the passage of the water to the next intersection(s).

[5] To model this process in abstract, we make use of a separation of time scales. We will consider the in-flow at the top of the network (driving) to be slow and delivered in small "drops" while movement within the network (relaxation) is fast and threshold based when it occurs. Each intersection can be considered as behaving as a "bucket" that operates under a simple local threshold rule: collect

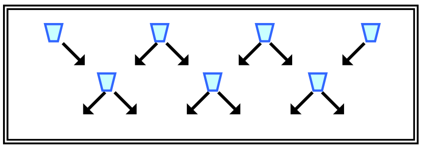

Figure 1. Network of fracture intersections represented by buckets, each bucket connects to two on the row above and two on the row below.

water entering from above and discharge it below only when the water volume reaches or exceeds a maximum value. For this combination of slow drive and rapid local relaxation in a directed network, we obtain a DSP similar to that first studied by *Dhar and Ramaswamy* [1989], which we will call the "Tipping Bucket Model", or TBM. When all the buckets split their outflow, the analogy is complete and the model will evolve into a self organized dynamical state, but far enough from its critical point that we obtain instead the dimension-independent mean-field self-organized behavior [*Jensen*, 1998; *Dhar*, 1999].

[6] For our simulations, we idealize the fracture network as a regular, two-dimensional array of intersections arranged on a diamond lattice. Every intersection, or bucket, is connected to two others on the row above and on the row below (see Figure 1). When the amount of water in a bucket reaches a predetermined value of 10, it tips, passing all of its volume to one or both of its neighboring buckets in the row below. A fraction of buckets, 1-Φ, are chosen at random to distribute flow equally; for the remainder of the buckets, flow goes right or left (but not both) with equal probability. The distribution behavior of any particular location does not change once fixed at the beginning of a simulation. This amounts to adding in quenched disorder into the network, and we anticipate from other work a great sensitivity of the results to Φ [*Tadic and Ramaswamy*, 1996]. Water is added in unit increments at random to buckets along the top row and exits the network from the bottom row. Periodic boundary conditions are implemented along the vertical edges of the network such that the network wraps in the horizontal. In this way, we minimize lateral edge effects in a relatively narrow but tall network (50×1000).

[7] Each simulation is initiated by distributing water at random within the network followed by the slow random addition of unit volumes of water along the top. When a bucket tips, we stop adding water and instead allow the problem to fully relax based on local rules. The tipping of one bucket passes water to the row below where it may cause connected buckets to tip. We follow the tipping process, or "avalanche", downward row by row until no further buckets are found over threshold. We then resume the addition of water along the top. Each simulation is conducted for 10^7 unit additions. For each avalanche, we kept track of the number of buckets that tip and the volume of water that exits the bottom of the network.

3. Spatial-Temporal Structure

[8] As Φ increases, the region swept by avalanches over the course of an entire simulation transitions smoothly from the entire domain to that of a small number of single bucket wide channels (Figures 2 and 3). For Φ near 0, each bucket distributes its flow to both connected buckets below and thus events tend to widen with depth. Past events that have cleared a zone of water will influence the general structure of the edges of an individual event and create a complicated "sculpted" pattern that can occasionally force the narrowing of an individual event (Figure 2a). As Φ increases, local convergence begins to create channels. The swept region takes on a braided structure within which individual events traverse (Figure 2b). The end member of a purely convergent hierarchical structure where all avalanches are constrained to an ever decreasing set of pathways (Figure 2c) is found as Φ approaches 1. The $\Phi = 1$ case corresponds to the Scheideggar model of river networks [*Scheideggar*, 1967].

[9] Channeling is accompanied by a smooth monotonic decrease in the fraction of the domain swept by avalanches (saturation, Figure 3). However, the average number of channels (channels, Figure 3) rises from a single "channel" the width of the network at $\Phi = 0$, to a maximum value at $\Phi \sim 0.8$, and then falls again as Φ increases to 1. This non-monotonic response in channel number is caused by the global dominance of convergence for large Φ. Below the maximum at $\Phi \sim 0.8$, channel number, while highly variable, has no trend with depth. However, above the maximum, convergence imposes a narrowing trend away from the top boundary that becomes most prominent above $\Phi \sim 0.95$.

[10] Every avalanche begins with the tipping of a bucket in the top row. For Φ small, outflow splitting causes the general dissipation of an avalanche with depth. In this divergent regime, many more small avalanches occur than large and so the number of times buckets tip is high at the top of the network and decreases with depth. In the braided regime, this behavior continues, however, at any particular depth, some pathways carry many more avalanches than others due to the happenstance of local convergence (e.g., Figure 2d). But as convergence becomes prominent and global within the hierarchical regime, the concentration of flow is taken to the extreme such that as Φ nears 1, the number of times a bucket tips increases with depth.

[11] Avalanche frequency versus size s compiled over entire simulation periods is shown in Figure 4a while Figure 4b illustrates a small portion of an avalanche size time series. For $\Phi = 0$, avalanche size is erratic in time and the avalanche size distribution $P(s)$ can be expressed as $P(s) \sim s^{-\tau}\exp(-s/s_0)$ [*Jensen*, 1998]. For $\Phi = 0$ we found τ to be 1.5, as expected for the mean field case [*Dhar*, 1999]. The roll-off indicated by the exponential term was not seen in our simulation for $\Phi = 0$, but as it is size dependent, and may require much larger systems to observe. The roll-off also depends upon Φ, and as Φ increases, the roll-off position is forced downward as channeling removes sites from participation (see arrows in Figure 4a). With convergent flow, the power-law scaling itself may also begin to break as channels constrain event sizes to a limited number of values. By Φ of ~ 0.8, this breakdown is clearly evident as a roughening of the distribution (Figure 4a) and as Φ approaches 1, event size distribution transitions through a Gaussian to approach the single value of 1000 where every event cascades through the hierarchical network along single bucket wide channels.

[12] As expected from the behavior of maximum avalanche size, maximum outflow volumes exiting the network at its bottom also decrease with Φ (Figure 5). Network outflow volume approaches a constant minimum value of

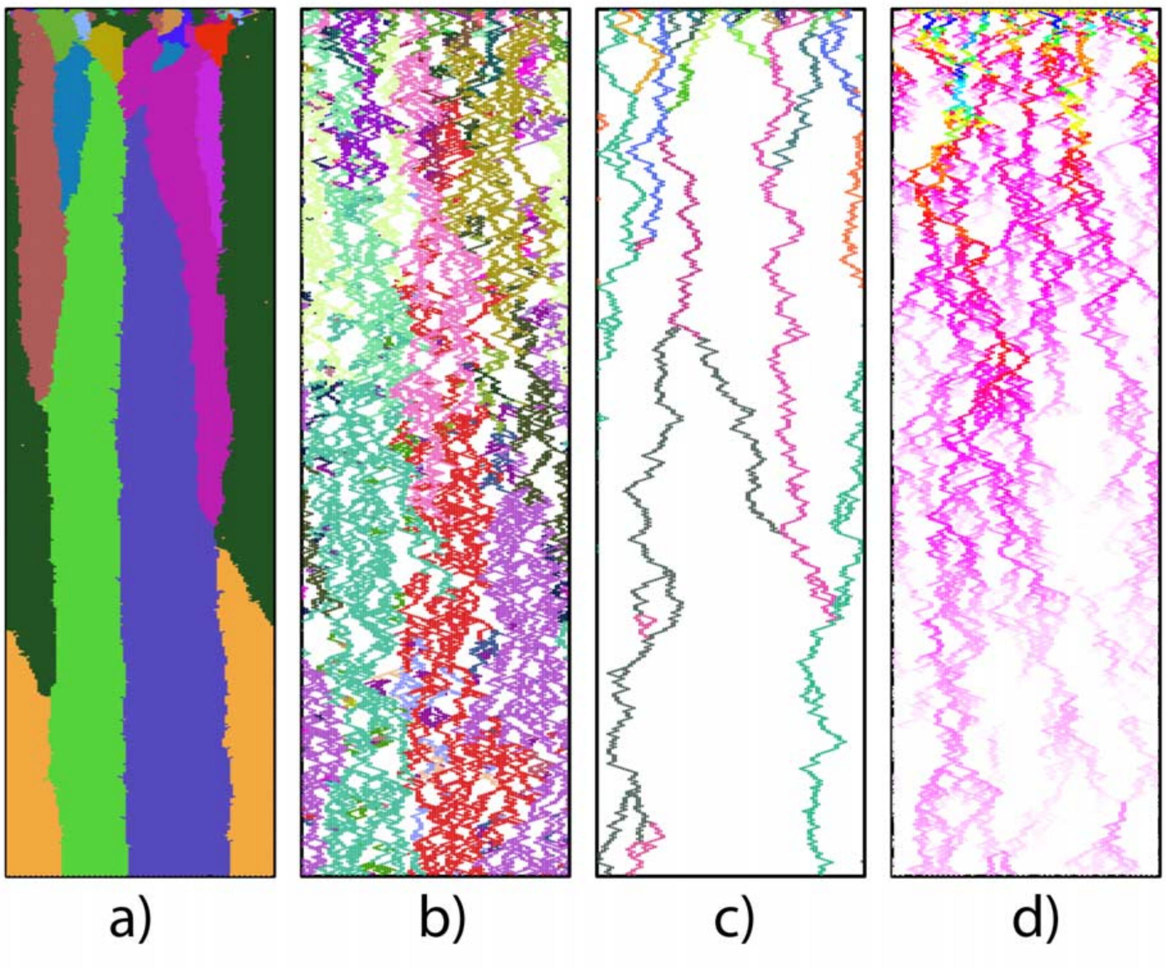

Figure 2. Spatial structure regimes: Avalanches are shown with color separation for a) divergent, $\Phi = 0.05$, b) braided, $\Phi = 0.8$, and c) convergent, $\Phi = 0.98$, spatial regimes. The number of tips for each bucket over the course of the simulation is shown with color gradation for d) $\Phi = 0.8$. Full 50×1000 networks are shown; viewing on a computer screen and zooming in allows individual buckets to be seen.

11 as Φ approaches 1 and convergence dominates. Interestingly, a power-law behavior is not found at any Φ due to the filtering of small avalanches. Filtering occurs because of the downward directed nature of the TBM where input occurs only along the top boundary and imposes a Gaussian character to the outflow volume distribution.

4. Discussion and Conclusion

[13] We considered the influence of two experimentally observed intersection behaviors on flow within fracture networks: integration, and singly directed outflow. We find that integration imposes avalanche behavior that yields an erratic temporal response. The increase in singly directed outflow within the domain causes a transition in spatial regimes from divergent, to braided, to convergent hierarchical and influences the temporal structure by forcing avalanches to stay within constrained structures. Thus, for the set of simple, experimentally supported, local intersection behavior embodied within the TBM, self organized spatial-temporal structure emerges within the network in context of slow random addition of unit volumes of water along the top. As the fraction of singly directed outflows increases, the power-law scaling that dominates the $\Phi = 0$ case is diminished by the convergent flow that begins to constrain the size of events within a braided structure. In the hierarchical regime, flow approaches one dimensional with pulses fully channeled within slender pathways, numerous and small.

[14] We expect that in natural fractured Vadose Zones, a number of the assumptions embodied within the TBM will be violated. For example, threshold volumes for integrating fracture intersections will likely not all be the same as we have currently assumed. However, such variability either with or without spatial correlation should not qualitatively

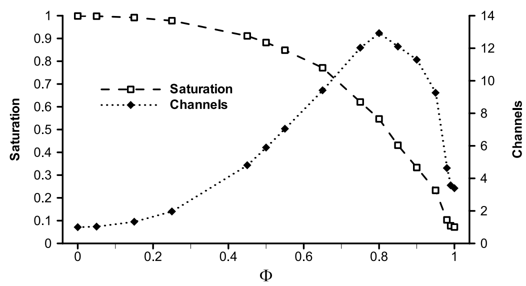

Figure 3. Fraction of domain swept (saturation) and number of channels as functions of Φ.

influence results. It is also likely that when a bucket is passed an "overwhelming" volume, its outflow distribution will tend to split rather than be directed to a single fracture. In combination with variable threshold volumes, pathway switching should result at a number of critical intersections. Pathway switching has been found in several experiments [e.g., *Glass et al.*, 2002a, 2003], and at present, we do not know if it's inclusion in the TBM will significantly alter results. Finally, it is expected that some intersections will not integrate but simply conduct. Replacement of integrators with conductors will not influence the generation of spatial structure but it should eventually dampen temporal fluctuations if a large enough fraction of conductors are present. We note that for the end member where every intersection acts as a conductor, all avalanches will reach the bottom of the network and be forced to the maximum size for the given structure. In this limit, the outflow volume at the bottom of the network will converge to the single "drop", that is, the signal supplied at the top will come out the bottom without modification.

[15] In conclusion, we have found that the local action of fracture intersections can create a range of self organized spatial-temporal structure at the scale of the fracture network. Such macro-scale structure is of great importance to applied hydrological problems in the fractured Vadose Zone where understanding the possible spatial-temporal structure for flow and transport is required.

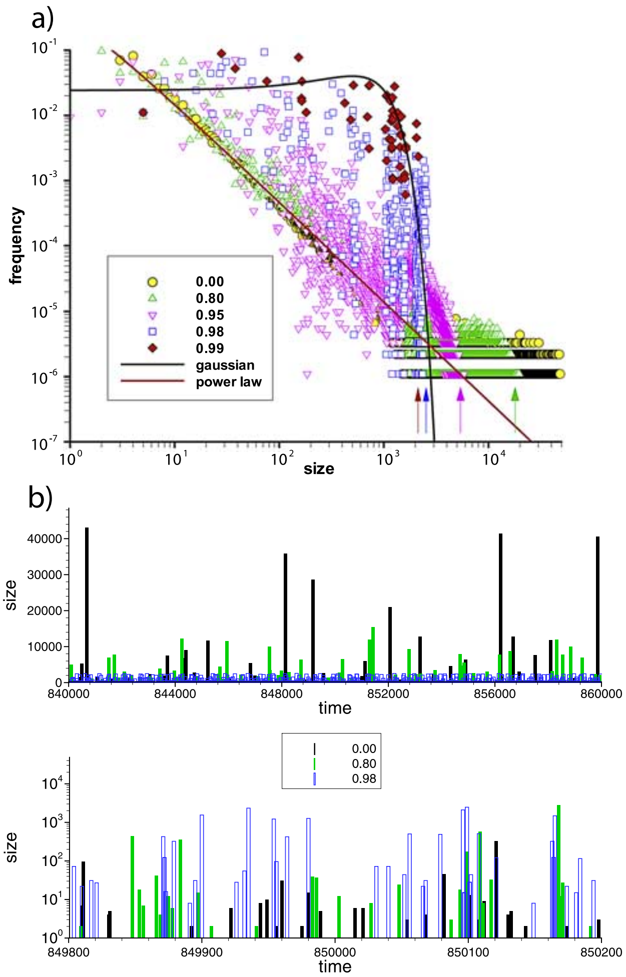

Figure 4. Temporal structure avalanche size: a) Log frequency vs. log avalanche size for entire simulations with example power-law and Gaussian distributions. Arrows denote roll-off values. b) Avalanche size in time for two nested cycle periods further illustrating the highly erratic temporal behavior (Φ near 0) which is suppressed as Φ increases.

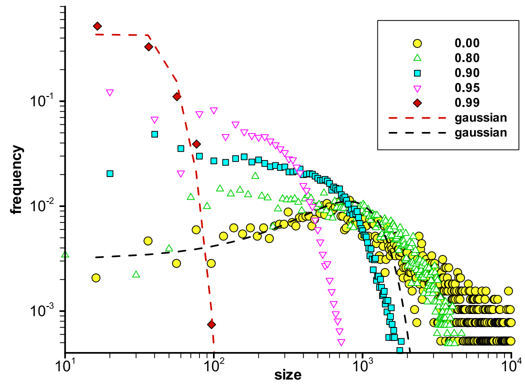

Figure 5. Temporal structure bottom outflow volume: Log frequency vs. log bottom outflow volume for entire simulations. Two example Gaussian distributions are shown for comparison.

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