Document Retrieval and Ranking using Similarity Graph Mean Hitting Times

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ABSTRACT

We present a novel approach to information retrieval and document analysis based on graph analytic methods. Traditional information retrieval methods use a set of terms to define a query that is applied against a document corpus to identify the documents most related to those terms. In contrast, we define a query as a set of documents of interest and apply the query by computing mean hitting times between this set and all other documents on a document similarity graph abstraction of the semantic relationships between all pairs of documents. We present the steps of our approach along with a simple example application illustrating how this approach can be used to find documents related to two or more documents or topics of interest.
ACKNOWLEDGMENT

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1. INTRODUCTION

Given a corpus, or collection, of text documents, analysts often use information retrieval methods to search, organize, and make sense of the contents of the corpus as a whole. Traditional information retrieval involves an analyst providing a set of search, or query, terms of interest and using various methods to find documents that include, or are related to, those terms [4, 27, 30]. However, in many analysis applications an analyst may not know a priori what terms are of most interest. In such applications, methods for analyzing text corpora are needed that provide both summary information about the corpus as a whole as well as a means for retrieving documents based on subsets of the corpus that are determined as part of an exploratory analysis process.

In this report, we describe a novel use of existing text analysis techniques coupled with a graph analysis technique for addressing the problem of document corpus analysis without the use of predetermined sets of search terms. Specifically, we leverage standard methods used in text retrieval based on natural language processing and topic modeling to identify summary information about a document corpus. This summary information is provided as a set of topics, each of which is a collection of weights identifying the importance of each term from the full document corpus in that topic. By analyzing this topic summary and the subsets of documents related to each topic, analysts then identify a set of documents of interest for further investigation and analysis. Finally, a graph abstraction of document similarities based on the topics relationships is used to provide a rank ordering of the similarity of each document in the corpus to the set of documents of interest.

This approach to text retrieval and analysis differs from traditional search techniques in that no search terms are required before analysis begins. Analysts are provided with topical summaries of a corpus, from which they can identify subsets of documents of interest. Note that document clustering is a standard approach for finding subsets of documents related by term or topic usage [23, 31]. However, standard clustering methods leverage only pairwise document relationships, whereas our approach can identify relationships between sets of documents. For applications where all possible term, topic, and document relationships cannot be identified from a given corpus, and where an analyst’s previous experience and expertise with term and topic relationships in a given subject area must be leveraged, our approach provide a means for identifying document relationships based on such information.

The remainder of this reports is structured as follows. In Section 2, we describe the methods used in our approach. In Section 3, we illustrate the use of our method applied to a simple document collection. Finally, in Section 4, we provide conclusions related to our approach as well as some ideas for future research in this area.

2. METHODS

In this section, we describe the methods used in our approach for analyzing text corpora described in Section 1.

The overall process for analysis is as follows:
1. Convert text documents into a vector space representation.

2. Weight document vectors.


4. Present summary of document topics to analyst.

5. Have an analyst indicate documents of interest using topic summary information.

6. Create document similarity graph using topic model.

7. Compute document similarities to set of documents of interest using graph analysis method.

We have implemented the methods in Python using the following packages: pandas (for data manipulation), numpy (dense linear algebra), scipy (sparse linear algebra), scikit-learn (vector space representations), matplotlib (visualization), and hitmix (graph analysis). In the descriptions of the methods below, we identify which packages and capabilities we leverage in our implementations, along with any new software developed specific for the analyses presented here.

2.1. Vector Space Representations of Documents

A corpus of documents can be represented using an $n \times m$ document-term matrix, $A$, where $n$ is the number of documents and $m$ is the number of terms in the corpus [30]. Such a representation supports a wide variety of analyses of document corpora, as we have demonstrated for retrieval [7, 20], topic modeling [5, 12, 13], named entity recognition [32], part-of-speech recognition [8, 16], clustering [12], classification [3], summarization [9, 17, 19], multilingual analysis [1, 2, 6, 7], and visualization [10, 11] tasks.

We use the CountVectorizer class in the scikit-learn Python package to create a document-term matrix. In the case studies presented in Section 3, we apply simple preprocessing of the documents to extract whitespace-delimited terms, convert all terms to lowercase, and remove numbers and URLs from the final set of terms used. More advanced document processing based on lemmatization [21] and stemming [29], has demonstrated utility in several text analysis applications [5, 19] and can be applied using the CountVectorizer class. However, we limit our use here to simple document processing to illustrate the creation of vector space representations and not provide an exhaustive or optimal matrix creation process.

The document-term matrix created using the process above contains the count of term $j$ appearing in document $i$ as the entry at index $(i, j)$. 
2.2. Document and Term Weighting using Pointwise Mutual Information

Using the raw terms counts per document across many text analysis applications has been shown to introduce biases based on document length, novel term usage, and common term usage [4, 18, 19, 25]. Weighting of the document-term matrix entries often leads to improved analyses. Specifically, the weighting scheme based on term frequency and inverse document frequency (TF-IDF) is by far one of the most popular forms of weighting document-term matrices [24].

More recently, Chew et al. demonstrated that the use of pointwise mutual information (PMI) for document-term matrix weighting provides several advantages over TF-IDF and other weighting schemes for retrieval tasks [6]. Using PMI, the weighted entry of the document-term matrix is given by

$$w(i, j) = \log_2 \left( \frac{p(i, j)}{p(i) \times p(j)} \right),$$

where $p(j)$ is the probability that a randomly selected term from the corpus is term $i$, $p(i)$ is the probability that a randomly selected term from the corpus comes from document $i$, and $p(i, j)$ is the joint probability that a randomly selected term from the corpus is term $j$ and that it came from document $j$. These probabilities are estimated using the term and document counts provided by the CountVectorizer class described in Section 2.1.

2.3. Topic Modeling using Latent Semantic Analysis

Topic models of documents may help to address several challenges associated with using raw term vectors, even weighted as above using PMI, when analyzing relationships between the contents of documents across a corpus. Specifically, topic models define a set of features, called topics, that represent combinations of terms to model latent semantic relationships between sets of terms as they occur within and across the documents. The number of topics is often chosen to be much less than the number of terms and documents in the corpus, leading to a reduction in the dimensionality of the document-term matrix.

We use the Latent Semantic Analysis (LSA) [14, 15] to model topics in this work. LSA computes a truncated singular value decomposition of the weighted document-term matrix [4]. Specifically, for an $m \times n$ document-term matrix $A$, the truncated SVD with $t$ topics is defined as

$$A_t = U_t \Sigma_t V_t^T,$$

where $U_t \in \mathbb{R}^{m \times t}$, $V_t \in \mathbb{R}^{n \times t}$, and $\Sigma_t \in \mathbb{R}^{t \times t}$ is a diagonal matrix. The diagonal elements of $\Sigma_t$ are scalar weights (called the singular values of $A_t$) and represent the contributions of each topic to the overall semantic content of the corpus.

In previous work, we have demonstrated that the LSA topic model has helped improve document analysis across many applications, including retrieval [7, 20], clustering [12, 13].
classification [3], summarization [9, 17, 19], multilingual analysis [1, 2, 6], and visualization [10].

Computation of the truncated SVD in our implementation is performed using the svds method of the scipy.sparse.linalg Python package. The choice of \( t \), i.e., the number of topics, is left to the analyst.

2.4. Presenting Topic Summaries to Analysts for Choosing Documents of Interest

Following our recent work on document clustering [5], we summarize the topic models by displaying \( \tilde{n} < n \) terms and \( \tilde{m} < m \) documents for each of the \( t \) topics. Topics are ordered by the singular value associated with each topic in decreasing order, thus displaying the topics contributing to the corpus contents the most at the top of the list (see Section 2.3). The ordering of the terms and documents within each topic are also presenting in decreasing order of the values from the column vectors of \( V_t \) and \( U_t \), respectively, thus displaying the most important terms and documents associated with each topic.

An example of this topic model summary display for the Twitter BBC Health dataset from the publicly available UCI Machine Learning Repository\(^1\) for top 3 topics (with \( \tilde{n} = 10 \) and \( \tilde{m} = 3 \)) is as follows:

Topic 0 (102.495): video (0.327) to (0.286) ebola (0.261) for (0.236) nhs (0.235) in (0.224) the (0.180) of (0.176) health (0.169) cancer (0.156)

Doc 1445 (0.041): VIDEO: NHS staff to help in Ebola areas
Doc 966 (0.037): VIDEO: NHS staff set off to help fight Ebola
Doc 2627 (0.037): VIDEO: The effect of floods on mental health

Topic 1 (76.529): health (0.595) mental (0.462) child (0.143) care (0.116) services (0.082) cuts (0.077) warning (0.077) nhs (0.069) needs (0.051) priority (0.042)

Doc 1107 (0.092): Child mental health services ‘unfit’
Doc 279 (0.088): NHS child mental health care pledge
Doc 115 (0.087): VIDEO: Child mental health services ‘broken’

Topic 2 (74.147): ebola (0.522) health (0.360) mental (0.283) uk (0.186) in (0.145) leone (0.101) vaccine (0.087) sierra (0.083) liberia (0.071) child (0.066)

Doc 153 (0.064): UK military health worker has Ebola
Doc 2846 (0.061): UK troops’ mental health ‘resilient’
Doc 436 (0.060): UK health worker monitored for Ebola

The values in parentheses in the display for topic \( t \) following “Topic \( t \)”, each term, and each document are singular value \( t \), values from the column \( t \) of \( V_t \), and values from column \( t \) of \( U_t \), respectively. From this display, analysts are asked to identify one or more documents that are of

\(^1\)https://archive.ics.uci.edu/ml/machine-learning-databases/00438/Health-News-Tweets.zip
interest and will serve as the set of documents for which all document similarities will be computed. Note that documents from different topics can be chosen to form this set.

### 2.5. Document Similarity Graphs

To analyze the relationships between documents, we construct a similarity graph where vertices represent documents and edges represent the semantic similarities between documents [26]. Such graphs have proven useful in many document clustering [5, 33] and topic model visualization [10, 11, 12, 13] applications.

The weight on the edge between vertices (i.e., documents) \( i \) and \( j \) using the topic model \( A_t \) is computed as follows:

\[
e_{ij}(t) = \frac{\langle U_i^t \Sigma_t, U_j^t \Sigma_t \rangle}{\|U_i^t \Sigma_t\|_2 \|U_j^t \Sigma_t\|_2},
\]

(3)

where \( U_i^k \) is column \( k \) of \( U_i \), \( \langle \cdot, \cdot \rangle \) denotes the inner product of two vectors, and \( \| \cdot \|_2 \) denotes the \( L^2 \) (or Euclidean) norm of a vector. This amounts to the Pearson correlation (or cosine similarity) between the documents using the topic model vectors. Thus, these weights take on values in the range \([-1, 1]\), with the sign corresponding to negative or positive correlation, and the magnitude corresponding to the strength of the correlation between the documents.

In this work, we limit the scope of our analysis to positive correlations only; hence all negative edge weights are replaced by weights of 0. We further allow for analyst-determined thresholding of the weights to create \( \epsilon \)-neighborhood graphs [33], which contain only the most significant document relationships.

### 2.6. Document Similarities using Graph Mean Hitting Times

Using the analyst-identified set of documents of interest from Section 2.4 and the document similarity graph from Section 2.5, we can now compute the relationships of interest to the analyst—i.e., the relationships between a set of documents of interest and all other documents in the corpus. To achieve this, we will compute the mean hitting times between the set of vertices associated with the documents of interest and all other vertices. Specifically, we leverage the HITMIX method of computing hitting time moments [22], originally developed to solve seed-set expansion problems on graphs, to compute the mean hitting times. HITMIX includes a highly-scalable, linear algebraic approach to computing mean hitting times between a vertex set (called the hitting set) and all other vertices, avoiding costly computations based on random sampling and simulation. In the document analysis context here, the mean hitting times represent the relationships between the documents of interest (i.e., the hitting set) and all other documents, with smaller values denoting stronger semantic relationships.

The original HITMIX method assumes a fully connected graph, which is often not the case for document similarity graphs created using the approach defined in Section 2.5, especially when using thresholding. Thus, our implementation of computing hitting time moments includes an
initial step of finding the connected components of the graph that contain the vertices associated with the documents of interest. We use the `connected_components` method of the `scipy.sparse.csgraph` Python package for this computation, which implements the efficient, scalable method by Pearce [28].

Our implementation of the HITMIX mean hitting times can be found at `https://github.com/sandialabs/hitmix`. The output of this implementation is a vector of size $m$ (i.e., the number of documents in the corpus) whose values are either 0 (indicating the document is in the hitting set), $\infty$ (indicating that there is no path in the document similarity graph that connects the vertex to the hitting set) or the mean hitting time of the vertex to the hitting set. These values are used to provide a rank-ordered list of documents as they related to the documents of interest, sorted by semantic similarity (as approximated by the mean hitting time values).

### 3. CASE STUDY

In this section, we present a case study describing the application of our information retrieval approach using a small number of documents that are very simple in terms of the size of the vocabulary used across the documents and the size of individual documents. Due to the simple structure of these documents, it is easy to see similarities and differences across the corpus, and illustration of each of the analysis steps described in Section 2 is also easy to follow. The full set of documents is presented in Table 3-1.

<table>
<thead>
<tr>
<th>ID</th>
<th>Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Document zero is about lions.</td>
</tr>
<tr>
<td>1</td>
<td>Document one is about tigers.</td>
</tr>
<tr>
<td>2</td>
<td>Document two is about bears.</td>
</tr>
<tr>
<td>3</td>
<td>Document three is about lions, tigers.</td>
</tr>
<tr>
<td>4</td>
<td>Document four is about lions, bears.</td>
</tr>
<tr>
<td>5</td>
<td>Document five is about tigers, bears.</td>
</tr>
<tr>
<td>6</td>
<td>Document six is about lions, tigers, bears.</td>
</tr>
</tbody>
</table>

Table 3-1 The complete set of simple documents used in the case study.

Across the documents, we see that the documents contain some common terms (i.e., `document`, `about`, and `is`), unique terms (i.e., document numbers such as `one`, `two`, etc.), and different combinations of shared terms (i.e., `lions`, `tigers`, and `bears`). We provide detailed results of the steps of the methods from Section 2 to illustrate how the relational structures associated with these different terms are identified, used, and captured in our analyses.

Figure 3-1 presents the document-term matrix of raw term counts in the top portion of the figure (Section 2.1) and PMI-weighted term values (Section 2.2) in the bottom portion of the term values. Note that the common terms all have the same PMI distributions across documents and
the values are either low or negative, indicating they are not very useful in distinguishing the relationship among the documents. The unique terms have high PMI values, as is expected. Finally, the shared terms also have relatively high PMI values, indicating their importance across documents.

For this small corpus we compute an LSA topic model (Section 2.3) using $t = 6$ topics. In many applications where the document corpus is much larger, we recommend using fewer topics than the maximum that can be computed, but for this example, we compute all available topics that can be computed using the `scipy.sparse.linalg.svds` method (which is one fewer than the minimum of the number of documents and number of terms). Using this topic model, we present the topic summaries (Section 2.4) below:
After some analysis, we see that the topics relate most strongly to documents about 0) individual animals, 1) tigers, 2) bears and tigers, 3) common terms in the shorter documents, 4) tigers and lions, and 5) lions. For this case study, we choose the documents of interest to be documents 0 and 1—i.e., a document about lions and a document about tigers. The goal is to find other documents that are related to both of these documents by treating them as a set.

Figure 3-2 presents the values in the document similarity graph (Section 2.5). No thresholding was applied in this use case. However, since we use only positive weights in the document similarity graphs (see Section 2.5), the values of 0 correspond to negative weights (i.e., negatively correlated document relationships).
Finally, we compute the mean hitting times (Section 2.6) of the similarity graph. Table 3-2 presents the documents ordered by mean hitting times. Recall that since documents 0 and 1 are in the set of documents of interest (i.e., hitting set), the mean hitting times for those documents are 0, as expected. The document closest to the documents of interest, with respect to mean hitting times, is document 3 about lions and tigers but not bears. Then, documents 6 is about all three animals; thus it is about lions and tigers but also bears. The next two documents, 4 and 5, are about lions or tigers and also bears. And then finally, the farthest document from the hitting set in terms of mean hitting time is document 2, which is not about lions or tigers and only about bears. This seems like a reasonable ordering of the documents if an analysts were searching for documents about lions and tigers, as indicated by the documents in the hitting set. If the analysts were to search for documents about lions and tigers separately through two queries of the corpus using traditional information retrieval methods, that would results in two ordered list that would need to be manually merged into a single ordered list. Using our approach described here, a single ordered list is returned that accounts for the relationships of documents to a set of documents, not to a set of terms or a single document.
4. CONCLUSION

We have presented an alternative approach to information retrieval based on document-document relationships rather than the traditional approach of using term-document relationships. We defined the steps of our approach that leverages standard natural language processing and topic modeling to construct a document similarity graph of the semantic relationships between documents. The main contribution of this work was the introduction of using graph mean hitting times between a set of documents of interest and all other documents in a corpus to provide an ordered list of documents that are most related to the set. We illustrated the application of this approach to information retrieval on a simple, synthetically generated small corpus, demonstrating that identifying a set of documents—rather than terms—is a viable approach to document retrieval.

Since the application of the HITMIX for text analysis and information retrieval presented here is a novel use of graph mean hitting times, future work will also include application to a variety of document corpora from multiple topical domains to better assess the effectiveness of this approach more generally.
REFERENCES


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