

# CHAPTER 19

## STABILITY ANALYSIS OF ENERGY STORAGE INTEGRATION IN POWER SYSTEMS

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### Abstract

Energy storage systems (ESSs) are increasingly being integrated into power systems because they can provide a wide array of unique services. ESSs and other renewable generation such as photovoltaics (PVs) and wind are integrated with power systems using power electronic converters (PECs) that can have different physical topologies and control loops governing their operational characteristics. With ESSs (including other PECs) starting to dominate the generation mix, the dynamics of their converters and control systems play a major role in the stability of modern power systems. Hence, specific modeling and stability analysis techniques are needed to accurately study and evaluate the performance of such systems. This chapter presents stability analysis tools and techniques for power systems with ESSs, including trends in this area.

### Key Terms

Control layers, direct method, electromagnetic transient (EMT), frequency, frequency stability, long-term stability, microgrid, phase locked loop (PLL), point of common coupling (PCC), power conversion system (PCS), power electronic converter (PEC), power system stability, primary control loop, rotor angle stability, short-term stability, small signal stability, stability analysis, stable operating point, synchronous machine, time-domain simulation, timescale, voltage source converter (VSC), voltage stability

## 1. Introduction

Power system stability is commonly defined as: *the ability of an electric power system to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact* [1]. Stability analysis is a key ingredient for reliable and secure power system operations to ensure continuous electric service without failure. Instability issues typically arise in a power system due to major disturbances such as natural disasters, faults of the major components in the network (e.g., generators, transmission lines, etc.) and /or cyberattacks on control and communication infrastructure. Energy resources such as PVs, wind, and ESSs are being increasingly integrated into power systems because they pave the path for the clean grid of the future. The power generation from these resources often encompass large stochastic variations which is another form of disturbance in power systems that can lead to instability.

Furthermore, these resources are integrated with the power system using PECs, which have different operational characteristics compared to conventional energy resources that are often based on rotational electromechanical generators. The dominance of PEC-based systems (ESSs and other renewable generation) is expected to result in the overall reduction of power system inertia. Power systems with low inertia are more susceptible to disturbances that can cause unacceptable variations in the voltage and frequency. In addition, the new generation mix of

modern power systems brings a higher degree of uncertainty and increases the complexity of the involved stability analysis.

ESSs typically require a PEC such as a voltage source converter (VSC) or some derivate of it to be interfaced with the power system [2]. Depending on the application, the VSC can have different physical topologies and control loops that enable various functionalities of the ESS. The operational characteristics of ESSs thus depends on the specific physical topology used and the way the control loops are designed and tuned [3]. Furthermore, ESSs can respond and operate much faster compared to rotational generators. This is mainly because the PECs used within an ESS are based on fast-acting semiconductor switches. As a result, the ESS can charge/discharge at rates much faster than rotational generators, which are typically quite slow. ESSs interact with power systems at different timescales which introduces nuances to stability analysis of power system with ESSs. The operating characteristics of an ESS and the entire power system to which it is being integrated are intrinsically different. Consequently, different models and approaches to stability analysis are required for power systems with ESSs.

As power systems continue to transition towards a more distributed form of operation, microgrids are another important sector where ESSs are gaining popularity. Microgrids are defined as a group of interconnected energy resources (including PVs, wind, ESSs among other energy generation resources), plus power system loads that can operate as a single controllable entity either in parallel with the grid or in islanded mode [4]. Microgrids can operate remotely serving a relatively small number of customers or connected to the larger integrated power system with the potential to isolate when needed. This distributed nature of microgrids allows the power system to be operated with a high degree of flexibility.

Stability analysis of microgrids is quite different compared to larger power systems because of their different characteristics. Typically, microgrids operate at low-to-medium voltage ranges in contrast to high voltage operation of a conventional power system. Furthermore, microgrids are typically laid out over a smaller geographical region. These factors lead to varied operational characteristics. The traditional assumptions that were used in the stability analysis of conventional power systems are invalid for microgrids. For example, the assumption that the voltage and frequency of power system are independent and can be studied separately no longer holds true for most microgrids [5]. Additionally, the limited generation and load in these systems result in a higher degree of uncertainty in the operation (large variability). The power generation mix is also different and typically dominated by PEC-based resources in contrast to the bulk power system, which is still dominated by rotational synchronous generation in the form of conventional fossil fuel, nuclear, and hydropower generation. As a result, traditional definitions, models, and stability analysis approaches may not be suitable for microgrids.

In this chapter, approaches for stability analysis of power systems in the presence of ESSs are discussed. The chapter starts with an overview of conventional definitions used to study power system stability. Next, a discussion on the differences in analyzing stability of large power systems versus smaller microgrids with ESSs is presented. Modeling of ESSs from the perspective of stability analysis is then introduced followed by a description of various stability analysis techniques that can be leveraged for power systems in presence of ESSs. Finally, challenges and emerging technologies for stability analysis and modeling are discussed.

## 2. State of Current Technology

### 2.1. Stability Analysis of Conventional Power Systems

Stability of a power system can be compromised due to disturbances such as natural disasters, variations in power system load/generation, faults in major components in the network (e.g., generators, transmission lines, etc.), and/or cyberattacks on control and communication infrastructure. In traditional stability studies for conventional power systems (such as transmission networks with synchronous generators as major generating units), stability phenomena are classified based on the *size* of the disturbance and the *timescale* of the dynamic response of the power system's physical quantities such as rotor angles, voltages, and frequency following a disturbance.

- *Size of disturbance*: Small disturbances include consistent generation and load fluctuations that occur in a power system. The consistent variations in the generation of stochastic resources such as PVs and wind are also potential sources of small disturbances in the power system. A power system must be able to operate within satisfactory operating ranges during such small disturbances while adequately supplying the load demand. Large disturbances on the other hand are typically a result of major component failure such as generator malfunctions or short circuits on transmission lines which, in turn, result in larger stability impacts such as cascaded failures and wide-area blackouts.
- *Timescale*: Timescale of stability analysis is categorized into *short-term* and *long-term* stability. Fast acting components of power systems (e.g., PEC-based loads and generators, and ESSs) have response timescales from milliseconds to seconds. This is because PEC-based components can act comparatively faster in power systems compared to traditional rotational generators. *Short-term stability* analysis studies whether these fast dynamics retrieve a stable operating point after the disturbance (also known as post-disturbance system stability). *Long-term stability* analysis involves investigation of power system stability due to slow acting components (e.g., large rotating generators, governors) whose temporal response lasts from seconds to minutes (much slower compared to PEC-based components).

The size and the timescale of disturbance allows the power stability to be studied through classification into three major stability phenomena: *rotor angle stability*, *voltage stability*, and *frequency stability* [1]. See Figure 1 for a visual description of stability based on different timescales. As the amount of PEC-based generation and load starts to increase in power systems, converter-driven stability (e.g., *harmonic stability*) also becomes an increasingly important aspect in modern power system. Converter-driven stability issues are observed at much shorter timescales as evident from Figure 1.

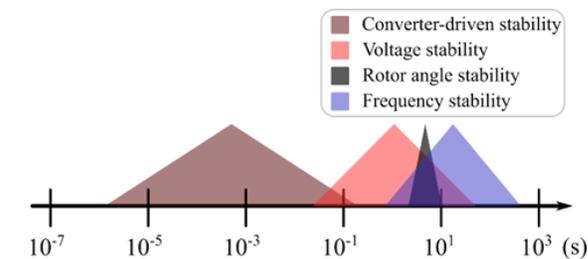


Figure 1. Stability based on different timescales

### 2.1.1. Rotor Angle Stability

Rotor angle stability is the ability of the synchronous machines in power systems to remain synchronized. Synchronism indicates that relative rotor angles of each synchronous machine interconnected in the power system maintain equilibrium with each other. In power systems, one or multiple groups of synchronous generators become unstable if relative rotor angles of the generators get out of synchronism from the rest of the generator groups due to large disturbances such as line outages or short circuits. This can damage generators and may lead to cascading failures if proper remedial actions are not followed (such as opening the circuit breakers to isolate the source of instability). Rotor angle stability is usually a short-term phenomenon and involves studying nonlinear dynamics of a power system for large disturbances. Rotor angle stability mainly appears in conventional bulk power systems with synchronous generators and is considered as a major stability phenomenon. However, as discussed in Section 2.3, in modern distribution systems and microgrids, voltage and frequency stability, and stability of controllers in PEC-based components, are increasingly becoming more of a concern.

### 2.1.2. Frequency Stability

Most components in power systems are designed to operate at certain frequencies (60 Hz in the North America, Central America, most of South America, parts of Japan and the Middle East, and 50 Hz in some European countries). Large deviations from these values will invoke protective actions (e.g., tripping generators or loads), which may lead to unwanted faults or cascading failures in the network. The main sources of frequency instability include loss of generating units, unplanned disconnection of certain areas of a power system (islanding), a sudden load shedding, or unplanned outages of equipment, which introduce imbalance between generation and load. Frequency in a power system is maintained to an extent by inertial responses of large rotating generators in bulk power systems. The kinetic energy stored in the rotors of the synchronous generators can absorb or release energy to counteract the imbalance between generation and load. This means that large synchronous generators are the first safeguard against potential frequency instability before major control layers are deployed to keep system frequency stable. These regulatory control layers have different time scales at which they act (e.g., primary [ $\leq$  seconds], secondary [seconds to minutes], and tertiary [ $\geq$  minutes]), driven by governor control response and various generation reserves (mostly fast ramping generating units such as diesel/gas turbine generators). Due to these different timescales of control layers as shown in Figure 2, frequency stability includes both short-term and long-term stability phenomena. The reduction in power system inertia due to replacement of conventional mechanical generation by PEC-based generation (such as ESSs) can lead to poor frequency stability as the load and generation imbalance can lead to higher frequency fluctuations in the power system [6].

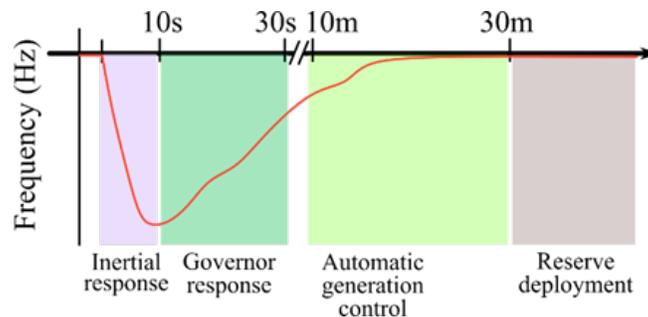
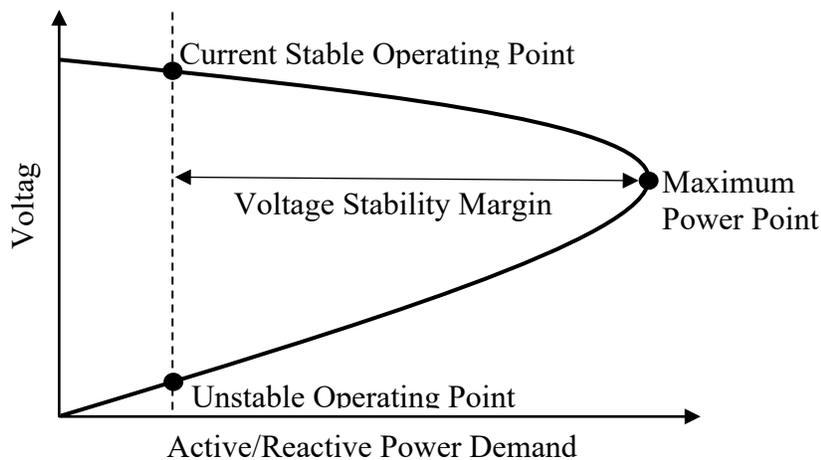


Figure 2. Frequency events in different timescales

### 2.1.3. Voltage Stability

Voltage stability refers to the ability of a power system to maintain steady-state voltages after a disturbance. Bus voltages in a grid should be maintained within a certain limit around the rated operating point. For example, ANSI C84.1—a national standard for service voltage regulation – recommends maintaining voltage magnitudes within 0.95 to 1.05 per unit in normalized quantity. Voltage is directly related to power quality, hence large voltage changes beyond the limits lead to failure of electricity supply to demand and loss of loads. Also, protective systems will trip transmission lines to isolate the area of the network showing abnormal voltage variations, which may cause wide-area cascaded failure in the system if proper control actions are not followed to maintain active/reactive power balance in the network after the isolation.

Voltage instability is mainly caused by heavy loading conditions due to increasing load demands beyond the capability of generating units and the transmission network. Figure 3 shows a power-voltage curve for a two-bus power system [7]. The power system, which is initially operating at a stable operating point, reaches the maximum power point as the load increases. The maximum operating point defines the voltage stability margin of the power system. Eventually, the bus voltage starts to collapse and becomes unstable when it reaches the unstable operating point. Voltage stability includes short-term and long-term stability analysis depending on the influence of fast acting components (e.g., network dynamics, PEC-based components) and slow acting component (e.g., tap-changing transformers or generator current limiters).



**Figure 3. Power-voltage curve showing voltage stability margin and maximum power point**

Although each stability definition described in this section is treated as an independent, individual phenomenon, it is important to acknowledge that rotor angle, frequency, and voltage stability are interdependent due to nonlinear interactions of machines, controls, and protection systems. These interdependencies become noticeable in the case of severe disturbances that invoke large deviations in the dynamic states of the system. For example, loss of synchronism in rotor angle stability can induce voltage drop when the relative angle difference between groups of machines is large [1,7]. Also, a large disturbance in the generation and load balance can induce frequency deviation and voltage drop simultaneously, which can potentially lead to cascading failure. This coupling between voltage and frequency dynamics of the power system is becoming increasingly dominant in low voltage power systems such as microgrids (discussed in further detail in Section 2.2). Thus, all aspects of power system stability need to be analyzed carefully. Enhancing one stability aspect should not hamper the other.

## 2.2. Stability Analysis of Microgrids with Energy Storage Systems

Stability analysis of microgrids with ESSs presents some unique differences compared to that of bulk power systems described in the previous section. Microgrids tend to have a high share of PEC-based energy resources like PVs and ESSs. Furthermore, as PEC-based generation resources are static and do not provide any mechanical inertial response, which is crucial for maintaining frequency stability of the system. Thus, typically microgrids are low inertia power systems where frequency stability issues are more common than transient and voltage stability problems.

Another key characteristic that effects the stability analysis of microgrids is the reactance-to-resistance ( $X/R$ ) ratio of the feeder lines. In conventional power systems, the voltage and frequency stability are analyzed in a decoupled fashion owing to the fact that the  $X/R$  ratio is very high. Most microgrids, however, are operated at lower voltage with short feeder lines which results in low  $X/R$  ratios and the conventional assumption that voltage and frequency of the power system are decoupled is no longer valid. Thus, it is difficult to classify the stability problems as either “frequency” or “voltage” instability as described in Section 2.1 in case of microgrids.

In this context, the IEEE Task Force on Microgrid Stability proposed a different definition and classification to analyze stability of microgrids [5]. The proposed classification is presented in Figure 4. Instead of classifying instability in terms of measured signals such as rotor angle, frequency, and voltage, the stability analysis is classified into phenomena related to 1) equipment control systems (control system stability) and 2) active and reactive power sharing and balance (power supply and balance stability). Control system stability issues arise due to improper tuning of the control loops of ESSs and other generation resources in the microgrid (either conventional electric machine-based generation or PEC-based generation). Thus, control system stability is further classified into electric machine stability and converter stability. Power supply and balance stability is further classified into voltage stability and frequency stability. Voltage stability is further classified into system voltage stability and DC-link voltage stability. Both system voltage stability and DC-link voltage stability are further classified into small disturbance and large disturbance. Both small disturbance and large disturbance are further classified into short term and long term.

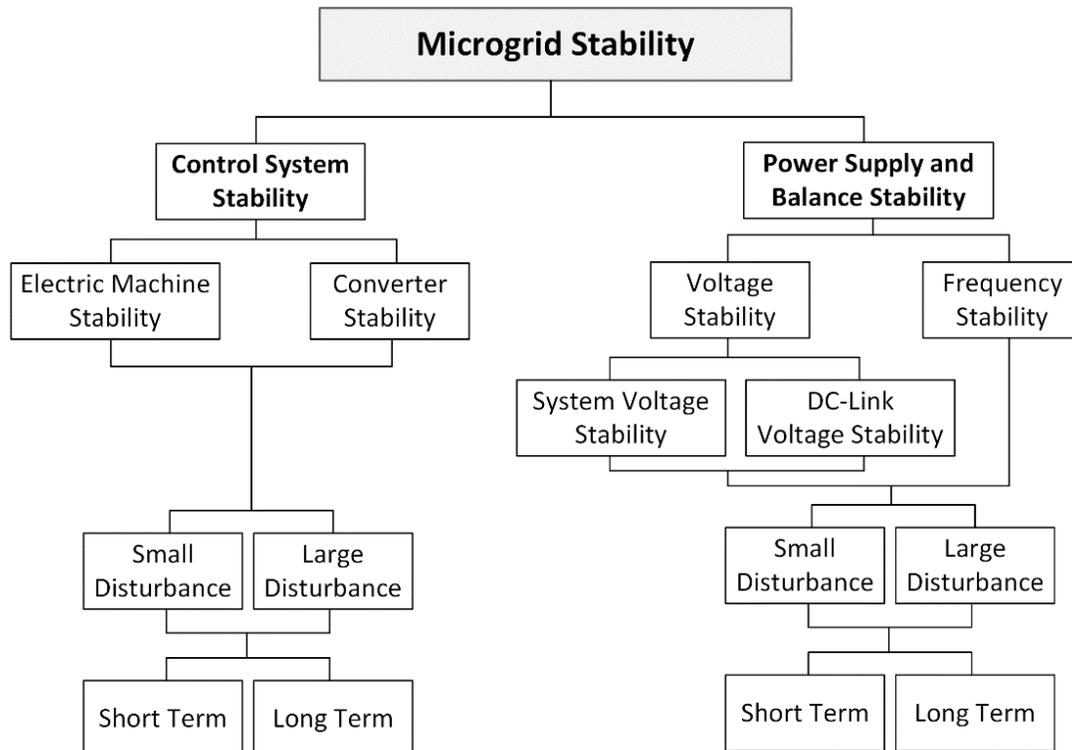


Figure 4. Classification of power system stability (adapted from [5])

Electric machine stability issues typically occur in a fraction of Hz and can be impacted by large and small disturbances. The slow interaction of converter-driven stability issues typically occurs at less than 10 Hz and encompasses controls for power sharing dynamics [8]. Between tens to thousands of Hz, the fast interaction converter-driven stability issues occur because of the interaction of switching modulation control and voltage and current waveform dynamics. These interactions can result in distortions in the current and voltage waveforms of PEC-based generation (e.g., ESSs) causing harmonic instability [9], poor power quality, and disruptions in power supply. Poor tuning and design of the phase locked loop (PLL) used to synchronize the PEC-based generation to the power system can lead to control system instability.

The power supply and balance stability, on the other hand, is associated with ability of the system to maintain power balance and sharing among the energy resources in the microgrid. Active and reactive power imbalances can lead to voltage instability and frequency instability in microgrids. Voltage instability refers to voltage regulation issues with either the power system (on the AC side) or the voltage at the DC-link of the PEC. In general, stability issues can be manifested through small-scale or large-scale disturbances and have either short-term or long-term consequences in the operation of the microgrid.

With the growing interest in DC microgrids and the potential of coupling them with AC power systems, hybrid DC/AC microgrids are becoming increasingly attractive for integrating ESSs. Hybrid systems do not require multiple stages of power conversion because ESSs can be directly integrated into the power system without the need for a DC to AC conversion. This in turn increases the efficiency of power delivery. A typical schematic of a hybrid DC/AC system is shown in Figure 5. A bidirectional PEC interfaces the DC and AC side of the system. This converter plays a critical role in maintaining the AC side voltage and frequency and the DC side voltage within permissible ranges. There is, however, a need to establish procedures to analyze the stability of such systems. Although many research papers propose the use of small-signal stability techniques, the non-linearities of the converters and control structures require more advanced methods. Some examples of stability analysis in hybrid DC/AC systems can be found in various other published papers [10, 11, 12].

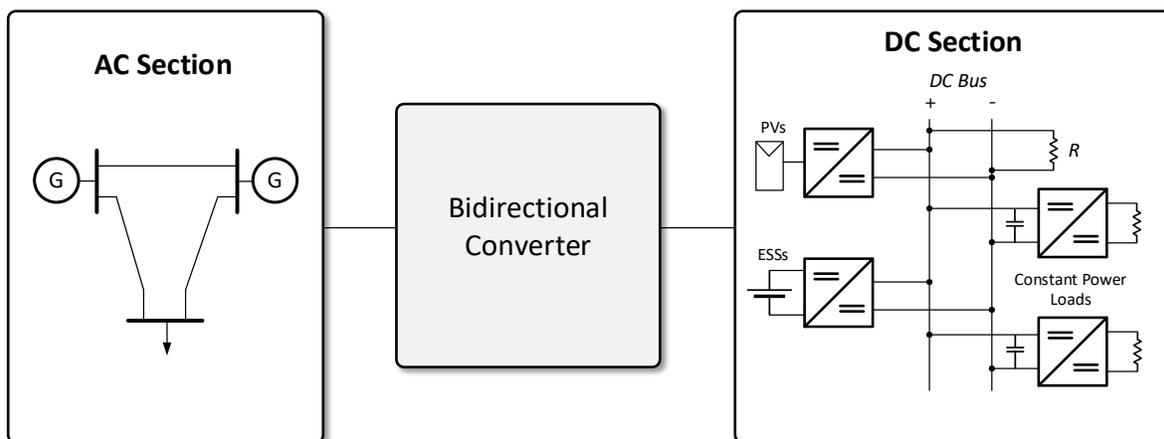
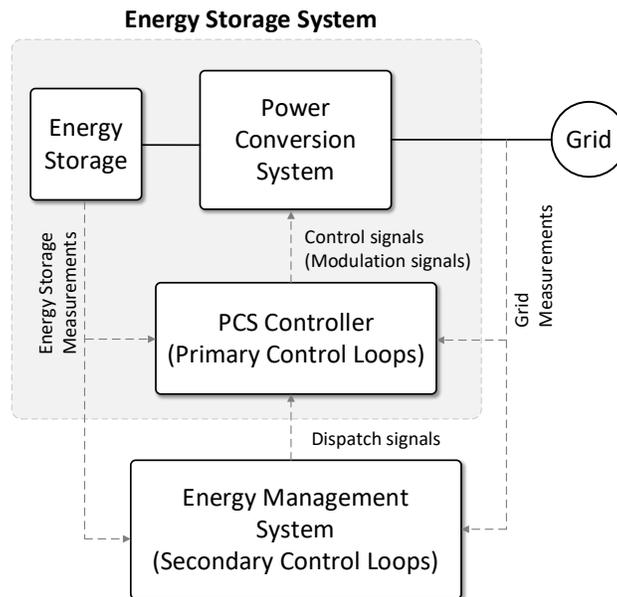


Figure 5. Schematic Diagram of a hybrid AC/DC microgrid (adapted from [10])

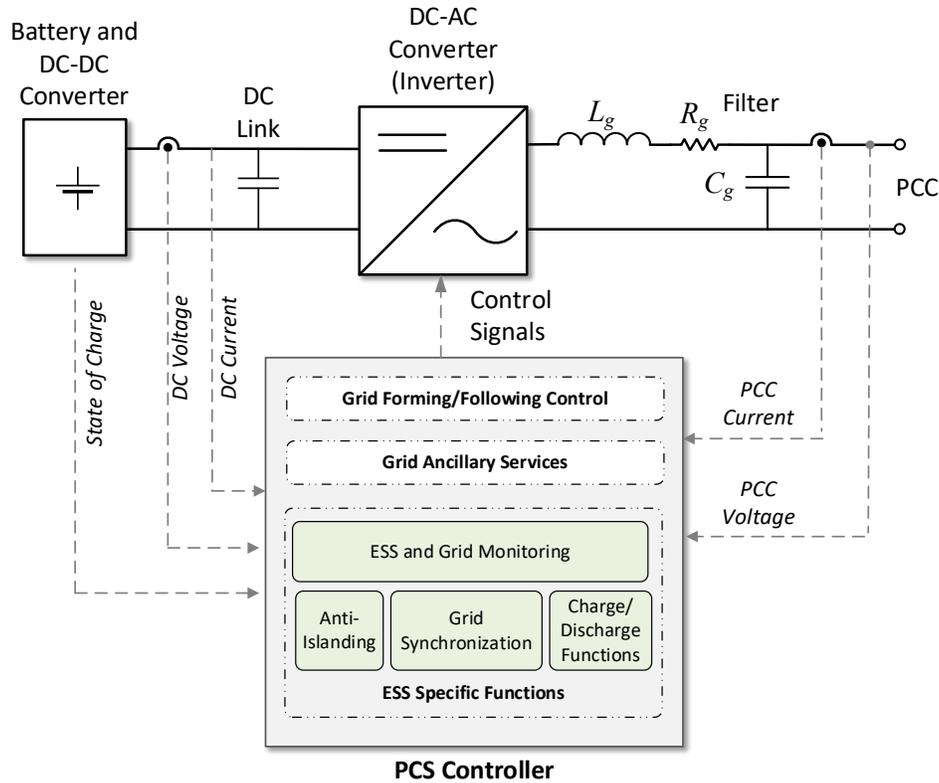
### 2.3. Modeling Converters in Energy Storage Systems for Stability Analysis

Stability analysis of power systems with ESSs entails the need to properly model and represent its various components. The major components of a typical ESS are illustrated in Figure 6. ESSs are integrated with power systems through a power conversion system (PCS) consisting of semiconductor switches. The PCS is a device that transforms the power from the energy storage unit into suitable AC power compatible with the grid. In DC power systems, the PCS transforms the power from the energy storage unit into suitable DC voltage using a DC-to-DC conversion. The operation of the semiconductor switches is controlled using signals generated through feedback control loops. These control signals are in-turn converted into high frequency gating signals for the semiconductor switches with the PCS using a modulation technique. The feedback control loops take measurements from the grid and the energy storage unit to make decisions on the operation of the ESS. The control loops governing the functioning of an ESS include the 1) secondary control loop (energy management system) that generates the power dispatch signals for the ESS and the 2) PCS controller (primary control loop) that utilizes voltage and current control mechanisms to track the reference power dispatch signals provided by the secondary control loop. Chapter 15, *Energy Storage Management Systems*, in this Handbook, discusses this in further detail [13]. The control mechanism implemented within the ESS governs its functionality, thus its operational characteristics. For power system stability analysis, the primary control loop shown in Figure 6 plays a critical role and needs to be properly modeled to accurately represent its interactions with power system dynamics.



**Figure 6. Components of an energy storage system**

The PCS controller is also responsible for a wide array of other functions that provides a smooth integration of ESS with the power grid. Figure 7 shows a more detailed diagram of a battery ESS. This is a generic implementation example and variations of these implementations are further discussed in Chapter 13, *Power Conversion Systems*.



**Figure 7. Schematic diagram of a grid-interfaced energy storage system along with its PCS controller**

Typically, the storage unit is first interfaced with a buck-boost type DC-DC converter, which in turn is interfaced with a DC-AC converter (inverter) unit. The inverter is then interfaced with a filter before being integrated with the grid at the point of common coupling (PCC). The combination of the inverter and filter converts the DC power from the battery to suitable AC power for the grid. An LC/LCL type filter is typically used to connect the ESS with grid at the PCC to mitigate the switching noise from the power electronic converter. When the ESS is operating in the charging mode, the inverter can also perform the function of a rectifier that converts AC power from the grid to suitable DC power. As stated before, the primary function of the PCS controller is to implement the primary control loops to track dispatch signals generated by the secondary control loop. This may be accomplished by either operating the inverter in the grid-forming or the grid-following/supporting mode. Based on multiple conditions (e.g., grid voltage, frequency, state of charge), the ESS can operate in either of these modes. If required, the operating mode can also be set manually by the system operator. Depending on the mode of operation, the primary control loop will have a distinct structure and will impact the stability of the power system differently.

In the grid-forming control mode, the ESS regulates the voltage and frequency at the PCC to reach desired reference values which are set points to the control loops. The ESS can be the master voltage and frequency controller of the power system [4] in such scenarios. When multiple ESSs are connected to the power grid, to avoid coordination issues a droop-based control strategy is typically used to ensure power sharing while supporting voltage and frequency control [14]. In the grid-following control mode, the ESS only regulates the active and reactive power injection into the power system. The ESS does not directly control the voltage and frequency of the power system under this control mode. A PLL is used to keep the ESS synchronized with the power grid when

operating in this control mode. The parameters and the design of the PLL play a crucial role in the stability of an ESS operating in the grid-following control mode. Therefore, the PLL needs to be properly modeled for accurate stability assessment. Several published articles [15, 16, 17] provide an excellent overview of the various PLL designs and models that can be utilized for stability studies. The grid-supporting control mode is very similar to the grid-following mode except that the active and reactive power output are now controlled with an objective to minimize the voltage and frequency variation of the power system. These functions are layers within the primary control loop as shown in Figure 7 and are known as grid ancillary services. Grid ancillary services are special functions built into the ESS to keep the voltage and frequency of the power system within acceptable limits. All these functionalities and control loops of an ESS need to be properly modeled for accurate stability analysis. These are typically implemented based on the IEEE 1547 standard [18].

Different modeling approaches and simulation tools can be used for stability analysis. The choice largely depends on the desired level of accuracy and the computational resources available. Ideally, all the components of the ESS (including semiconductor switches, modulation techniques, and control loops) must be modeled for an accurate stability analysis. However, such a detailed model of ESS can quickly become computationally intractable. This is mainly because such a detailed model must represent high frequency switching dynamics introduced by the operation of the semiconductor switches. To accurately represent these switching dynamics, the models must be very detailed and solved using timesteps in the order of microseconds, greatly increasing the computational workload involved.

Considering these factors, the two major approaches/tools that are typically used to study such power system dynamics are: (a) phasor-based simulation tools, and (b) electromagnetic transient (EMT) tools. Both these approaches have their advantages and shortcomings. Phasor-based tools are computationally inexpensive but lack the modeling detail, potentially compromising the accuracy of the results. Hence, phasor-based approaches are only suitable for higher-level system study and impact analysis. The phasor-based simulation models and tools only model the fundamental frequency of the power system (60 Hz) and assumes balanced system operation. This is quite a strong assumption as power systems are often unbalanced especially when analyzing the stability of distribution systems and microgrids. Phasor-based models are not capable of capturing the fast dynamics introduced by the switching mechanisms and the primary control loops of PEC based ESSs [19]. Without modeling these critical aspects, the stability analysis may lead to erroneous conclusions. Thus, under the influence of PEC-based resources and ESSs, phasor-based tools are becoming increasingly invalid for stability analysis. EMT tools on the other hand provide a high degree of granularity in the model thus ensuring accuracy but need higher amounts of computational resources and time to solve.

EMT type simulation tools are more suitable for proper analysis of ESS-based systems where all the switching dynamics and control loops must be properly represented. EMT models can model dynamics over a wide range of timescales including the transients/harmonics that are essential to analyzing the stability of power systems with PEC-based ESSs. Within EMT modeling approaches, two major sub-modeling techniques are employed – detailed modeling and average modeling [4]. Detailed modeling is an accurate approach that models the switching behavior associated with the power electronic converters, in addition to other dynamics due to passive/parasitic resistance and reactance of the power electronic switches. In this approach, individual semiconductor switches are modeled along with the complete detail of the primary

control loops including the modulation algorithms. The detailed models lend themselves to accurate time-domain simulations of ESSs. However, because the models need to be solved in a very small time-step (in the range of microseconds or less), the computational resources are typically high when using the detailed models to analyze and simulate ESSs.

To reduce the computational cost, average modeling techniques are introduced. Average modeling use “small signal” and “large signal” models of the ESS that help in modeling the low frequency dynamics of the system while ignoring the fast dynamics caused by the switching of the semiconductor switches [8, 20]. The system dynamics that arise due to the parasitic elements of the semiconductor switches of the PCSs are also neglected when using these modeling techniques. In these approaches, the semiconductor switches can be represented by simple controlled current and voltage sources that significantly reduce the computational time to run simulations. The primary control loops are still modeled but the modulation algorithms may be neglected. These are time-varying, non-linear models that provide a good balance between accuracy and computational cost. Because the switching behaviors are not incorporated in average models, these models are not as accurate as the detailed models. Note that the models used for the stability analysis must match the specific dynamic power system stability issues of interest, the relevant timescale, and the mode of operation of the ESS [8].

## 2.4. Stability Analysis Techniques for Grid-Connected Energy Storage Systems

As discussed in the previous sections, converter models for ESSs are developed to study stability of grid-connected ESSs. Stability analysis studies how the system behaves when abnormal operation or failure of components in the grid (contingencies) occurs, disturbing the system from normal operating condition and producing one or multiple stability phenomenon as described in Section 2.1 (e.g., oscillations in rotor angle, frequency, and voltage). Stability analysis methods widely used by energy industries, utilities, and system operators, and reliability coordinators mostly include small signal stability and time-domain simulation-based approaches. These approaches are divided into two categories:

- **Simulation-based approaches** are typically used for planning long-term operation and protection of power systems. High-fidelity power system models with detailed grid, ESS, and control models are simulated for a large set of possible contingency scenarios, and dynamic responses are observed and studied for each contingency.
- **Analytic approaches** include small signal stability and direct methods. Small signal stability is used for natural oscillations such as constant fluctuations of load power where the oscillations are not too large. On the other hand, direct method can study forced oscillations caused by major critical disturbances or contingency events.

The main difference between a simulation-based approach and direct method of an analytic approach is that the direct method can be easily applicable to online contingency classification while simulations are not often used for online applications. However, the direct method is in the research stage due to its algorithmic complexity.

The following sections discuss details of small signal stability, time-domain simulation, and direct method approaches and their applications to stability analysis of grid-connected ESSs.

### 2.4.1. Small Signal Stability

Small signal stability analysis is based on the local linearization of the original power system models at its current operating point. Stability and analysis tools are well developed for linear systems and are scalable and practical for large power systems. More specifically, linearization is based on first-order approximation of the model given a general power system model  $\dot{x} = f(x)$ :

$$f(x) \approx f(\bar{x}) + \left. \frac{\partial f}{\partial x} \right|_{\bar{x}} \Delta x + \delta f(x)$$

where  $\bar{x}$  is represent the current operating point,  $\Delta x = x - \bar{x}$  is the disturbance around the operating point, and  $\delta f(x)$  represents nonlinear terms that are ignored. Small signal stability analyses are effective when the size of the disturbance  $\Delta x$  is relatively small such as load fluctuations. However, since the resulting dynamics are simplified (i.e., linearized), small signal analysis fails to cover nonlinear effects of the power system dynamics, i.e.,  $\delta f(x)$ , in the case of large disturbances (e.g., large generator failure, line-to-ground faults). Nonetheless, small signal stability is a standard practice of power system stability analysis to guarantee robustness against constant power fluctuation and noise in the network [1]. The examples of utilizing small signal stability techniques have been demonstrated in many grid-connected ESS applications. For example, small signal stability study is used for grid-connected ESSs with different energy storage technologies such as:

- enhancing power system stability
- optimal energy storage sizing for stability improvement
- voltage stability improvement for wind power penetrations using ESS
- stability of low inertial AC/DC microgrids [21, 22, 23, 24, 25, 26, 27]

However, with the increasing utilization of converters in power systems with nonlinear converters and controllers, small signal models might not accurately represent the power system and lead to misleading conclusions [8, 7].

### 2.4.2. Time-domain Simulation

Time-domain simulation is a simple and effective approach to analyze stability for different contingency scenarios. In this approach, repeated time-domain simulations are performed using high-fidelity models to find scenarios that may cause instability in the power system. This involves computing system trajectories conducted through numerical integration of power system models over a finite time horizon for different contingency scenarios. One such technique, Monte Carlo, is often used in time-domain simulations to account for randomness of various contingency cases [28]. Typically, this approach requires observation of the system trajectories over a long time to get sufficient information and verify whether the system will become unstable. Hence, parallel computing simulation is often leveraged to overcome computational challenges for the application in practical large-scale power systems [29]. Also, stability of power electronics can be studied leveraging hardware-in-the-loop (HIL) experiments. In HIL experiments, the PECs are connected to real-time simulators mimicking the interconnection to the bulk power grid or distribution network. The output waveforms of PECs are observed to investigate the stability of interconnecting the PECs to the grid. Below is a list of time-domain simulations that are used to evaluate stability of different ESSs and their control designs:

- Generator angle/voltage stability [30]
- Frequency stability [31]
- Transient stability [32] [33] [34] [35] [36]
- Voltage stability and power transfer capability of power system with PV, BESS and STATCOM [37]
- Voltage and Transient stability for power systems with VSC-based energy storage systems and DC link [38]
- Transient voltage/frequency stability [39]

### 2.4.3. Direct method

Direct method is used to analyze stability without resorting to repeated time-domain simulations. Hence, the direct method for stability analysis is much faster and robust compared to small signal stability and time-domain simulation methods, and suitable for online stability monitoring. It is considered a promising tool for online dynamic security assessment in the research community. However, it involves nontrivial tasks of verifying complex mathematical conditions (e.g., Lyapunov stability theorem [40, 41, 42, 43]) and has challenges to be adopted for practical use in power systems. There are existing methods to utilize direct method for power system transient stability such as transient energy function methods [40, 43, 44, 45, 46, 47, 48] and polynomial optimization methods [49, 50, 51]. Direct method is mostly used in conventional power systems with synchronous generators. However, its extension to non-conventional power systems with renewable generations, distributed energy resources, and ESSs is still an ongoing research area.

## 3. Challenges and Emerging Technologies

Stability analysis of modern power systems is becoming increasingly complex. Modern power systems are equipped with increased PEC-based generation in addition to conventional rotational generation. Furthermore, a distributed operation paradigm is taking priority with recent advancements in microgrids, distributed generation, and communication infrastructure. The emergence of microgrids that can operate in islanded mode (disconnected from the main power grid) raises several challenges related to low-inertia, voltage and frequency coupling, and voltage imbalance. Existing analytical tools for stability analysis do not allow the option of examining all possible conditions and operating scenarios of such modern complex power systems. In addition, the need to model the PECs and control loops of ESSs along with other renewable energy sources such as PVs and wind energy systems demands higher computational resources. In this section, the challenges and emerging technologies for stability analysis of such modern power systems are discussed.

### 3.1. Emerging Technologies in Stability Analysis

Analytical tools developed for conventional power systems often fail to address stability issues in a heterogeneous power system setting with renewables and control systems. In this section, we discuss some of the emerging technologies that have received significant attention as alternative approaches to overcome the challenges for quantifying stability of today's rapidly changing power systems.

### 3.1.1. Data-Driven Methods

Most existing tools for stability analysis (e.g., simulation) require a model that describes the behavior of grid components (e.g., electricity network, generators, controllers). However, these models, in most cases, only approximate the real power system dynamics. Data-driven methods utilize measured data to analyze stability. The relevant data are collected from advanced metering and measurement devices such as relays and phasor measurement units, which are prevalent in today's power systems. Data-driven methods leverage data analytical tools such as machine learning and deep neural network modeling approaches to learn complex power system models and classify and predict abnormal activities in electricity grid [52, 53, 54, 55, 56, 57, 58]. These methods (not requiring a model to describe the power system) are suitable for adaptive and online applications and often improve scalability compared to existing analytical model-driven methods. However, they have several challenges: 1) Researchers must preprocess large volume of data before using it, although the data is not always available at that time; 2) results are sensitive to quality of data (i.e., measurement noise); and 3) machine learning models (e.g., neural network) are heuristic to some extent and require fine tuning of model parameters.

### 3.1.2. Robust Stability Analysis

In today's power systems, generation and load profile are variable and uncertain due to increasing renewable generation, electric vehicles, ESSs, etc. Deterministic approaches do not cover the effect of those uncertain sources. As a result, the notion of robustness is becoming more important. To make the analysis comprehensively valid for variable operating conditions, stability analysis should incorporate all uncertainties. There are several recent approaches including probabilistic methods (e.g., stochastic optimization), reachability analysis, and machine-learning-based approaches such as reinforcement learning, which promote robustness of the analysis. Probabilistic analysis investigates input-output (IO) correlations through different probabilistic distributions of the uncertainties [59, 60, 61, 62]. In most cases, these methods resort to (near-) linear approximation of a power system model to circumvent challenges in finding exact probabilistic outputs. Reachability analysis propagates an uncertainty set through power system trajectories and the result provides worst-case impact of uncertainty on power system dynamics [63, 64, 65, 66, 67, 68]. These probabilistic and worst-case results can be utilized to analyze potential contingency scenarios that can induce instability in power systems [68]. Further, convex optimization-based algorithms can be utilized to find robust stability margin of power system dynamics [69, 70].

## 3.2. Challenges and Opportunities in Modeling of Energy Storage Systems for Stability Studies

Time-domain simulations have been widely used for stability analysis lately and require adequate models to be effective. Traditional modeling techniques and simulation tools have been developed under the premise that the power system is largely dominated by rotational-synchronous generations. However, with widespread integration of PEC-based resources in the form of PVs, wind, and/or ESS, the dynamics of the power system are significantly different. The existing PEC model types and their applicable uses are presented in "Review of Dynamic and Transient Modeling of Power Electronic Converters for Converter Dominated Power Systems" [8]. The dynamics of the primary control loops and the PLL play a vital role in the way ESS interacts with the grid [71]. Detailed EMT simulation tools are desirable for modeling and simulation of ESS-based power systems, especially when analyzing stability of weak, unbalanced power systems and

power systems under faults. EMT models and simulations tools, as stated before, come at a higher computational cost. When simulating power systems with larger number of nodes the simulation gets computationally intractable. Hence, new modeling techniques and tools are being developed that are more suitable for modern power systems. A comprehensive review on methods to accelerate EMT simulations for power systems with a PEC is presented in “Review of Methods to Accelerate Electromagnetic Transient Simulation of Power Systems” [72].

Switching function models of power electronic converters are an alternative to detailed models to achieve computational gains in simulating the power system models [73]. With the use of switching models, the converters are represented as multi-port, black-box circuits with controlled voltage and current sources. The signals controlling these sources are switching functions defined according to the converter’s operating behavior. Because it avoids the use of physical nonlinear models of the semiconductor switches, the simulation times can be significantly reduced. However, this limits the application to steady-state and large signal transient analysis at the system level. Another significant challenge in modeling and control of power systems with ESSs is that the dynamics largely depend on the control loops and are not governed simply by physical laws as in synchronous generator dominated systems. Because different control mechanisms can be employed to design the control loops, accurate parameters to model these converter-based ESSs are difficult to obtain because of the proprietary nature of the converter design. Hence, data-driven methods that extract the model dynamics without knowledge of the underlying design of the converter are also gaining popularity [74, 75, 8].

Real-time simulation techniques provide another solution for the needs of large computational power to model and simulate ESSs [76, 72]. Real-time digital simulators are configured to be scalable such that several computational units can be used in parallel for computational gains in simulating the EMT type ESS models. Furthermore, if the size of the power system is limited, the simulations can be performed in real-time. This allows the simulation models to interact with external controllers and/or hardware components through IO modules. These types of testing are referred to as HIL and/or power-hardware-in-the-loop (PHIL) type simulations. Such technologies allow for improved control design and stability analysis of ESS systems as the interactions can be studied in almost realistic grid conditions without having to solely rely on mathematical models of the system. Novel controllers for ESSs such as adaptive machine-learning-based controllers and/or predictive controllers can be validated in a realistic and safe testing environment before being deployed in real systems. This helps in rapid control algorithm development cycles and potentially reduces the cost of testing the control design [77]. Opal-RT Technologies and RTDS Technologies manufacture some of the most popular real-time digital simulators in commercial use.

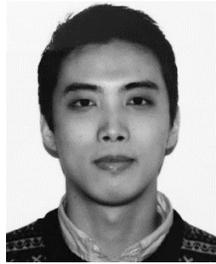
#### 4. Concluding Remarks

The dynamics of modern power systems with fast-acting components such as PEC-based loads, generators, and ESSs are significantly different from traditional power systems dominated by rotational generators. Therefore, the stability of the power system is largely affected by PECs and their underlying control loops that govern their functionalities. Additionally, short-term stability issues pertaining to fast-acting components of the power system are more common. Large-signal analysis using time-domain simulations are preferable for accurate stability analysis compared to small-signal analysis utilized in traditional power systems. This is especially true for microgrid systems where the conventional stability definitions and classification need rethinking. New

methods and tools for accurate modeling of all the components and sophisticated computational tools in the form of real-time digital simulations are essential for proper stability analysis of power systems with ESSs.



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