Growing public concern about potential contaminant transport in water distribution systems has increased the use of models to assess risk and detect sources of contamination. The movement and distribution of contaminants depend largely on mixing at pipe junctions, where different flow rates and contaminant concentrations can exist. This article presents experimental observations of solute mixing in various pipe junction configurations. Analytical models are derived for each configuration, and results are compared with experimental data. A key finding—that impinging fluid streams within a junction often do not mix completely—is contrary to the most common assumption of complete mixing in pipe junctions. This study finds that if concentrations of two incoming fluid streams differ, they tend to bifurcate and reflect off one another, affecting subsequent solute distribution and mixing of fluid streams. The authors introduce a new bulk-advective mixing (BAM) model that has been shown to accurately represent this behavior and lead to more accurate water quality assessments.

Evaluation of solute mixing in water distribution pipe junctions

BY CLIFFORD K. HO
AND LESLIE O'REAR JR.

Mixing in pipe junctions can play an important role in the movement and distribution of solutes and contaminants in water distribution pipe networks. The flow pattern and geometric configuration of these junctions govern the mixing behavior of solutes that enter them. Typical pipe junctions allow one or more incoming fluid streams to be combined or split into one or more outlet pipes (Figure 1). Water distribution analysis software such as EPANET (Rossman, 2000) typically assumes complete and instantaneous mixing within a junction such that the outlet concentrations are all equal. However, recent studies have shown that flow in cross junctions can result in incomplete mixing under a wide range of conditions (Austin et al, 2008; Ho, 2008; Romero-Gomez et al, 2008; Ho et al, 2007, 2006; McKenna et al, 2007; Webb & van Bloemen Waanders, 2006; O'Rear et al, 2005; van Bloemen Waanders et al, 2005). Impinging flows within a cross junction tended to bifurcate and reflect off one another rather than mix completely.

This article focuses on solute mixing and transport in cross joints, which are commonly used in water distribution network systems (Romero-Gomez et al, 2008). Several water utility engineers and public works directors have estimated that 75–80% of all modern intersecting pipes use a cross junction instead of a double-T (or paired-T) junction because of a number of factors, including cost, thrust restraint, and ease of construction (Howie, 2007). Previous studies have also investigated solute mixing in double-T junctions (Ho et al, 2007, 2006).

The previous studies of solute mixing in cross joints focused on conditions with adjacent inlets and outlets. This article explores the effect of alternative configurations on solute mixing in junctions: (1) adjacent inlets with equal pipe sizes, (2) adjacent inlets with unequal pipe sizes, and (3) opposing (180º) inlets with equal pipe sizes. Using images of mixing from experiments representing each of these configurations, the authors explain the salient flow and mixing processes. They then present and derive models for the different configurations, and compare the model predictions with the experimental data.

## DESCRIPTION OF PHYSICAL MIXING PROCESSES

Methodology for earlier experiments. A number of experiments have been performed to characterize the mixing behavior within individual pipe joints (Austin et al, 2008; McKenna et al, 2007; O'Rear et al, 2005). Pumps were used to supply water through pipes joined by a cross junction. The flow rates were controlled at both the inlets and outlets of the pipes using valves, and flow meters were used to monitor the flow rate through each pipe. The pipes were constructed of polyvinyl chloride (PVC) with prescribed diameters, and the inlet and outlet pipe lengths were sufficiently long (20–100 pipe diameters) to ensure the water was well mixed within each pipe section before entering the junction and before being monitored by the electrical conductivity sensors in the effluent pipes. Water entering the system was pumped from two supply tanks (a well-mixed tracer supply tank and a clean water supply tank), and water leaving the system was emptied into two effluent tanks. Figure 2 shows a photograph and sketch of the test apparatus used by McKenna et al (2007) and O'Rear et al (2005).

For all experiments, sodium chloride (NaCl) was mixed with water in the tracer supply tank. The amount

FIGURE 1 Typical pipe junctions allow one or more incoming fluid streams to be combined or split into one or more outlet pipes

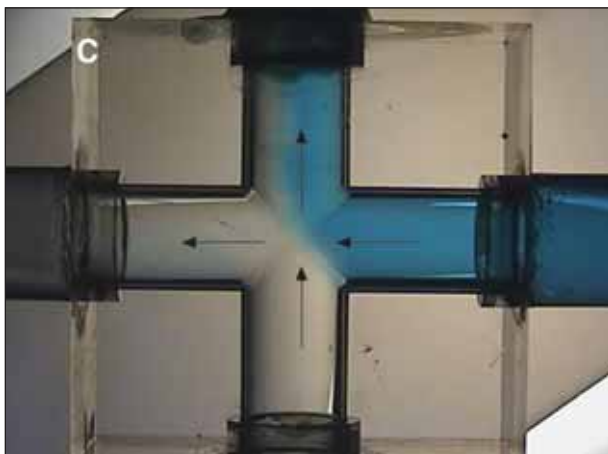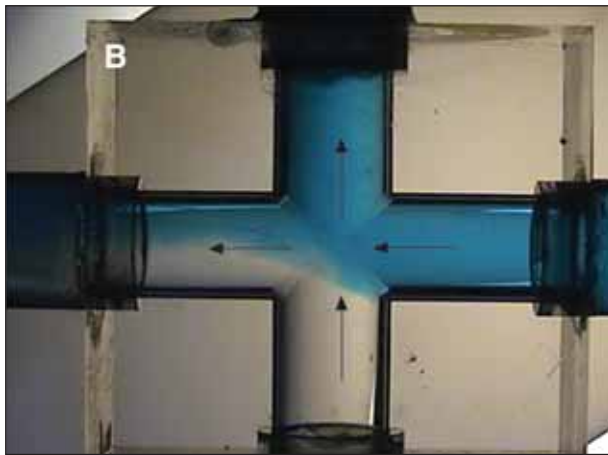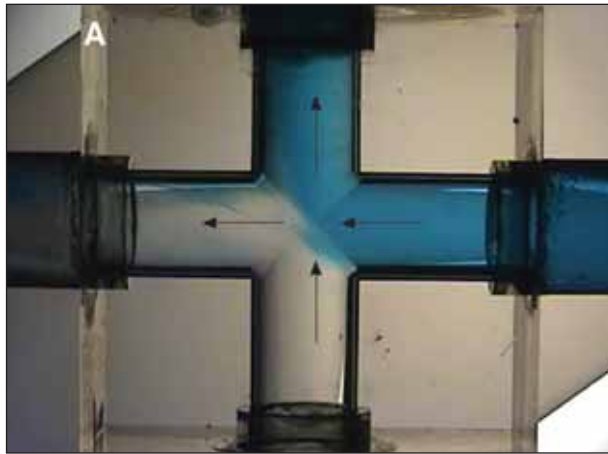Cross junction   T-junction
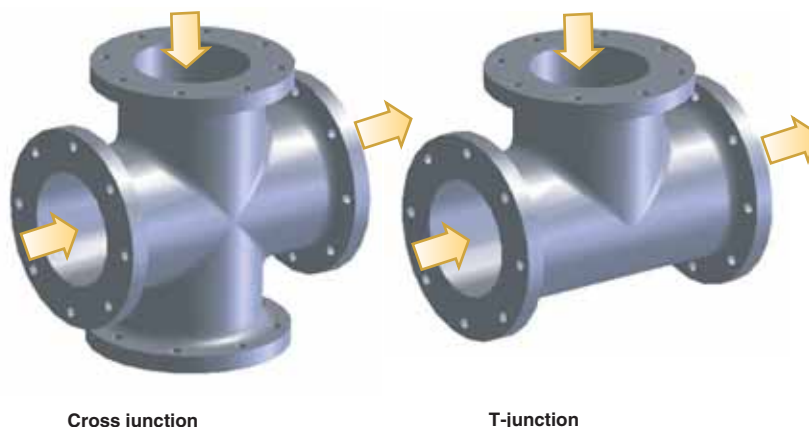
FIGURE 2 Photograph and schematic of single-joint apparatus used in experiments to characterize mixing processes
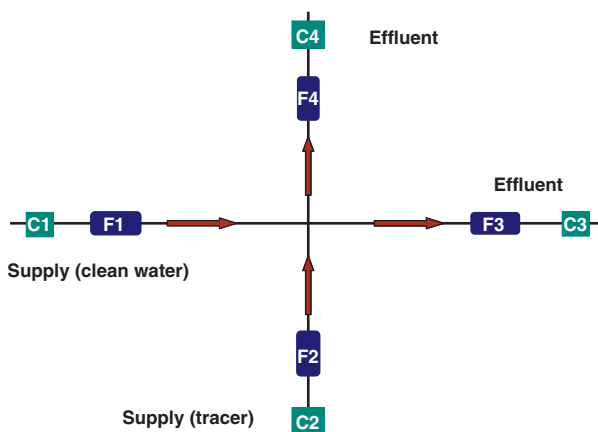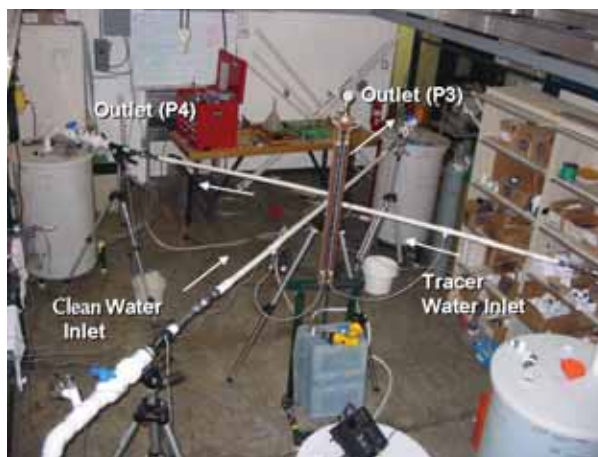
Source: McKenna et al, 2007 and O'Rear et al, 2005

C—conductivity meter, F—flow meter, P—pipe

These video images show of solute mixing in a pipe junction with adjacent inlets. Part A shows 1:1 tracer-to-clean inlet flow rates, part B shows 3:1 tracer-to-clean inlet flow rates, and Part C shows 1:3 tracer-to-clean inlet flow rates.

of NaCl added was enough to raise the electrical conductivity of the tracer solution to two to four times above that of the "clean" water. The NaCl tracer was monitored in the effluent pipes using electrical conductivity sensors, and normalized concentrations of the tracer were calculated using the maximum value of the conductivity of the NaCl solution and the minimum value of the conductivity of the clean water (i.e., normalized concentration = [measured concentration – minimum concentration]/[maximum concentration – minimum concentration]). Thus the normalized concentration of the tracer water was 1, and the normalized concentration of the clean tap water was 0.

Methodology for this study's visualization experiments. For the visualization experiments presented in this study, the cross junction connecting the inlet and outlet pipes was fabricated from a clear block of acrylic. The PVC pipes were then fitted into the acrylic block such that the inner diameter of the pipes was flush with the inner diameter of the junction openings.

• For the equal-pipe-size tests, the inner diameter of the pipes and junction openings was 26 mm (~ 1 in.).

• For the unequal-pipe-size tests, the inner diameters of the pipes and junction openings were 26 mm (~ 1 in.) and 52 mm (~ 2 in.). Blue food coloring was added to the NaCl tracer, and a digital video camera was used to record images of the mixing and distribution of the dyed tracer within the clear junction.

Adjacent inlets with equal pipe sizes. The diagrammed video images on this page show solute mixing in a cross junction with adjacent inlets and equal pipe sizes (26 mm). The Reynolds number for the flow rates ranged from approximately 4,000 to 13,000, indicating that the flow was turbulent in all pipes. Tracer was introduced in the pipe on the right, and clean water was introduced in the pipe at the bottom.

• Image A was taken when the flow rates in all the pipes were nearly equal.

• Image B was taken when the tracer inlet flow was approximately three times greater than the clean water inlet flow (outlet flow rates were equal).

• Image C was taken when the clean water inlet flow was approximately three times greater than the tracer inlet flow (outlet flow rates were equal).

The images on the left show that the incoming fluid streams reflect off one another and, depending on the relative momentum flux of each stream, may cross over the junction into the opposing outlet pipe.

Equal flow rates. In image A on this page, the flow rates are approximately equal in each pipe, and the incoming tracer and clean water streams reflect off one another and exit through the adjacent outlet pipes. The averaged normalized concentration of the top outlet pipe adjacent to the tracer inlet was measured to be approximately 0.9, whereas the averaged normalized concentration of the left outlet pipe adjacent to the clean inlet was only ~ 0.1. Turbulent and transient instabilities along the impinging interface (where the two incoming fluid streams meet in the junction) cause some of the fluid to mix (Webb & van Bloemen Waanders, 2006), but the majority of the incoming flows stay separated.

Crossover effect from higher-momentum tracer flow. Image B above shows that the larger momentum flux of

the incoming tracer water on the right causes some of the flow to cross over the junction into the opposing outlet pipe on the left. This effectively blocks the incoming clean water from the bottom, forcing the clean water to exit through the adjacent outlet pipe on the left. The effluent exiting the top outlet pipe is composed entirely of the tracer water, and the measured normalized concentration was ~ 1. The normalized concentration of the effluent in the left outlet pipe was ~ 0.55.

Crossover effect from higher-momentum clean water flow. Image C on page 118 shows a similar but opposite effect when the clean water inlet flow rate is approximately three times greater than the tracer inlet flow rate. The higher-momentum clean water now flows across the junction from the bottom inlet to the top outlet and effectively diverts the incoming tracer water from the right into the top outlet as well. In this case, the measured average normalized concentration in the left outlet is 0, and the measured average normalized concentration in the top outlet is ~ 0.5.

Summary of mixing process: equal pipe sizes. Table 1 summarizes the tests with adjacent inlets and equal pipe sizes shown in the images on page 118. The normalized concentrations in the effluent pipes do not necessarily add to 1 because of the different flow rates. That is, mass flow, which is equal to the flow rate times the concentration, is conserved, but concentration is not.

Mixing models for adjacent inlets with equal pipe sizes, along with their processes and assumptions, are detailed later in this article.

Adjacent inlets with unequal pipe sizes. The diagrammed video images on this page are from experiments that evaluate mixing in a cross junction with adjacent inlets and unequal pipe sizes. The diameter of the larger vertical pipe (~ 52 mm) is twice the diameter of the smaller horizontal pipe (~ 26 mm). Tracer is introduced in the small pipe on the right, and clean water is introduced in the larger pipe on the bottom. The Reynolds number for the flow rates ranged from approximately 3,000 to 12,000 in the larger pipe and from approximately 3,000 to 9,000 in the smaller pipe.

Higher momentum in larger pipe. Image A on the right shows the mixing behavior when the flow rate in the larger pipe has a momentum that is 14 times greater than the momentum in the small pipe; the flow rate ratio between the larger pipe and the smaller pipe is ~ 7.6:1. The greater momentum causes the flow in the larger pipe to push across the junction, effectively blocking the incoming flow from the smaller pipe. Thus, the flow at the left outlet pipe is composed primarily of the fluid from the adjacent inlet at the bottom, and the flow at the top outlet pipe is composed of a mixture of fluid coming from both the bottom and right inlets.

Higher momentum in smaller pipe: wraparound effect. Image B above shows the mixing behavior when the momentum in the smaller pipe is four times greater than the momentum in the larger pipe; the flow rate
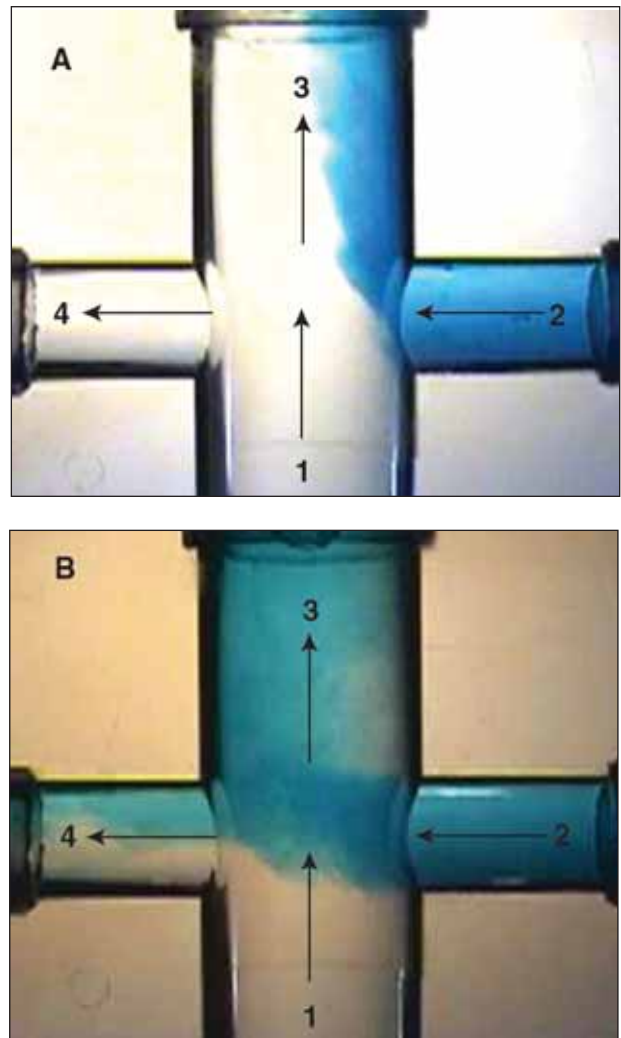
Video images of solute mixing in a pipe junction with adjacent inlets and unequal pipe diameters show the mixing behavior at a 1:14 momentum ratio (A: small pipe/large pipe) and at a 4:1 momentum ratio (B: small pipe/large pipe)

ratio is nearly equal to 1. In this case, the tracer from the smaller pipe inlet is seen to penetrate through the junction into the opposing outlet. However, rather than deflecting all of the clean water from the adjacent inlet, some of the clean water wraps around the flow emanating from the smaller pipe and also propagates through the junction to the opposing outlet, as indicated by the outlet concentrations for these tests (Table 2).

Summary of mixing process: unequal pipe sizes. In Table 2 pipes 1 and 2 denote the clean water (larger pipe) and tracer (smaller pipe) inlets, respectively; and pipes 3 and 4 denote the outlets opposing the clean water and tracer inlets, respectively. The results in Table 2 confirm the crossover behavior described for the equal pipe sizes, but also show that flow in a larger pipe can wrap around the flow from a smaller pipe when the momentum in the smaller pipe is greater than the momentum in the larger pipe.

TABLE 1   Summary of experiments with adjacent inlets and equal pipe sizes

Pipe	Average Flow Rate m³/s	Velocity m/s	Reynolds Number	Normalized Concentration
Case 1: Equal flow rates				
Tracer in (right)	7.9E-05	0.15	4,300	1.00
Clean water in (bottom)	7.5E-05	0.14	4,100	0.00
Effluent (left)	7.6E-05	0.14	4,100	0.12
Effluent (top)	8.3E-05	0.15	4,500	0.91
Case 2: Tracer inlet > clean water inlet				
Tracer in (right)	2.3E-04	0.43	12,000	1.00
Clean water in (bottom)	7.4E-05	0.14	4,000	0.00
Effluent (left)	1.5E-04	0.28	8,300	0.55
Effluent (top)	1.5E-04	0.29	8,400	1.00
Case 3: Clean inlet > tracer water inlet				
Tracer in (right)	7.4E-05	0.14	4,100	1.00
Clean water in (bottom)	2.4E-04	0.45	13,000	0.00
Effluent (left)	1.6E-04	0.29	8,600	0.02
Effluent (top)	1.6E-04	0.30	8,700	0.47

For example, in cases 1, 3, and 6, the momentum is greater in the smaller pipe, and although it penetrates across the junction, some of the clean water entering from the large pipe inlet wraps around and dilutes the outlet concentration in pipe 4, reducing the normalized concentration to a value < 1.

In contrast, for cases 2, 4, and 5, in which the momentum in the larger pipe (1 and 3) is greater than in the smaller pipe (2 and 4), the flow from the larger pipe completely deflects the incoming flow from the inlet of the smaller pipe, and the outlet concentration in pipe 4 is composed entirely of clean water (normalized concentration = 0).

Mixing models for adjacent inlets with unequal pipe sizes, along with their processes and assumptions, are detailed later in this article.

Opposing inlets with equal pipe sizes. the diagrammed video image on page 121 shows solute mixing in a cross junction with opposing (180º) inlets. The tracer enters the junction on the right, and clean water enters on the left. The incoming flows collide in the junction and then exit through the top and bottom outlet pipes. Different combinations of flow rates in each of the inlet and outlet pipes were investigated (Table 3). The Reynolds number ranged from ~ 6,000 to 30,000 to maintain turbulent flow conditions within each pipe.

The image on page 121 shows an extreme combination of flow rates in which the flow ratios in the tracer inlet, clean water inlet, top outlet, and bottom outlet were 5.5, 3.1, 7.6, and 1.0 relative to the lowest flow rate, respec-

TABLE 2   Summary of experiments with adjacent inlets and unequal pipe sizes

Case	Larger Clean Water Inlet Flow Pipe 1 m³/s	Smaller Tracer Inlet Flow Pipe 2 m³/s	Larger Outlet Flow Pipe 3 m³/s	Smaller Outlet Flow Pipe 4 m³/s	Relative Flow Ratio Smaller Pipe/ Larger Pipe (2+4)/(1+3)	Momentum Ratio* Smaller Pipe/ Larger Pipe (2+4)/(1+3)	Normalized Concentration in Outlet Pipe 3	Normalized Concentration in Outlet Pipe 4
1	2.45E-04	1.91E-04	2.53E-04	1.84E-04	0.75	2.27	0.64	0.16
2	2.39E-04	6.65E-05	2.44E-04	6.29E-05	0.27	0.29	0.30	0.00†
3	1.22E-04	1.81E-04	1.25E-04	1.78E-04	1.45	8.43	0.61	0.60
4	2.36E-04	1.17E-04	2.40E-04	1.14E-04	0.48	0.94	0.51	0.01
5	4.82E-04	6.59E-05	4.88E-04	6.22E-05	0.13	0.07	0.15	0.00†
6	1.19E-04	1.22E-04	1.20E-04	1.20E-04	1.01	4.10	0.65	0.40

*Momentum ratio > 1—momentum is greater in the smaller (tracer) pipe; momentum ratio < 1—momentum is greater in the larger (clean water) pipe.
†Larger pipe completely deflected incoming flow from smaller pipe.

tively. Even under these extreme flow ratios (and in all of the cases summarized in Table 3), the measured tracer concentrations in the top and bottom outlets were nearly equal. The complete-mixing model for opposing inlets with equal pipe sizes, along with its processes and assumptions, is described later.

MIXING MODELS FOR ALTERNATIVE JUNCTION CONFIGURATIONS

Based on the observed physical mixing processes detailed in the previous sections, the sections that follow present and/or derive the models that describe the salient processes of solute mixing in the alternative pipe-junction configurations. Comparisons to test data are provided for each of the configurations. The models for two configurations (adjacent inlets with equal pipe sizes and opposing inlets with equal pipe sizes) have been implemented in EPANET-BAM (Ho & Khalsa, 2008), which is open-source software and freely available to the public at www.sandia.gov/EPANET-BAM. The BAM-WRAP model for adjacent inlets with unequal pipe sizes has not yet been implemented in EPANET-BAM.

Adjacent inlets with equal pipe sizes. A model of solute mixing in a cross junction with adjacent inlets and equal pipe sizes has been derived by Ho (2008) and is summarized here. Figure 3 shows the assumed flow pattern in a cross junction with unequal flow rates. The flow in the pipe with the largest momentum is assumed to cross over the junction, deflecting the incoming flow from the adjacent inlet. This has been confirmed by the tests of adjacent inlets with equal pipe sizes in the previous section.

In this model, it is assumed that the constituents are well mixed before entering and after leaving the junction. Previous studies have shown that constituents are well mixed within ~10 pipe diameters downstream of the junction (Ho et al, 2006; Plesniak & Cusano, 2005). The following expression results from a mass balance on the solute leaving outlet pipe 4:

$$Q_4 C_4 = Q_{1 \to 4} = Q_4 C_1 \tag{1}$$

in which $Q_{1 \to 4}$ is the portion of the flow from inlet pipe 1 that flows into outlet pipe 4. Because none of the flow from inlet pipe 2 is assumed to cross over into outlet pipe 4, all of the flow leaving outlet pipe 4 is from inlet pipe 1 (i.e., $Q_{1 \to 4} = Q_4$). Therefore, the previous expression (Eq 1) states that the solute concentration in outlet pipe 4 is equal to the solute concentration in inlet pipe 1 as follows:

$$C_4 = C_1 \tag{2}$$

The concentration in outlet pipe 3 is derived by performing a solute mass balance on the entire cross junction:

$$Q_1 C_1 + Q_2 C_2 = Q_3 C_3 + Q_4 C_4 \tag{3}$$
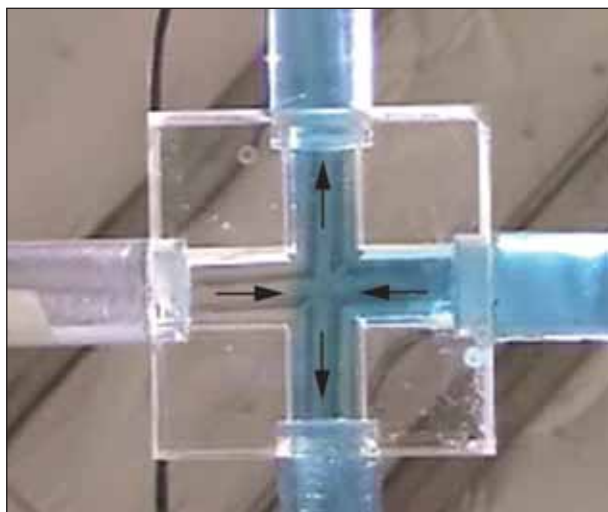
This video image shows solute mixing in a pipe junction with opposing (180º) inlets and extreme combination of flow rates. The clean water inlet is on the left, and the tracer inlet is on the right. The top and bottom are both outlets.

Using Eq 2 in Eq 3 results in the following equation for the solute concentration in outlet pipe 3:

$$C_3 = C_{BAM} = \frac{Q_2 C_2 + (Q_1 - Q_4)\ C_1}{Q_3} \tag{4}$$

Bulk-advective mixing (BAM) model. Equations 2 and 4 provide the solutions to the bulk advective mixing (BAM) model and solve for the outlet concentrations assuming the flow rates and inlet concentrations are known. In a network model, these solutions can be applied sequentially to each downstream junction starting with the upstream-most junction where the concentration boundary conditions are prescribed. The flow rate in each pipe is typically calculated beforehand in network or computational fluid dynamics (CFD) models based on prescribed boundary conditions of pressure and/or flow rates. In transient simulations, the bulk-mixing model solution can be applied at each time step with updated flow rates at each junction.

BAM model versus complete-mixing model. The BAM model neglects instabilities and turbulent mixing at the interface of the impinging flows. Therefore it provides a lower bound to the amount of mixing that can occur in this configuration. The complete-mixing model that is used in EPANET (Rossman, 2000) and other network models assumes that the outlet concentrations are equal as a result of complete and instantaneous mixing within the junction:

$$C_{complete} = C_3 = C_4 = \frac{Q_1 C_1 + Q_2 C_2}{Q_3 + Q_4} \tag{5}$$

Therefore, the complete-mixing model provides an upper bound to the amount of mixing that can occur in

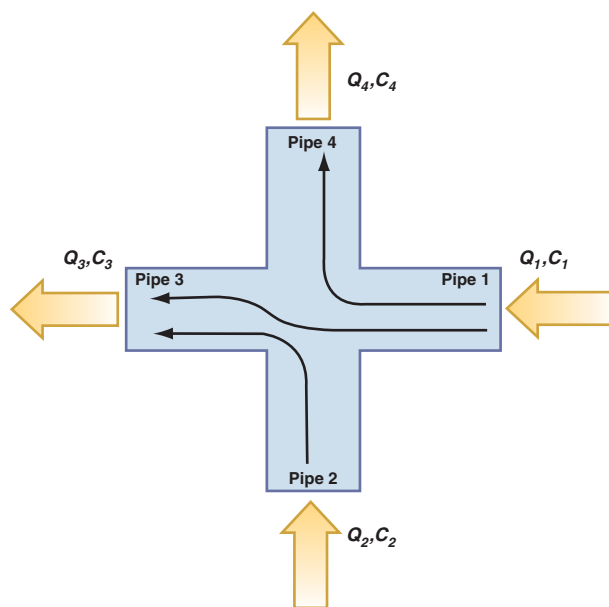TABLE 3 Summary of experiments with opposing inlets (180º) and equal pipe sizes

Case	Average Velocity m/s				Normalized Outlet Concentration	
	Tracer In (Right)	Clean Water In (Left)	Effluent (Top)	Effluent (Bottom)	Top	Bottom
1	0.24	0.23	0.24	0.23	0.52	0.52
2	0.23	0.44	0.34	0.34	0.36	0.36
3	0.23	0.45	0.22	0.46	0.35	0.35
4	0.22	0.92	0.58	0.59	0.20	0.20
5	0.22	0.88	0.44	0.68	0.19	0.21
6	0.24	0.89	0.23	0.92	0.22	0.22
7	0.22	0.94	0.93	0.24	0.18	0.16
8	0.23	0.44	0.27	0.40	0.35	0.37

this configuration whereas the BAM model provides a lower bound. The actual amount of mixing will fall in between these two bounds. Therefore, a scaling (or mixing) parameter, $0 \leq s \leq$, is defined to estimate a combined (intermediate) concentration, $C_{combined}$, in an outlet pipe based on the physically bounding concentrations calculated from the complete ($C_{complete}$) and BAM (C_{BAM}) mixing models described earlier:

$$C_{combined} = C_{BAM} + s\,(C_{complete} - C_{BAM}) \qquad (6)$$

in which C_{BAM} represents the concentration given by the BAM model in Eq 2 and 4. The value of the scaling parameter, s, may depend on fluid properties, flow conditions, and the geometric configuration of the pipe junction, all of which can contribute to local instabilities at the impinging interface and turbulent mixing within the junction that are not captured by the bulk-mixing model.

Figure 4 shows a comparison of the measured and predicted normalized concentration at the tracer outlet (adjacent to the tracer inlet) from different experiments with different combinations of flow rates. The measured concentrations are bounded by the BAM and complete-mixing models, and the majority of the data fall between predicted concentrations using a mixing parameter between 0.2 and 0.5.

EPANET-BAM. The BAM model has been implemented in a new version of EPANET (Rossman, 2000), called EPANET-BAM. The mixing parameter, *s*, has been added to the junction property field and can be adjusted at each junction between 0 (bulk mixing only) and 1 (complete mixing). EPANET-BAM has been used to predict solute transport and mixing in a number of single-joint and multi-joint laboratory-scale network experiments with good comparisons to data (Ho & Khalsa, 2008). EPANET-BAM evaluates all the junctions in the model at each time step and determines (based on nodal coordinates and flow directions) whether the junction is a cross-joint with adjacent inlets. If so, the BAM model is used. If not, the complete-mixing model is used.

Adjacent inlets with unequal pipe sizes. This section presents a model that accommodates mixing in unequal pipe sizes. Although many different combinations of pipe sizes can exist at a junction, the case presented here is of a larger pipe intersected by a smaller pipe (for example, a pipe main intersected by smaller pipes). Figure 5 shows a sketch of a cross junction with adjacent inlets and two pipe sizes.

Applying the BAM model: greater momentum in larger pipe. As described earlier, the mixing behavior depends on whether the momentum is greater in the larger or smaller pipe. If the momentum is greater in the larger pipe, it will cross over the junction and effectively deflect all of the flow from the incoming smaller pipe into the adjacent outlet pipe. This behavior is similar to the BAM model described in the previous section for equal pipe sizes. Therefore, the BAM model can be applied to unequal pipe sizes when the momentum is greater in the larger pipe.

Deriving the BAM-WRAP model. If the momentum is greater in the smaller pipe, the mixing experiments described in the earlier section indicate that the flow in

FIGURE 3   "Crossover" flow in cross junction when momentum is greater in pipes 1 and 3

Q_4,C_4

Pipe 4

Q_3,C_3 Pipe 3   Pipe 1 Q_1,C_1

Pipe 2

Q_2,C_2

Q—*flow,* C—*concentration*

the smaller pipe will cross over the junction, but the flow in the larger pipe can still wrap around the flow originating from the smaller pipe. As a result, the flow in both outlet pipes will be composed of a mixture of fluid from both inlet pipes. A model of this wraparound flow and mixing can be derived assuming that the flow and transport through the "core" and "wraparound" regions within the junction are proportional to the geometric areas of the different pipe sizes.

• The core region is defined by the size of the smaller pipe, and mixing in the core region is assumed to behave similarly to the processes described by the BAM model.

• The wraparound region is the area outside of the core region that is available for wraparound flow because of the extra volume of the larger pipe (Figure 5).

This model, denoted as BAM-WRAP, is derived using the pipe numbering scheme shown at the top of Figure 5. The zoomed cross-sectional area of the wraparound region in the larger pipe (crosshatched) shown in the lower part of Figure 5 can be expressed as a function of the diameters of the larger (D) and smaller (d) pipes (Spiegel, 1968):

$$A_{\text{wrap}} = \left(\frac{D}{2}\right)^2 (\theta - \sin \theta \tag{7}$$

in which $\theta = 2 \cos^{-1} (d/D)$.

The amount of flow in the larger pipe that wraps around the core region, $Q_{i,\text{wrap}}$, is assumed to be proportional to the wraparound area, A_{wrap}, and the total flow in the larger pipe, Q, as follows:
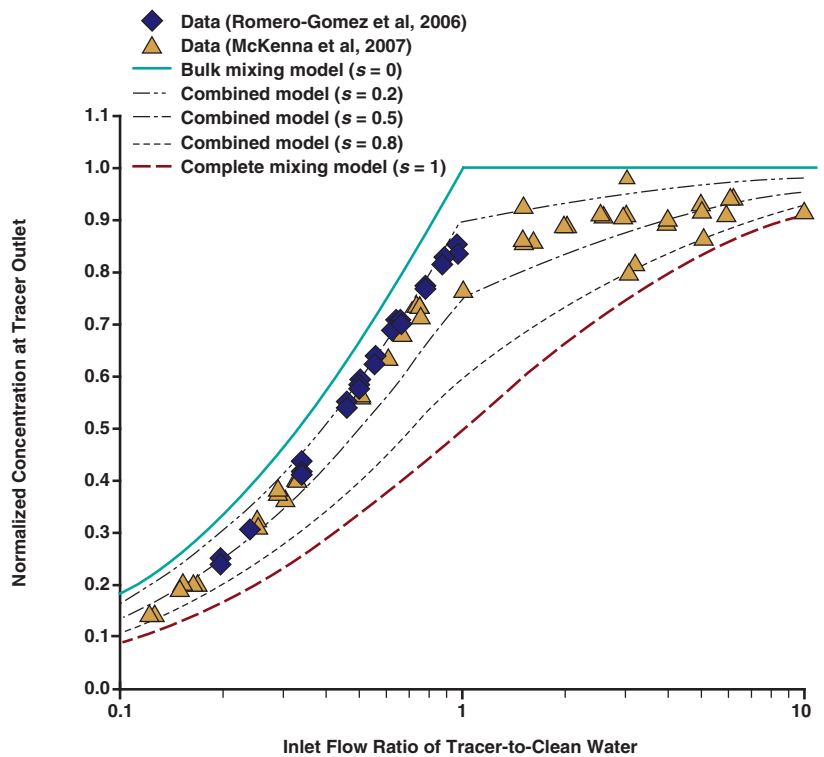
$$Q_{i,\text{wrap}} = \left(\frac{A_{\text{wrap}}}{\pi D^2/4}\right) Q_i \tag{8}$$

in which i denotes either the inlet (pipe 1) or outlet (pipe 3) of the larger pipe. The flow rate in the core region of the larger pipe is calculated as the difference between the total flow in the larger pipe and the wraparound flow:

$$Q_{i,\text{core}} = Q_i - Q_{i,\text{wrap}} \tag{9}$$

The concentration of the solute in the core region of the outlet pipes can then be calculated using the BAM model as follows:

FIGURE 4 Measured and predicted normalized concentrations at tracer outlet* for different inlet flow rates and equal outlet flow rates

Legend:
- Data (Romero-Gomez et al, 2006)
- Data (McKenna et al, 2007)
- Bulk mixing model ($s = 0$)
- Combined model ($s = 0.2$)
- Combined model ($s = 0.5$)
- Combined model ($s = 0.8$)
- Complete mixing model ($s = 1$)

y-axis: Normalized Concentration at Tracer Outlet
x-axis: Inlet Flow Ratio of Tracer-to-Clean Water

Source: Ho, 2008, with permission from the American Society of Civil Engineers (ASCE)

*Tracer outlet adjacent to tracer inlet

s—scaling parameter

$$C_{3,\text{core}} = C_2 \tag{10}$$

$$C_{4,\text{core}} = C_4 = \frac{Q_{1,\text{core}} C_1 + (Q_2 - Q_{3,\text{core}}) C_2}{Q_4} \tag{11}$$

The total concentration in the larger outlet pipe is calculated by adding the solute mass flow rates in both the core region and wraparound region and dividing by the total flow rate in the larger outlet pipe:

$$C_3 = \frac{Q_{3,\text{core}} C_{3,\text{core}} + Q_{3,\text{wrap}} C_1}{Q_3} \tag{12}$$

Equations 11 and 12 provide the BAM-WRAP solutions for adjacent inlets with unequal pipe sizes when the momentum is greater in the smaller pipe. If the momentum is greater in the larger pipe, the original BAM solution can be applied.
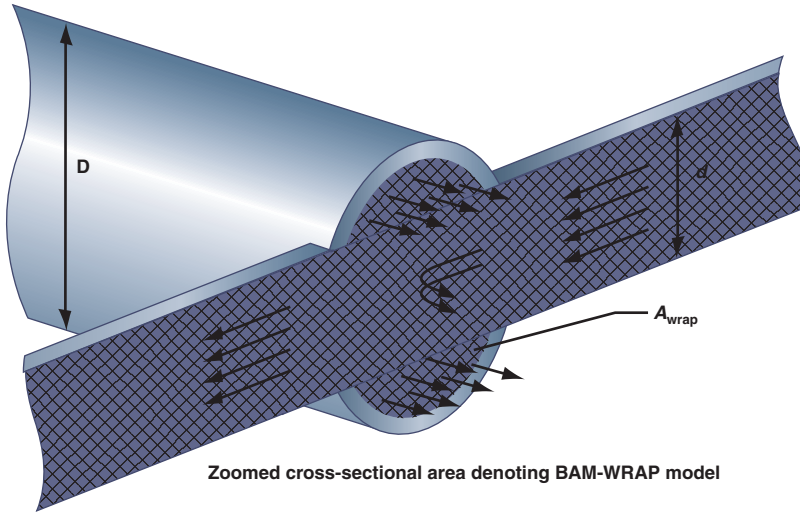
Results of BAM-WRAP and BAM models. Figure 6 shows the steady-state results of the BAM-WRAP and BAM models for the six test cases summarized in Table 2. The momentum flux was greater in the larger pipe (pipes 1 and 3) for cases 2, 4, and 5, so the flow from the larger
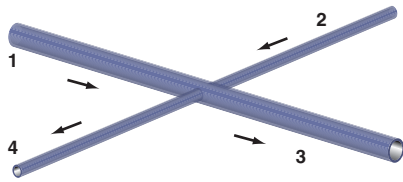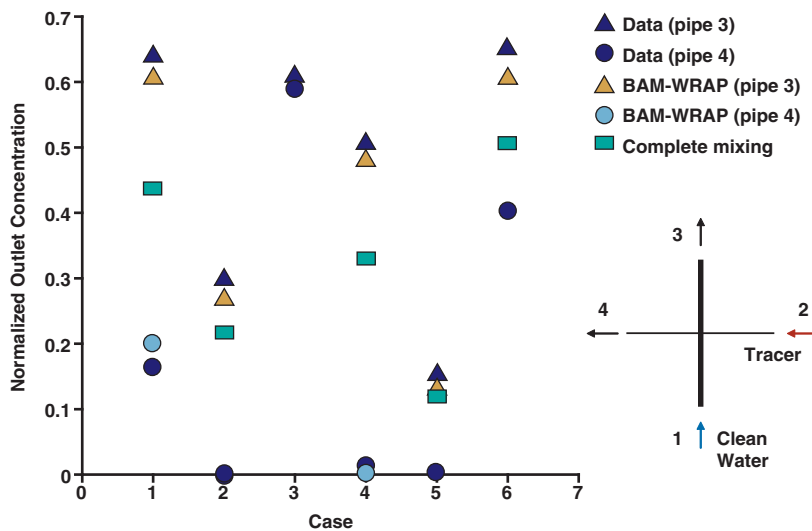
Configuration and numbering scheme for deriving BAM-WRAP model

Zoomed cross-sectional area denoting BAM-WRAP model

D—*diameter of larger pipe,* d—*diameter of smaller pipe,* A$_{wrap}$—*wraparound area*

FIGURE 6 Measured and predicted normalized outlet concentrations for mixing in a cross junction with adjacent inlet flows and different pipe sizes

Some data overlap (cases 2, 3, 5, and 6), giving the appearance of a single symbol. For cases 2, 4, and 5, the original BAM model was used.

BAM—bulk-advective mixing

pipe effectively blocked the incoming flow from the smaller pipe. For those cases, the original BAM model can be applied. The normalized outlet concentration at pipe 4 almost entirely comprised the clean water originating from the larger pipe, and the resulting normalized concentration at pipe 4 was close to 0. The normalized concentration in pipe 3 was a mixture of tracer and clean water. Its value depends on the relative flow rates of the tracer and clean water (Table 2). The BAM model accurately predicts the normalized concentrations at both outlets for these cases.

For cases 1, 3, and 6, the momentum flux was greater in the smaller pipe (pipes 2 and 4), so the tracer originating from pipe 2 was able to cross over the junction into pipe 4, deflecting some of the incoming clean water from pipe 1. However, because of the larger size of pipe 1, clean water was also able to wrap around the core region into the opposing pipe 3. Therefore, the flows at both outlets comprised fluid from both the clean and tracer inlets. The normalized concentrations predicted by the BAM-WRAP model match the trends and time-averaged data values quite well at both outlets for these cases (Figure 6).

Complete-mixing model versus BAM and BAM-WRAP models. The results of the complete-mixing model are also shown in Figure 6. The complete-mixing model assumes that the outlet concentrations are equal as a result of perfect and instantaneous mixing within the junction. Therefore, the predicted completely mixed concentrations are in between the actual outlet concentrations. It is interesting that the observed data do not align more closely with the complete-mixing model. Because the BAM-WRAP and BAM models do not include turbulent mixing and instabilities, the actual mixing was expected to fall in between the BAM predictions and the complete-mixing predictions, as was observed in tests with equal-sized pipes. How-

ever, for tests with unequal pipe sizes, this was not observed. In most cases, the data showed more separation from the complete-mixing model. This can be explained because of the different mixing behavior in the core and wraparound regions.

With case 1 as an example, the observed normalized concentrations in pipes 3 and 4 were ~ 0.64 and 0.16, respectively, and the complete-mixing model yielded a normalized outlet concentration of 0.44. The BAM-WRAP predictions yielded normalized concentrations of 0.61 and 0.20, respectively. In the core region, enhanced mixing caused by turbulent instabilities would act to decrease the concentration in pipe 3 and increase the concentration in pipe 4 (moving the results toward those of the complete-mixing model). However, in the wraparound region, enhanced mixing would act to increase the concentration in pipe 3 and decrease the concentration in pipe 4; the clean water wrapping around the tracer water in the core region would pick up more tracer at the fluid interface between the core and wraparound regions, and the tracer fluid passing through the core region into pipe 4 would be more diluted by clean water at this interface. Therefore the turbulence-enhanced mixing in the core and wraparound regions offset each other, and the observed results closely match those of the BAM-WRAP model, which neglects the impact of turbulence-enhanced mixing.

A new model is shown to accurately predict observed solute concentration distribution for a number of different flow-rate combinations.

Opposing inlets with equal pipe sizes. The earlier mixing experiments showed empirically that mixing in cross junctions with opposing inlets yielded nearly equal outlet concentrations for different combinations of inlet and outlet flow rates. Although this may seem intuitive for equal flow rates, it was not clear that opposing inlets would yield equal outlet concentrations when the flow rates in each pipe of the cross junction were significantly different.

Results equivalent to complete mixing. This section demonstrates why this junction configuration yields results equivalent to complete mixing, regardless of the flow rates and pipe sizes. Figure 7 shows the configuration and numbering scheme for the model.

The key process that results in equal outlet concentrations in this configuration is that the fraction of flow

FIGURE 7 Configuration and numbering scheme for model of mixing with opposing inlets

Q—*flow,* C—*concentration*
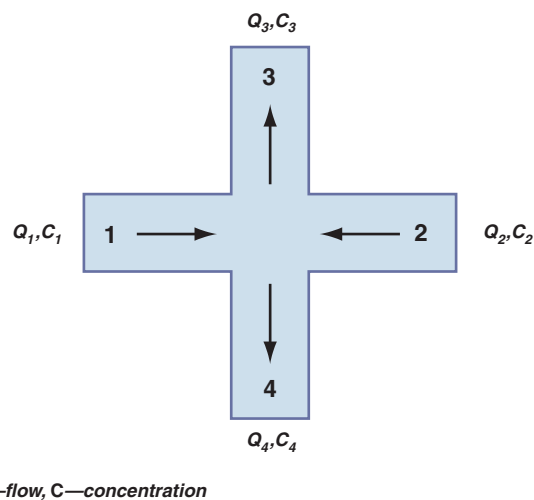
from an inlet pipe that exits a particular outlet pipe must be equal to the ratio of flow in that outlet pipe to the total outflow. For example, if 30% of the outflow is through pipe 3 and 70% of the outflow is through pipe 4, then 30% of the inlet flow from each of pipes 1 and 2 will exit pipe 3, and 70% of the inlet flow from each of pipes 1 and 2 will exit pipe 4. This condition ensures conservation of fluid mass and can be expressed as follows:

$$\frac{Q_{1\rightarrow3}}{Q_1} + \frac{Q_{2\rightarrow3}}{Q_2} = \frac{Q_3}{Q_3 + Q_4} \tag{13}$$

$$\frac{Q_{1\rightarrow4}}{Q_1} + \frac{Q_{2\rightarrow4}}{Q_2} = \frac{Q_4}{Q_3 + Q_4} \tag{14}$$

in which $Q_{i\rightarrow j}$ denotes the flow rate from pipe *i* to pipe *j*. The concentrations at the outlet pipes can be expressed as a function of the flow rates and inlet concentrations as follows:
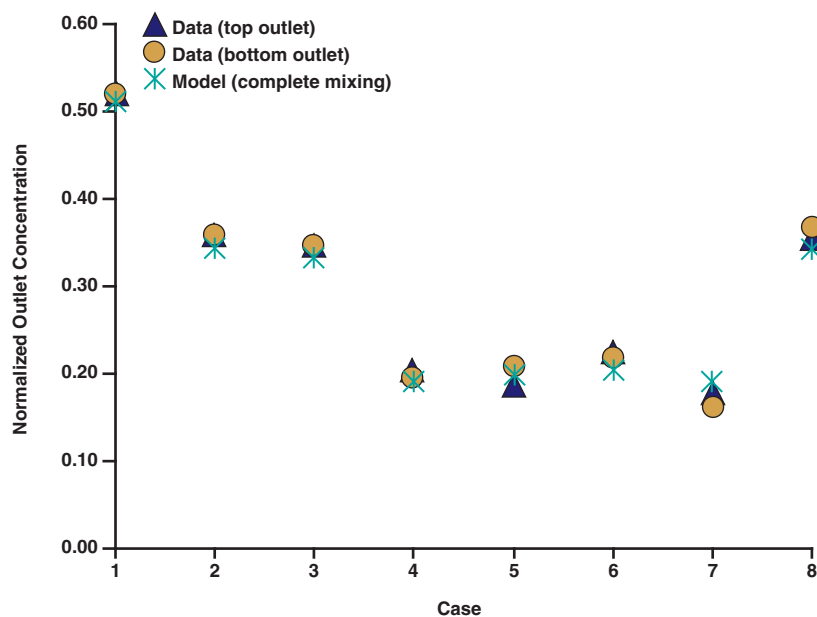
$$C_3 = \frac{Q_{1\rightarrow3}\,C_1 + Q_{2\rightarrow3}\,C_2}{Q_3} \tag{15}$$

$$C_4 = \frac{Q_{1\rightarrow4}\,C_1 + Q_{2\rightarrow4}\,C_2}{Q_4} \tag{16}$$

Plugging Eqs 13 and 14 into Eqs 15 and 16 yields the following expression for the outlet concentrations:

$$C_3 = C_4 = \frac{Q_1C_1 + Q_2C_2}{Q_3Q_4} \tag{17}$$

FIGURE 8 Comparison between measured and predicted normalized outlet concentrations for mixing in a cross junction with opposing (180°) inlets and equal pipe sizes

CONCLUSION

Understanding and predicting solute transport through water distribution pipe networks has become increasingly important, as the potential for accidental or intentional contamination of water distribution systems has become a growing public concern. The transport of contaminants through water distribution pipe networks depends on mixing at pipe junctions, where different flow rates and contaminant concentrations can exist. Most of the current models of flow and transport, though, assume complete mixing at pipe junctions. In many cases, this leads to a poor representation of the actual mixing behavior. Concentrations in various regions of the network can be significantly over- or underpredicted, assuming complete mixing.

This study examined the impact of solute mixing on water quality in a pipe network, using several different junction configurations. For junctions consisting of two adjacent inlets and outlets with equal pipe sizes, solute mixing has been shown to be incomplete because of the bifurcation and reflection of incoming fluid streams. In these instances, current water quality models incorrectly assume complete and instantaneous mixing. Therefore, a new bulk-advective mixing (BAM) model has been developed and implemented into EPANET-BAM, an augmented version of the widely used EPANET software (Rossman, 2000). The BAM model has been shown to yield good matches with data from single-joint and multijoint laboratory-scale network tests. Additional details regarding the implementation of the models in EPANET-BAM are provided in Ho and Khalsa (2008).

Junctions consisting of two adjacent inlets and outlets with unequal pipe sizes revealed the potential for wraparound flow and transport. If the momentum in the smaller pipe is greater than the momentum in the larger pipe, flow in the larger pipe can wrap around the core region defined by the size of the smaller pipe. A

Equal outlet concentrations. Eq 17 is equivalent to the results of the complete-mixing model and demonstrates that the outlet concentrations for this configuration are equal, provided that the fraction of flow from any inlet pipe that leaves a particular outlet pipe is equal to the ratio of flow in that outlet pipe to the total outflow. This conclusion is independent of relative pipe sizes.

Figure 8 shows the measured normalized outlet concentrations from the experiments summarized in Table 3 along with the results of the complete-mixing model. These results demonstrate that mixing in cross junctions with opposing inlets yields equal outlet concentrations, with different combinations of inlet and outlet flow rates.

Growing public concern about potential contaminant transport in water distribution systems has increased the use of models to assess risk and detect sources of contamination.

new model was derived for this case and was shown to accurately predict observed solute concentration distribution for a number of different flow-rate combinations. The BAM-WRAP model has not yet been implemented into EPANET-BAM.

Finally, junctions consisting of two opposing inlets with equal pipe sizes yielded nearly complete mixing for a number of different test conditions. An analytical solution was derived that confirmed complete mixing would occur in this configuration, and complete mixing would also be expected in any configuration with two opposing inlets, even with unequal pipe sizes.

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