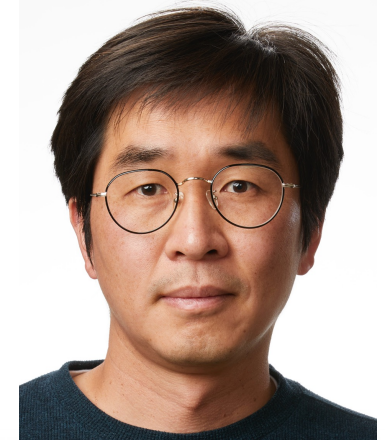


gLaSDI: Parametric physics-informed greedy latent space dynamics identification

6th Annual Sandia MLDL Workshop

July 25th, 2022



Youngsoo Choi



Awesome reduced order model team and collaborators



Dylan Copeland



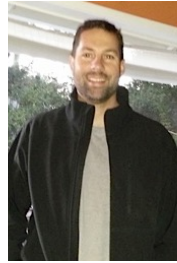
Kevin Huynh



Tony Cheung



Youngkyu Kim



David
Widemann



Tarek Zohdi



Jessie Lauzon



Jon Belof



Quincy Huhn



Michael
Juhasz



Kevin
Carlberg



Peter
Brown



Bob
Anderson



Sean
McBane



Karen
Willcox



Rob
Rieben



Debojyoti
Ghosh



Eric Chin



Keo
Springer



Dan
White



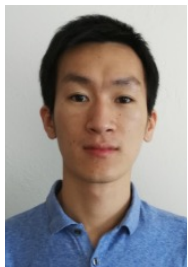
Geoff
Oxberry



Trenton
Kirchdoerfer



Karen Wang



Xiaolong He



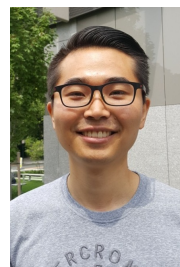
William Fries



Teeratom
Kadeethum



Nikolaos
Bouklas



Yeonjong Shin



Saad
Khairallah



Bedros Afeyan



Jean Ragusa

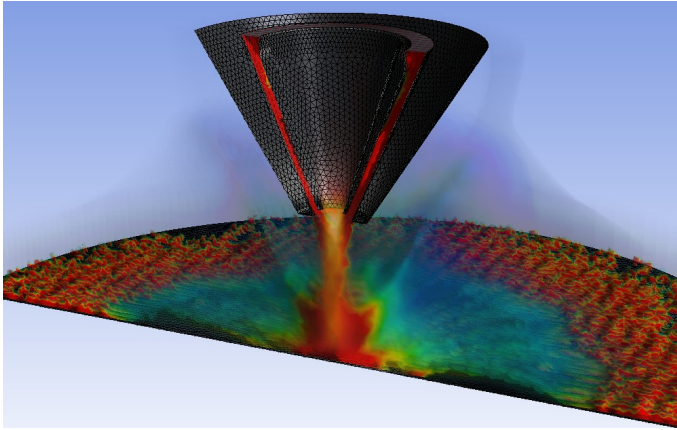


Charles
Fredrick Jekel

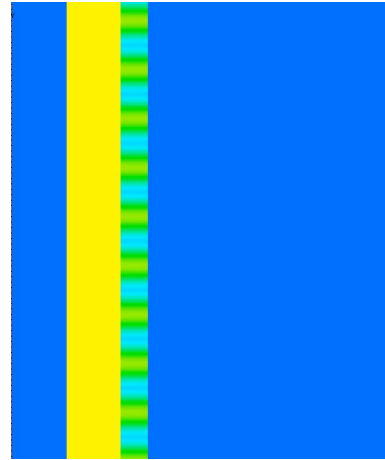


Xiaoting
Zhong

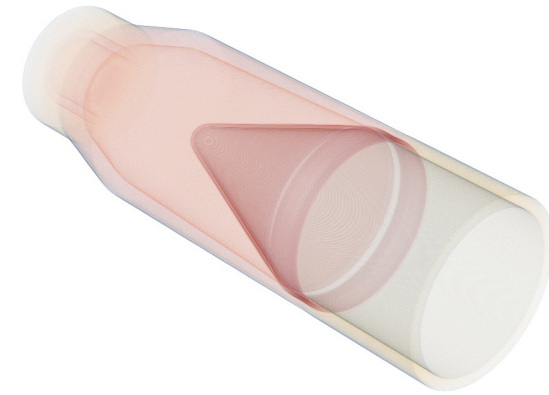
Physical simulations play an important role in modern science



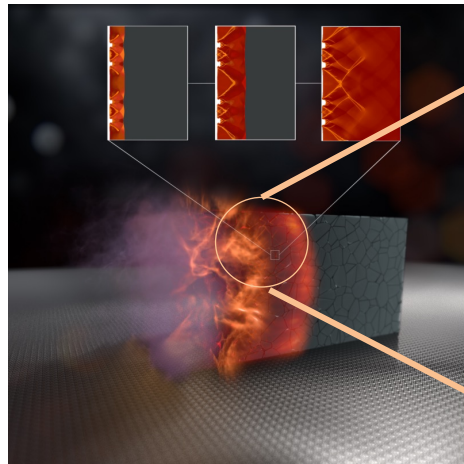
3D Direct energy deposition
(40 minutes on 4 cores) ANSYS



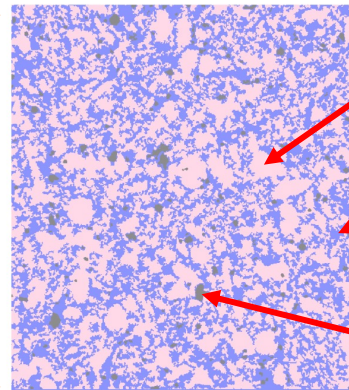
2D Shockwave
(1 hour on 40 cores)
BLAST



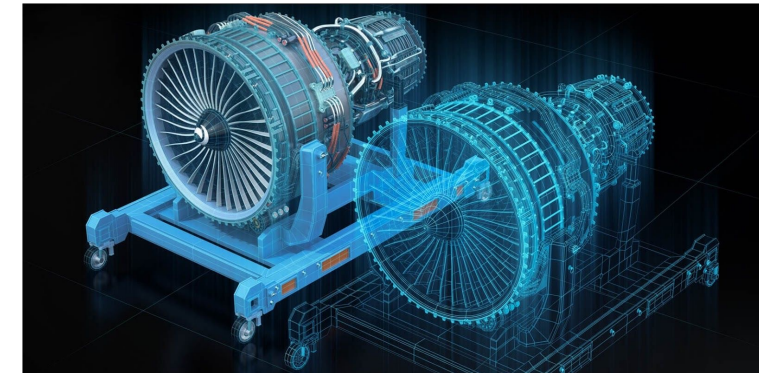
3D shaped charge
(23 hours on 96 GPUs)
BLAST



Pore-collapse (1 week on 1024 cores) ALE3D



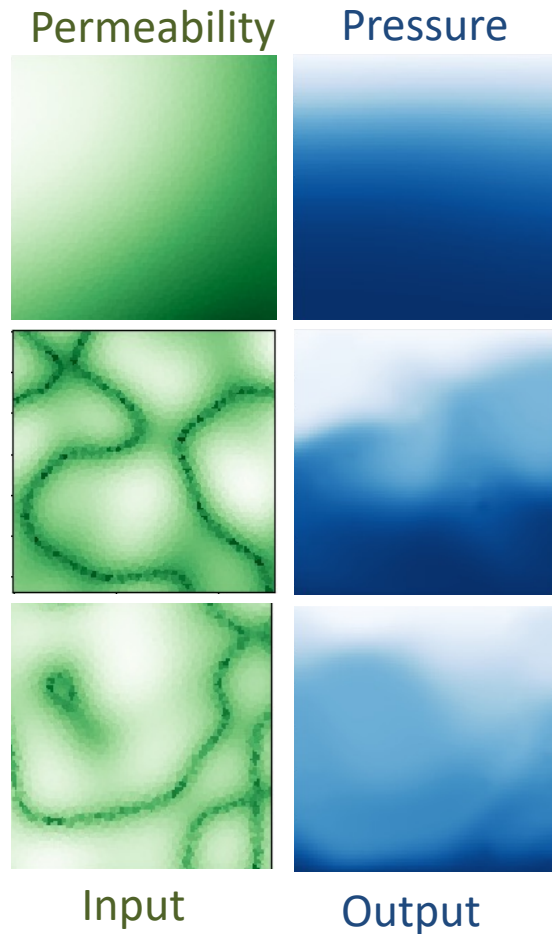
HE Xtal
Energetic Binder
Voids



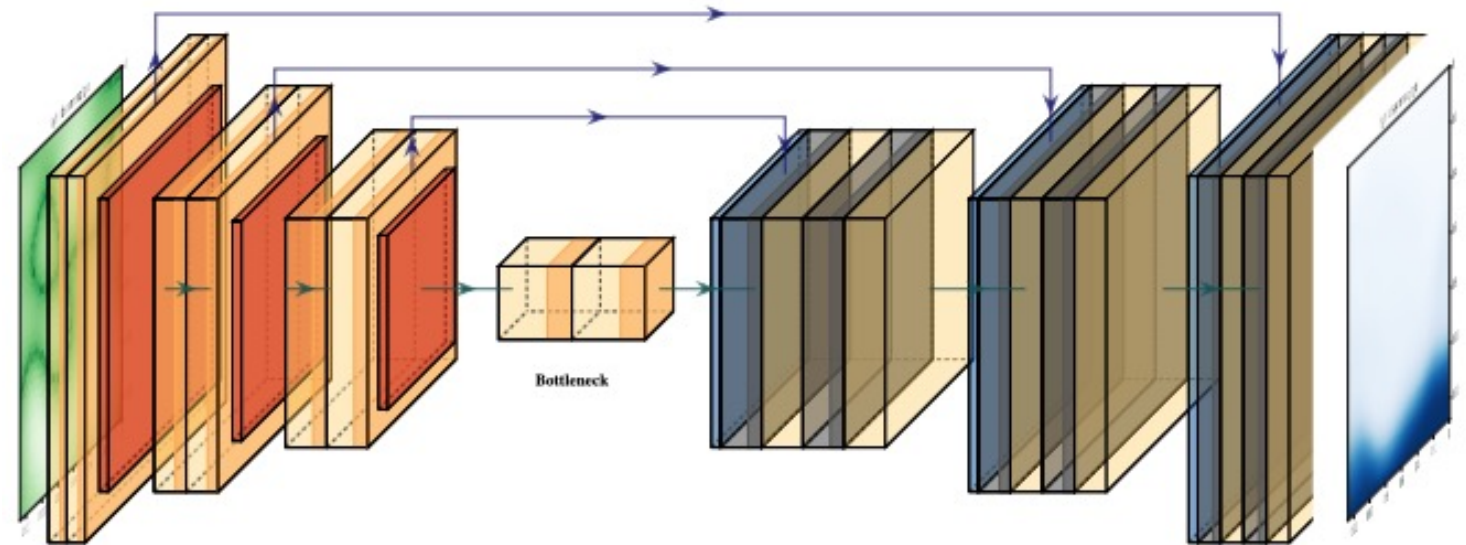
Digital Twin: source from
thedigitalspeaker.com

How can you accelerate existing physical simulations with data?

1. Generate Simulation data



2. Get the relation between input and output, e.g., training a neural network



Conditional Generative Adversarial Neural Network

Blackbox approach

*Kadeethum, O'Malley, Fuhg, Choi, Lee, Viswanathan, Bouklas. "A framework for data-driven solution and parameter estimation of PDEs using conditional generative adversarial networks." *Nature Computational Science*, 2021.

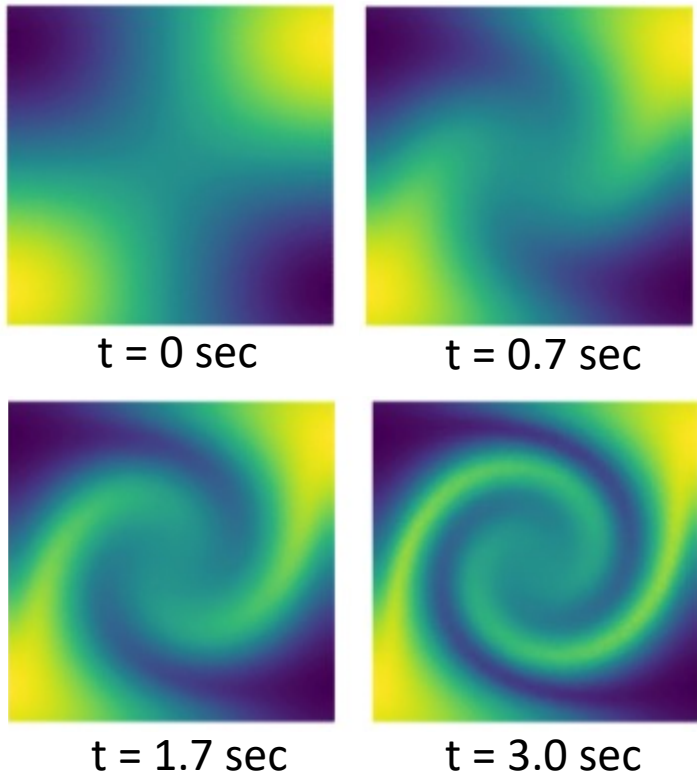
How can we get an interpretability? LaSDI

Radial advection:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{u} = \epsilon \quad \Omega = [-1, 1] \times [-1, 1], \quad t \in [0, 3], \quad \mathbf{v} = \frac{\pi}{2} d [x_2, -x_1]^T, \quad d = (1 - x_1^2)^2 (1 - x_2^2)^2$$

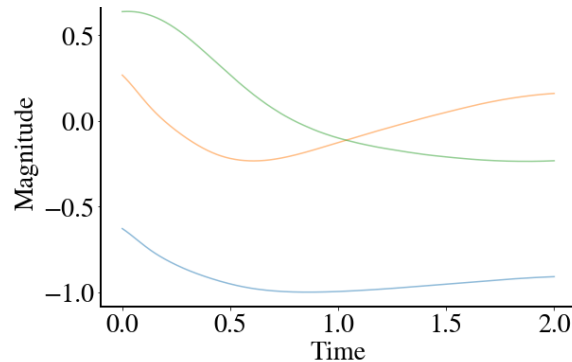
$$u(\mathbf{x}, t; \boldsymbol{\mu}) = 0 \quad \text{on} \quad \partial\Omega, \quad \text{Parameterized initial condition: } u(\mathbf{x}, 0; \boldsymbol{\mu}) = \sin(w_1 x_1) \sin(w_2 x_2)$$

high dimensional simulation data



compress

Latent space dynamics data
with dimension of 3



Linear compression: POD, SVD, ...
Nonlinear compression: Autoencoder

Dynamic mode decomposition

(DMD):

$$\frac{d\hat{\mathbf{u}}}{dt} = \hat{\mathbf{A}}(\boldsymbol{\mu}) \hat{\mathbf{u}}$$

Fit into
ODE

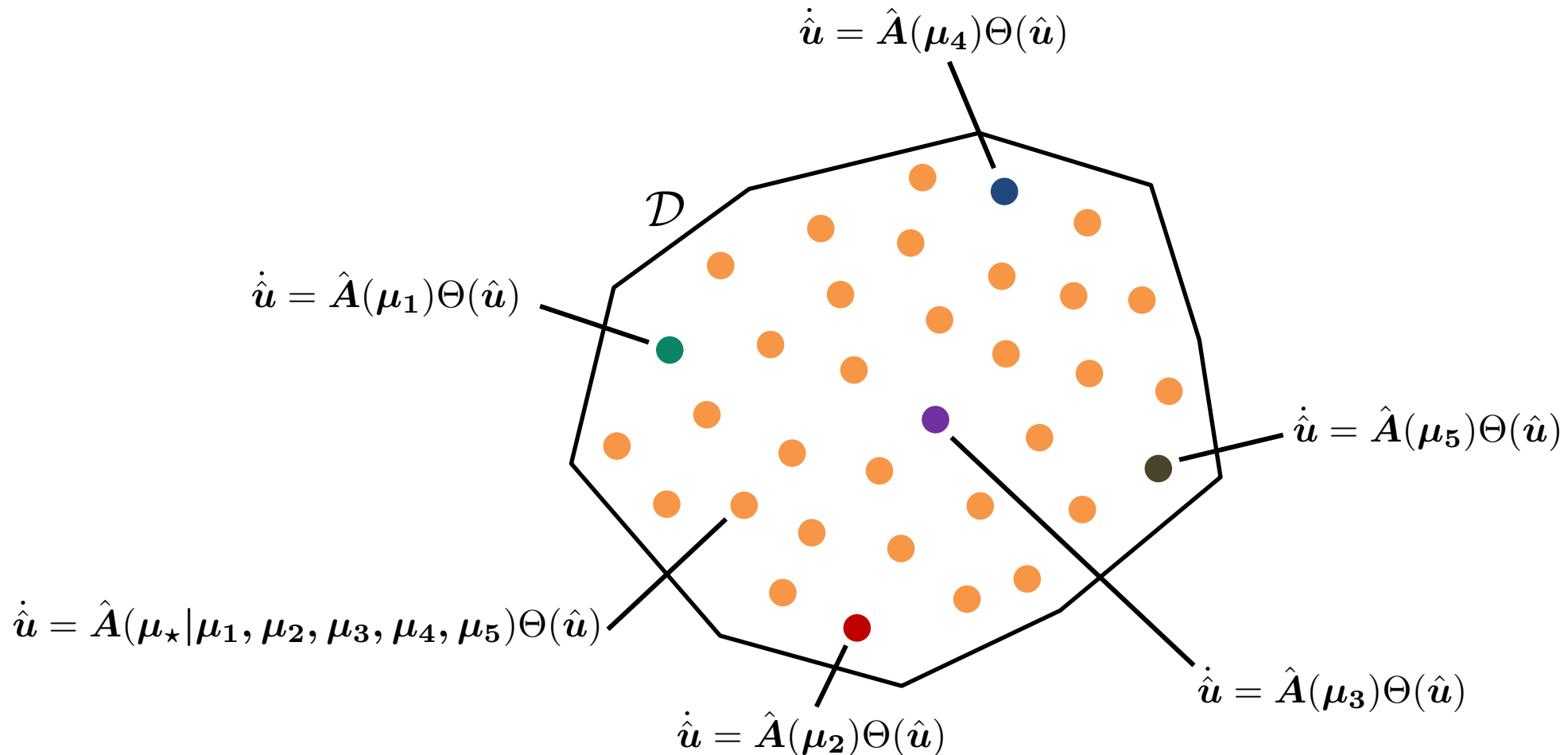
$$\frac{d\hat{\mathbf{u}}}{dt} = \hat{\mathbf{A}}(\boldsymbol{\mu}) \Theta(\hat{\mathbf{u}})$$

Operator Inference

(OpInf):

$$\frac{d\hat{\mathbf{u}}}{dt} = \hat{\mathbf{A}}(\boldsymbol{\mu}) \hat{\mathbf{u}} + \hat{\mathbf{H}}(\hat{\mathbf{u}} \otimes \hat{\mathbf{u}})$$

Parameterized latent space dynamics identification (LaSDI)



Fries, He, Choi, "Lasdi: Parametric latent space dynamics identification." arXiv:2203.02076, 2022.

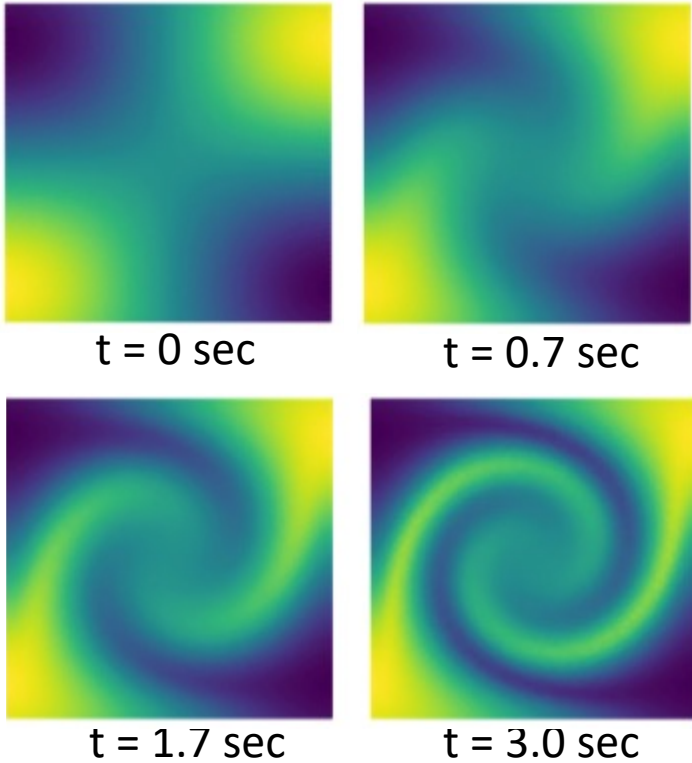
Performance of LaSDI to radial advection problem

Radial advection:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{u} = \epsilon \quad \Omega = [-1, 1] \times [-1, 1], \quad t \in [0, 3], \quad \mathbf{v} = \frac{\pi}{2} d [x_2, -x_1]^T, \quad d = (1 - x_1^2)^2 (1 - x_2^2)^2$$

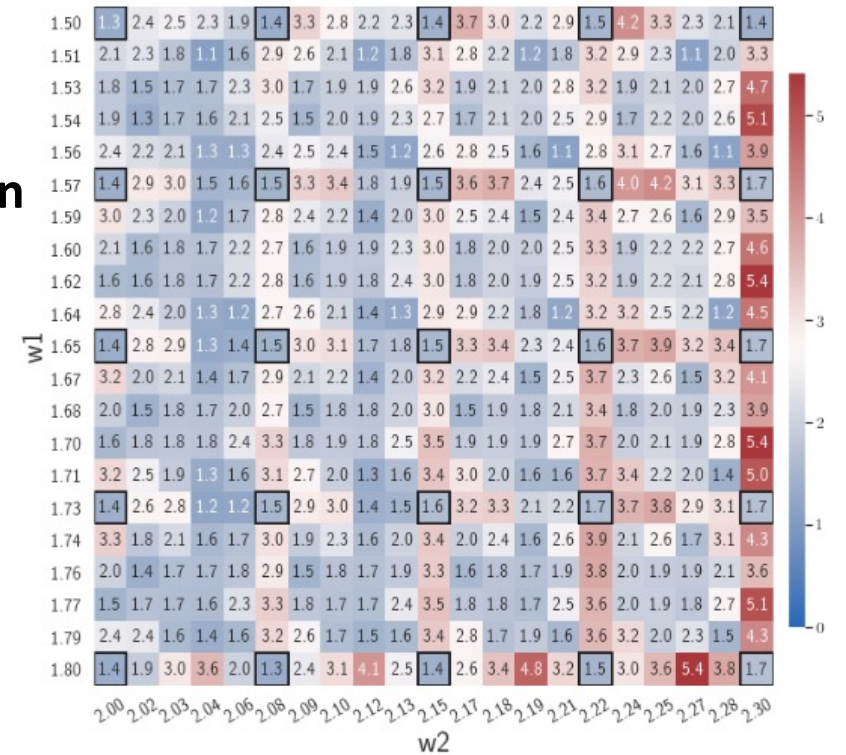
$$u(\mathbf{x}, t; \boldsymbol{\mu}) = 0 \quad \text{on} \quad \partial\Omega, \quad \text{Parameterized initial condition: } u(\mathbf{x}, 0; \boldsymbol{\mu}) = \sin(w_1 x_1) \sin(w_2 x_2)$$

High dimensional simulation data

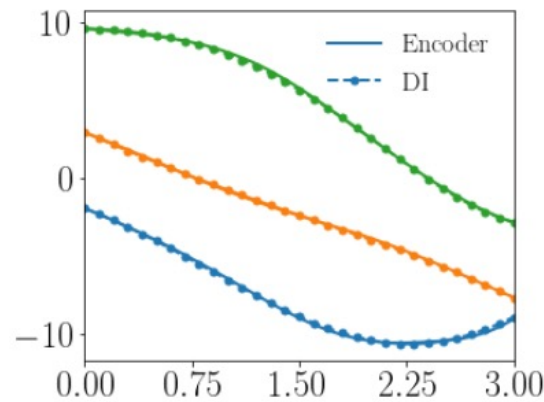


Maximum relative error:
5.4% with 25 uniformly sampled training points

Speed-up of 200x

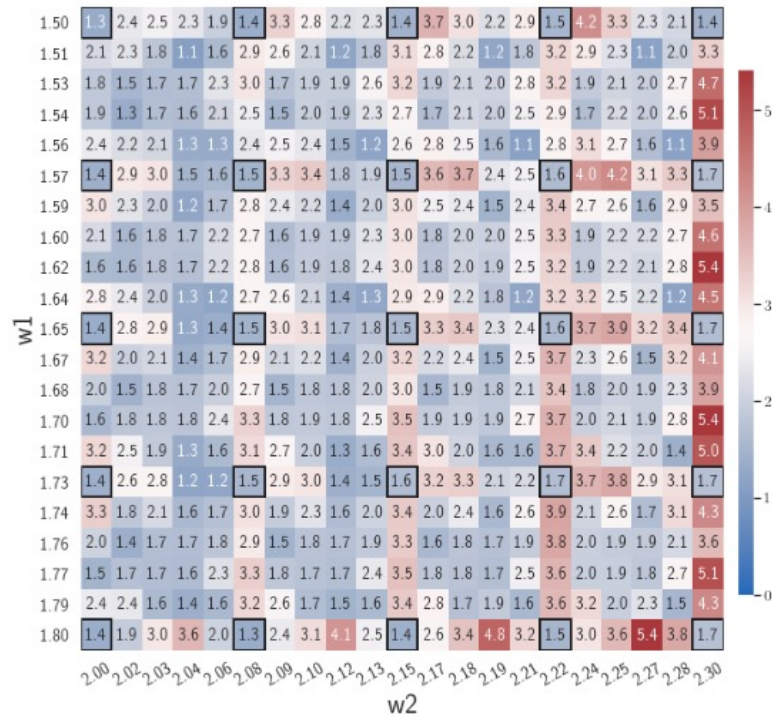


Latent space dynamics identification
with a dimension of 3



Is uniform sampling enough? No, so we need physics-informed greedy sampling!

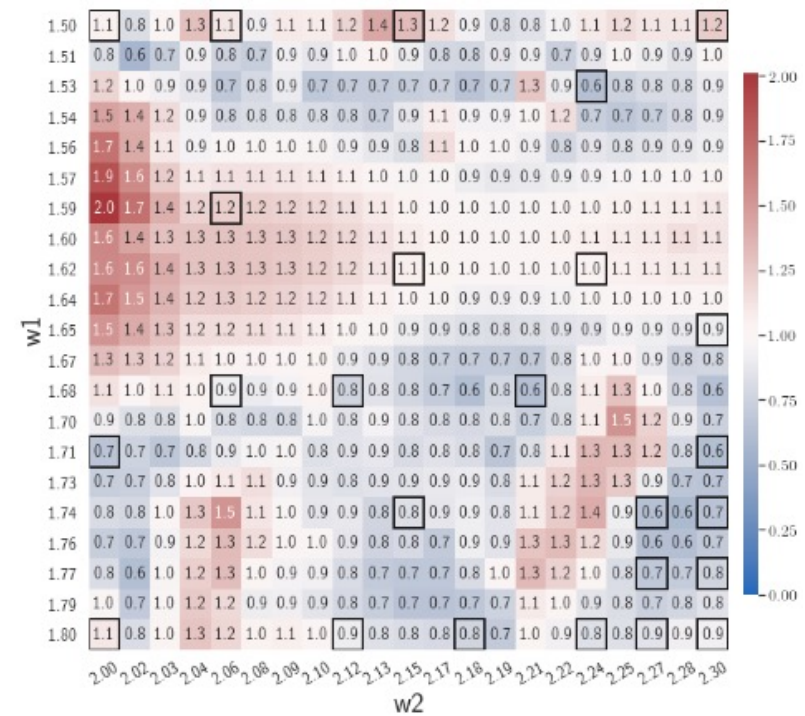
Uniform sampling



Maximum relative error:

5.4% with 25 uniformly sampled training points

Physics-informed greedy sampling

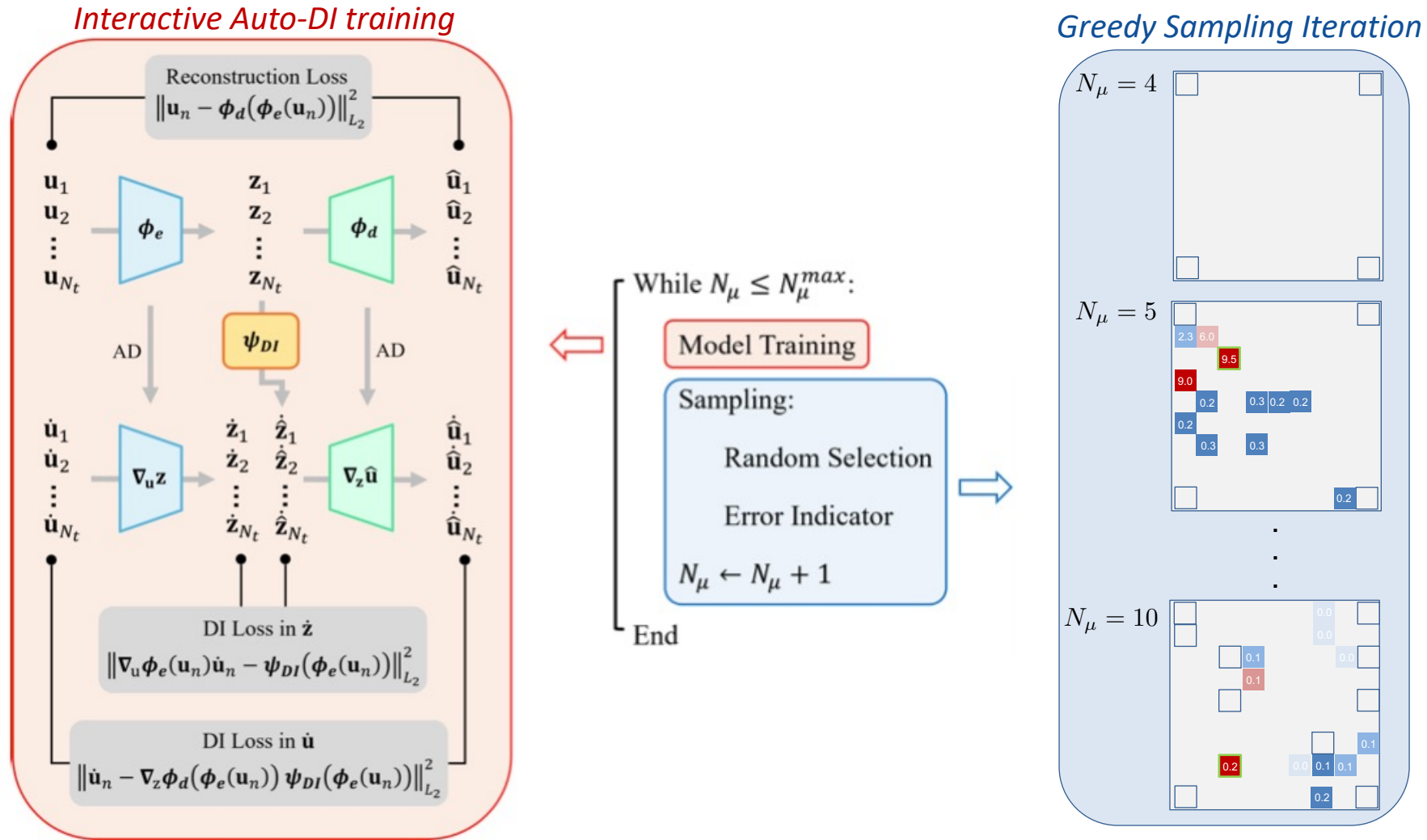


Maximum relative error:

2.0% with 25 greedy sampling points

*He, Choi, Fries, Belof, Chen. "gLaSDI: Parametric Physics-informed Greedy Latent Space Dynamics Identification." arXiv:2204.12005. 2022.

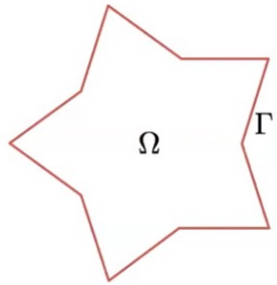
gLaSDI: physics-informed greedy latent space dynamics identification*



*He, Choi, Fries, Belof, Chen. "gLaSDI: Parametric Physics-informed Greedy Latent Space Dynamics Identification." arXiv:2204.12005. 2022

Curious about the physics-informed greedy procedure?

- Watch this YouTube video (less than 10minutes) : <https://youtu.be/A5JlIXRHxrl>



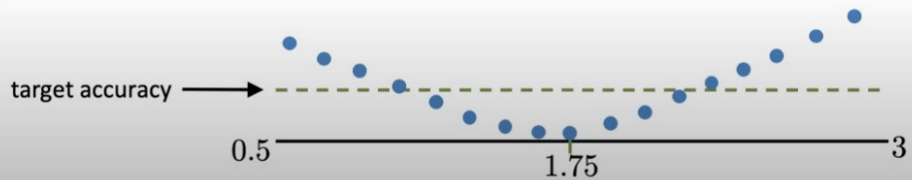
$$-\Delta u = f \text{ on } \Omega$$

$$u = 0 \text{ on } \Gamma$$

$$f(x) = \sin(\kappa(x_0 + x_1))$$

Goal: Find a near-optimal set of $\kappa \in [0.5, 3]$ whose reduced order model achieves relative error less than 1%

Greedy procedure



Physics-informed error measure: error indicator

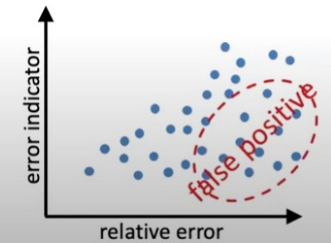
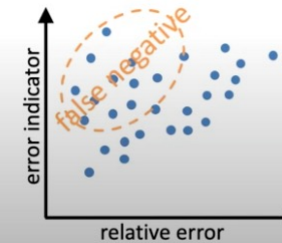
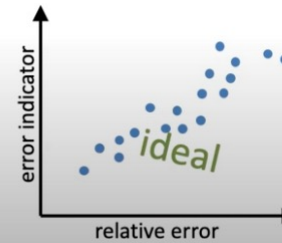
Two characteristics for efficient error indicators:

- Easy to evaluate

$$\frac{\|u - \tilde{u}\|}{\|u\|}$$

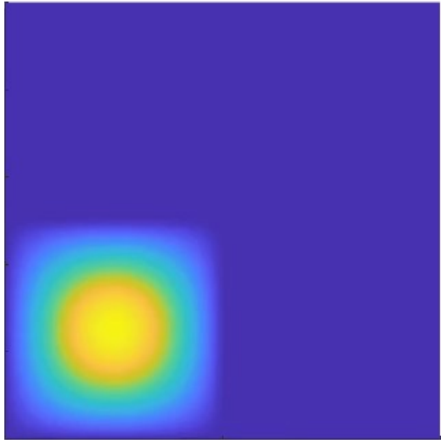
No full order model solution is allowed!

- Strongly correlated with an actual error measure



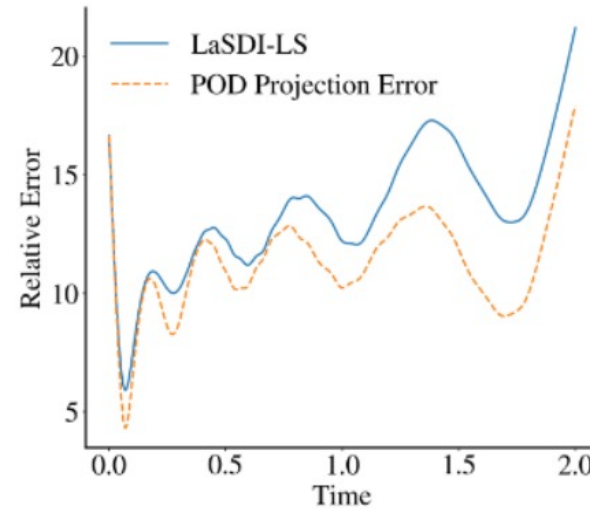
Does linear compression always work? No

Benefit of nonlinear compression

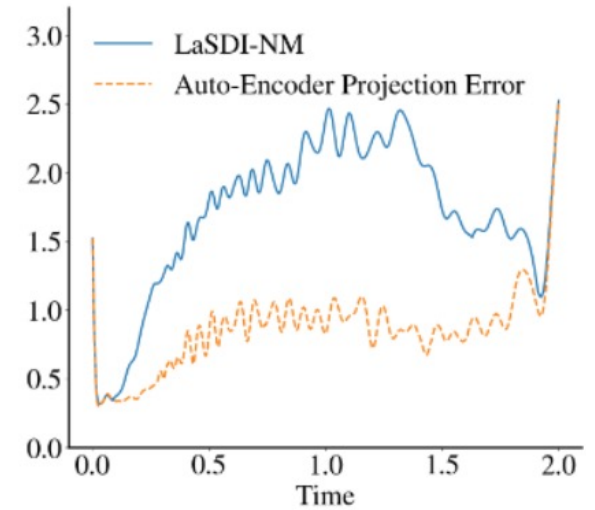


2D Burgers

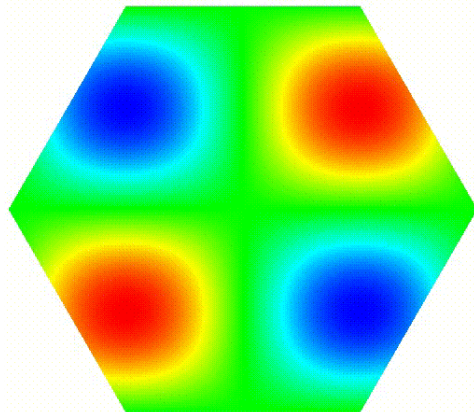
☺ Better projection error



Linear compression vs.

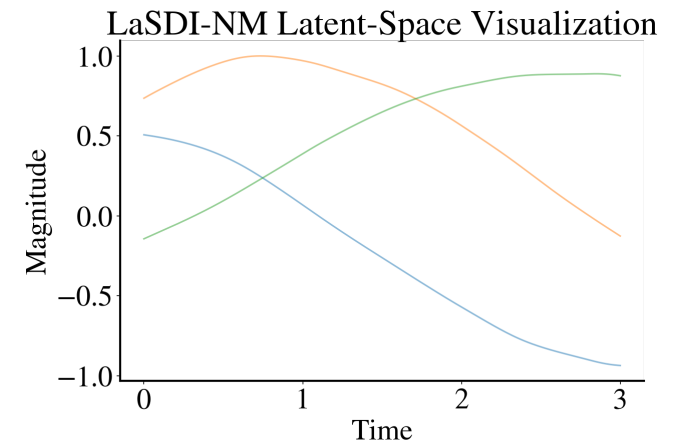
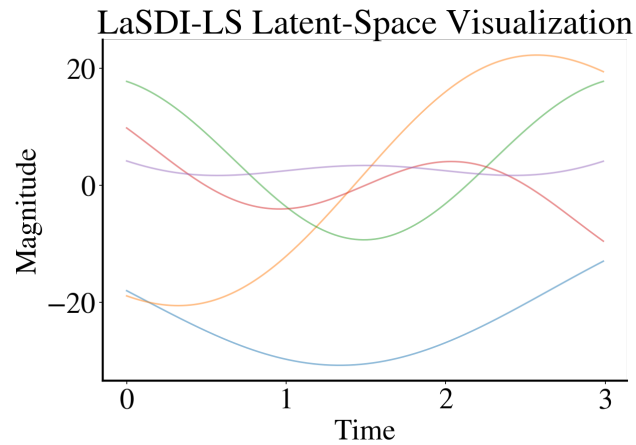


Nonlinear compression



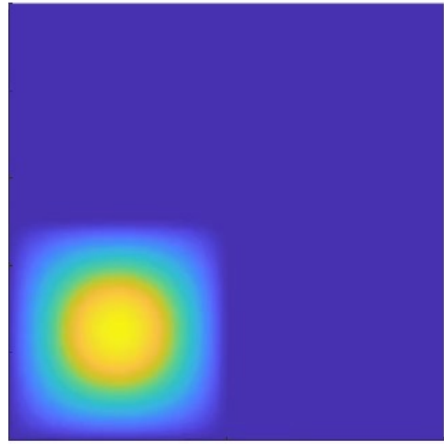
radial advection

☺ Simpler latent space dynamics

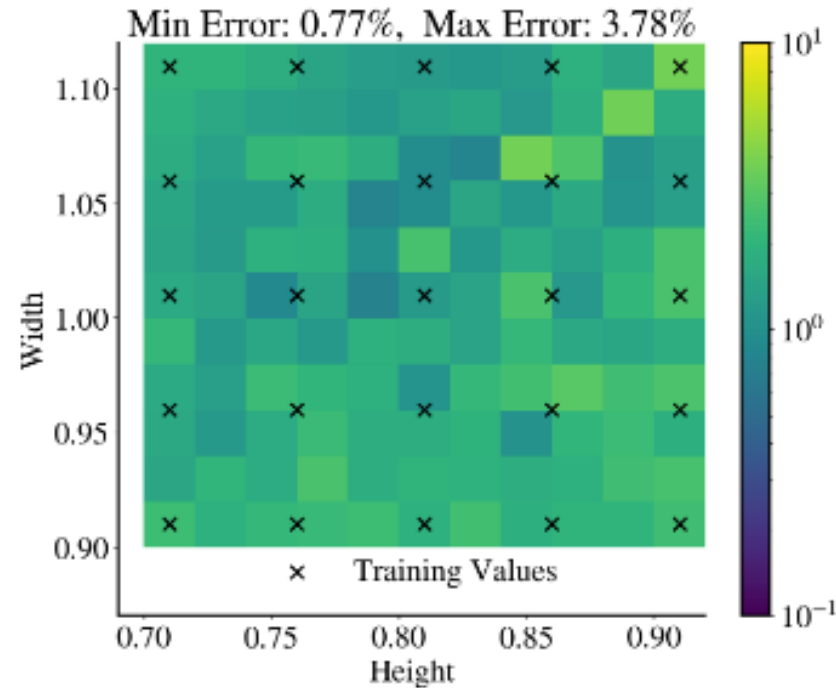


Nonlinear compression outperforms!

Nonlinear manifold



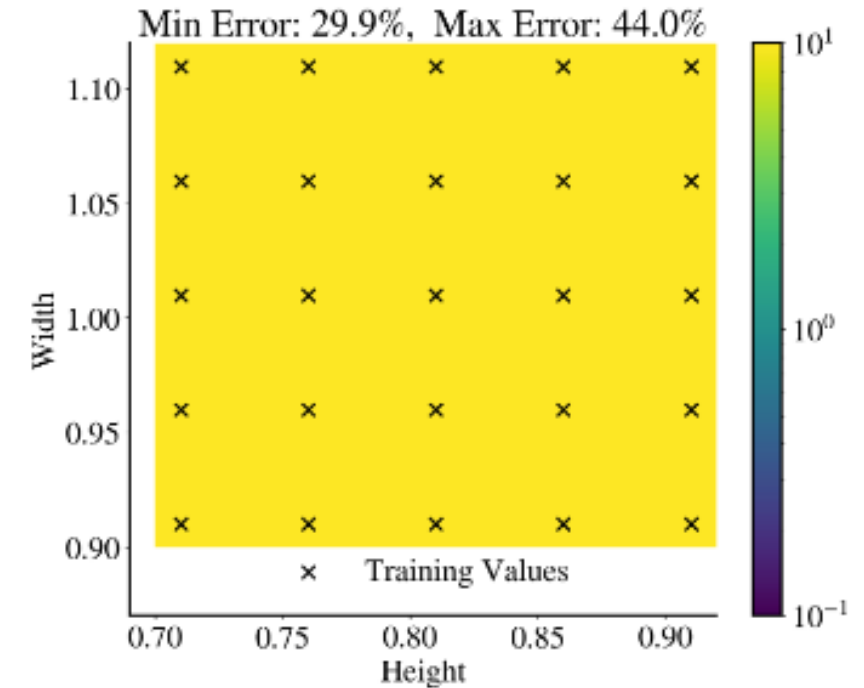
2D Burgers
advection-dominated



Latent space dimension of **three**

Cubic ODE for latent space dynamics model

Linear subspace



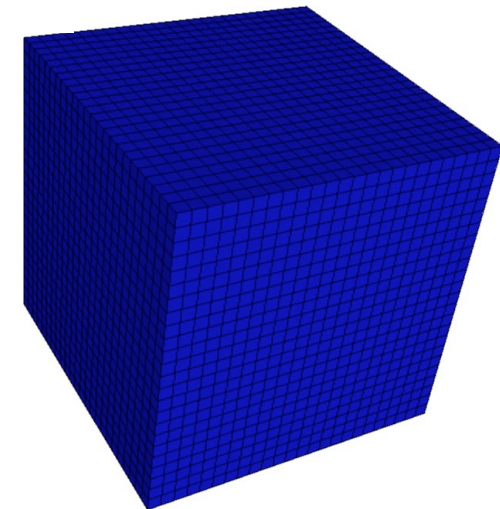
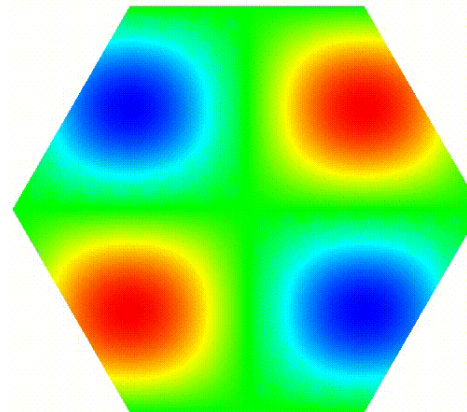
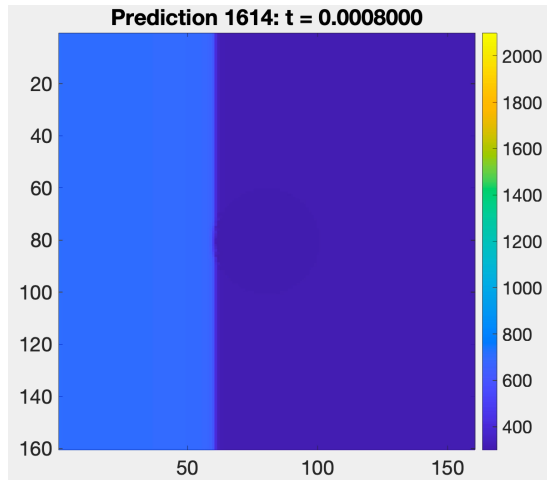
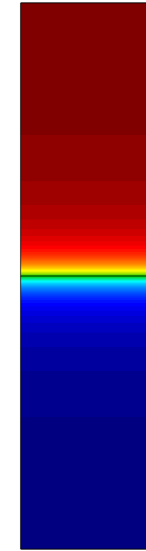
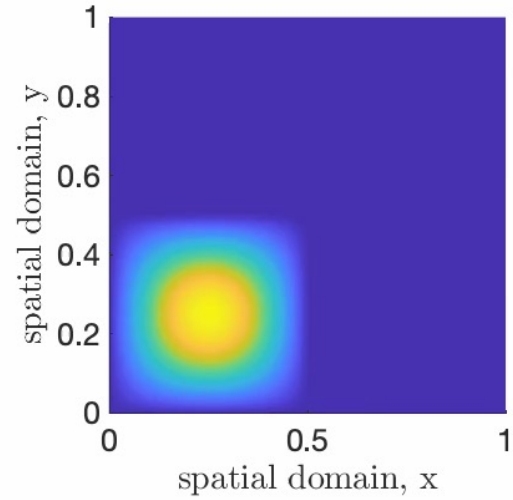
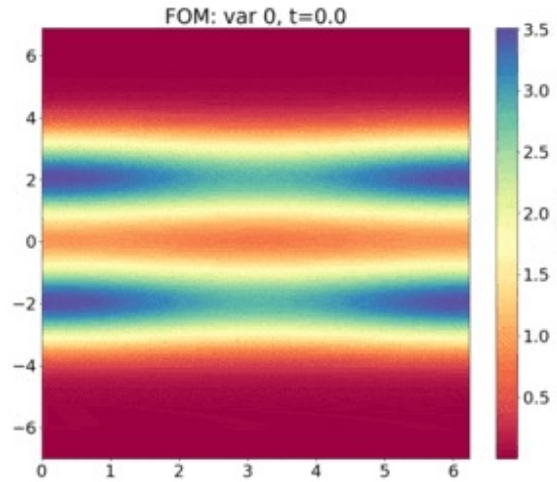
Latent space dimension of **five**

Linear ODE for latent space dynamics model

*Fries, He, Choi, "LaSDI: Parametric latent space dynamics identification." arXiv:2203.02076, 2022.

*He, Choi, Fries, Belof, Chen. "gLaSDI: Parametric Physics-informed Greedy Latent Space Dynamics Identification." arXiv:2204.12005. 2022

Questions? Email choi15@llnl.gov





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