Applications of Machine Learning in Plasma Physics

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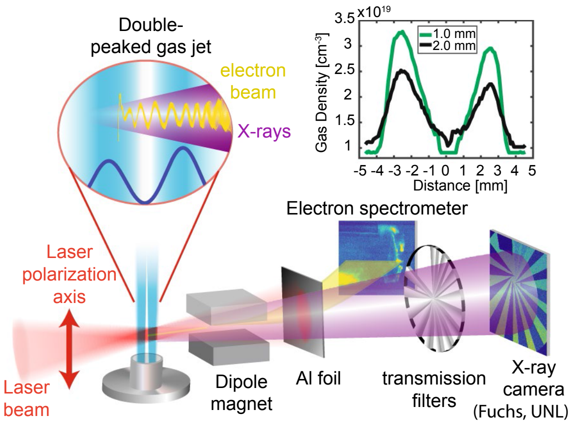
Overview

- Primarily interested in basic plasma physics but with significant technological applications.
- Can't tolerate "black box" solutions. Need interpretability (mostly).
- Applications are all parameter estimation. Classification seems to have no use case in our work.
- Our training data is produced by simulations.
- Use ML to produce an approximation that can then be refined or validated by direct computations.
- Only makes sense if ML is computationally more efficient than the direct calculations.
- Big return from using ML to find reduced models.

Laser-Driven X-Ray Sources: Betatron Radiation

- Generate x-rays by causing energetic electrons to oscillate (violently) in a spatially varying electromagnetic field.
- Same mechanism as in a free-electron laser but incoherent, *i.e.*, without feedback.
- The electrons and fields are produced by a laser pulse ionizing a gas and subsequently driving large amplitude waves in the plasma.
- "Table top" version of a multi-kilometer facility.



Multitude of imaging applications ranging from medicine, biological and agricultural research to materials physics and national security.

Betatron Radiation

Calculating the Emitted Spectrum

- Comparing the radiation spectrum to theoretical predictions is a key element in validating our understanding of this system.
- Given an electron trajectory, compute the fields from the Liénard–Wiechert potentials:

$$\text{trajectories} \Longrightarrow \boldsymbol{A}(\boldsymbol{r}, t) \Longrightarrow \widetilde{\boldsymbol{A}}(\boldsymbol{r}, \omega) \Longrightarrow \frac{d^2 I}{d\omega d\Omega}\,.$$

  Straightforward but expensive.
- Spectra from each particle are added incoherently.
- Neither electron trajectories nor the fields can be obtained from experiments.
- Obtaining the spectrum directly from the plasma physics simulations is not practical:
  - The radiation fields are much shorter wavelength than the mean plasma fields.
  - Using a grid to obtain the radiation from Maxwell's equations is not computational reasonable.
  - To sample the particle trajectory with enough temporal resolution results in unmanageable amounts of data.

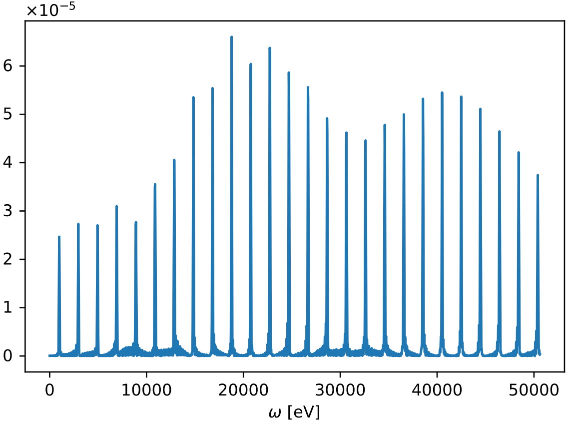
- Since we don't have a good representation of the electron trajectories, we can live with some error in the spectrum.
- Sample the fields and particle phase space from the plasma physics simulations and use this as inputs to a MLP to estimate the single particle spectrum.
- We have a proof-of-concept implementation.

Calculating the Emitted Spectrum

Single Particle Spectrum

- Magnetic field
  - $\boldsymbol{B} = B_0\,\widehat{\boldsymbol{y}}$.
  - $B_0 = 400\,\mathrm{T}$.
  - Period: 100 microns.
  - Extent: 1 mm.
- Electron
  - Moving in the z direction.
  - Energy: 307 MeV ($\gamma = 600$).
- Spectrum is a series of spikes with little other structure.
- Spikes depend on γ and B_0.
- For an electron beam with a range of energies, the spikes are blurred out in the complete spectrum.

Spectrum looking in the $-z$ direction

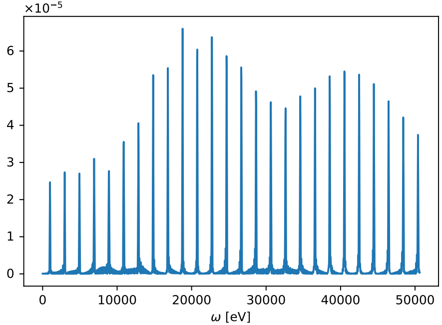


Calculating the Emitted Spectrum

ML Approach

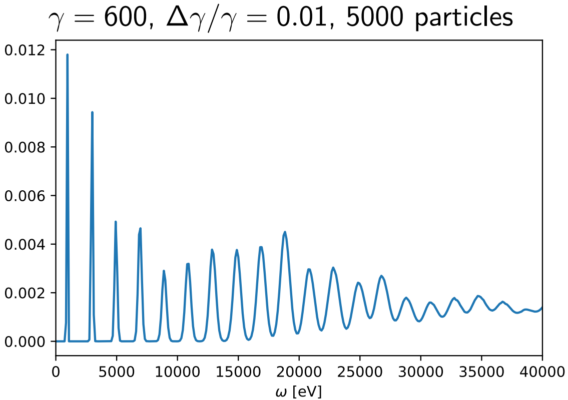
- Extract ω_k and a_k for the first 25 spikes.
- Training data generated for $100 \leq \gamma \leq 1000$.
- Separate MLPs for estimating $\{\omega_k\}$ and $\{a_k\}$.
- Construct the spectrum from an electron beam by a weighted average of the single-particle spectra.
- Need to generalize field structure and initial electron direction.

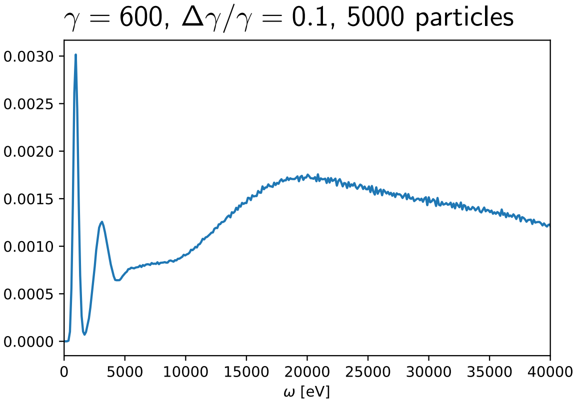
Spectrum looking in the $-z$ direction



Calculating the Emitted Spectrum

ML Results

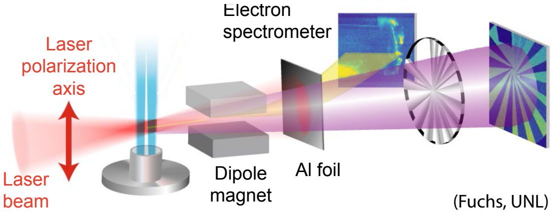




- Drop-off at large ω due to only keeping 25 harmonics.
- Several orders of magnitude faster than direct calculation from trajectories.
- More validation work needed.

Inverse Problem: Electron Beam Phase Space from the Energy Spectrum

- Knowledge of the electron beam phase space is critical to interpreting many experiments.
- Direct measurement of the beam phase space is not practical.
- Measure total charge.
- Measure energy with a magnetic spectrometer.
- Imaging of the beam in one spatial direction.



- All measurements are integrated over the beam head-to-tail.
- Determine macroscopic beam parameters (6-D correlation matrix, *i.e.*, beam emittance).
- Ill-posed inverse problem. Regularize using physics insight.
- Use ML parameter estimation to produce a starting point for a nonlinear optimization.

Vlasov–Maxwell System

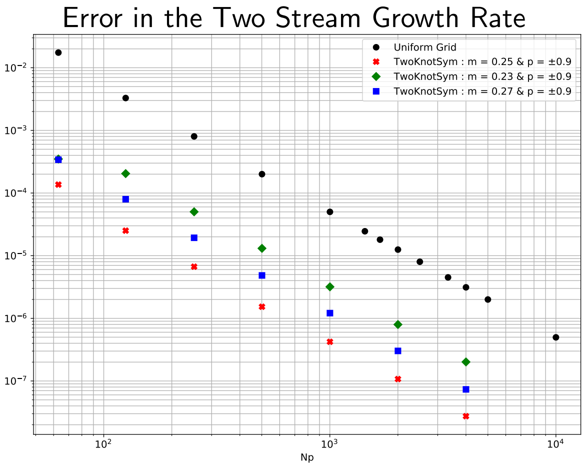
The Vlasov–Maxwell equation is a phase space transport equation that well-describes many low collisionality plasmas.

- 1-particle phase-space distribution function $f(\boldsymbol{r}, \boldsymbol{p}, t)$.
- The evolution of f is given by
$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + q\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right) \cdot \frac{\partial f}{\partial \boldsymbol{p}} = 0\,,$$
where
$$\boldsymbol{v} = \boldsymbol{p}/\gamma\, m \quad \text{and} \quad \gamma = \sqrt{1 + p^2/m^2c^2}\,.$$
- $\boldsymbol{E}$ and $\boldsymbol{B}$ are determined from the charge and current densities
$$\rho(\boldsymbol{r}, t) = q \int d^3\boldsymbol{p}\, f(\boldsymbol{r}, \boldsymbol{p}, t) \quad \text{and} \quad \boldsymbol{j}(\boldsymbol{r}, t) = q \int d^3\boldsymbol{p}\, \boldsymbol{v}\, f(\boldsymbol{r}, \boldsymbol{p}, t).$$
- Computationally demanding in two or more spatial dimensions.
- Not all of phase space contributions equally to the fields; some regions are more important than others.

Vlasov–Poisson System

Improving Computational Performance with a Non-Uniform Momentum Grid.

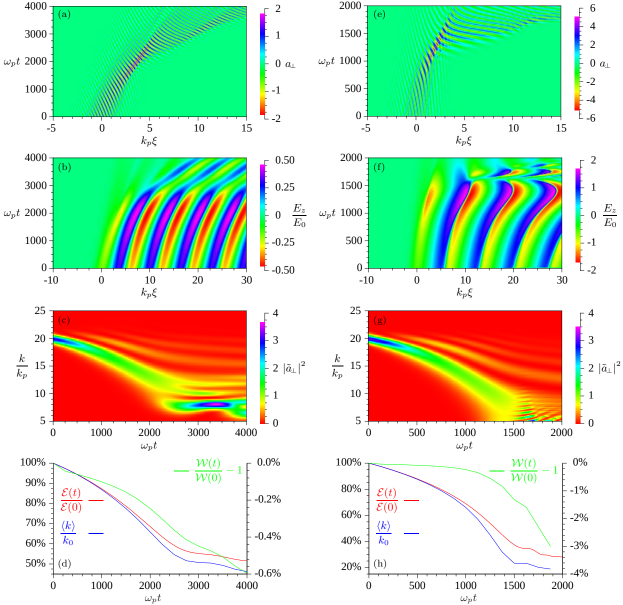
- Specialize to one spatial dimension, non-relativistic motion, and consider only electrostatic fields.
- Non-uniform grid in momentum space leads to significant computational savings.
- Use ML to predict grid parameters for a class of problems.
- How much training data is needed?
- How close can the predictions get to the optimal grid?
- This work is just beginning.



Reduced Models: Wakefield Excitation and Laser Pulse Evolution

- The laser pulse deposits energy into the plasma by exciting waves.
- The energy lost by the laser pulse manifests as a redshifting of the pulse.
- Not obvious; seems that the pulse amplitude should just decrease.
- An adiabatic invariant, wave action, forces the laser wave number to decrease as energy is lost:

$$\frac{\langle k \rangle}{k_0}\,\mathcal{W} = \frac{k_0}{k_p}\,\mathcal{E}\,.$$

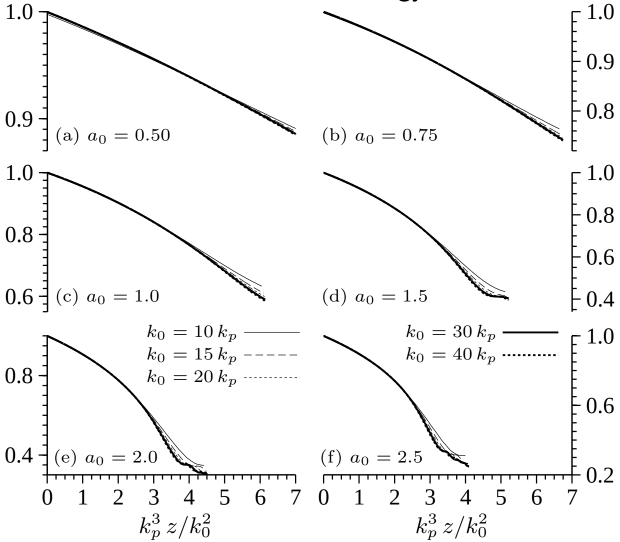


Reduced Models

- There is an set of universal scaling laws.
- Except for small k_0/k_p, the depletion is independent of k_0.
- The spatial scale length for energy depletion is

$$\frac{k_p^3 z}{k_0^2} .$$

Laser Pulse Energy



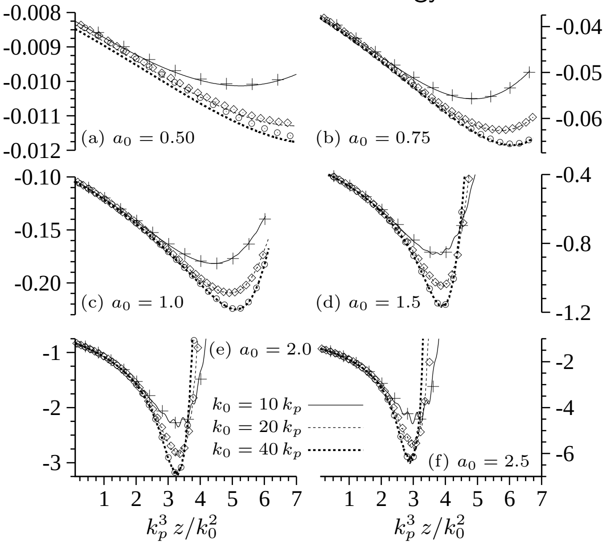
Reduced Models

- The rate of energy loss of the laser pulse is given by

$$\frac{d\mathcal{E}}{d\omega_p\, t} = -\left(\frac{k_p}{k_0}\,\frac{E_{\max}}{E_0}\right)^2 .$$

- We have evidence for additional "hidden" scaling laws but these have not yet been fully explored or understood.
- Could this have been found by ML using latent-space methods?

Rate of Laser Pulse Energy Loss



Conclusions

- Numerous interesting applications of ML in basic plasma physics.
- Proof-of-concept implementation for computing x-ray spectra in laser-driven light sources.
- Finding scaling laws and reduced models is a high-payoff application. How?
- Possible applications in real-time guiding of experiments?

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