

HYBRID MULTISCALE COUPLING WITH STRUCTURE-PRESERVING OPERATOR INFERENCE AND THE SCHWARZ ALTERNATING METHOD

IRINA TEZAU, ERIC PARISH, ANTHONY GRUBER, AND PATRICK BLONIGAN

ABSTRACT. High-fidelity simulations of complex multiphysics systems are often prohibitively expensive, motivating the development of hybrid multiscale methods that combine models of differing fidelity. In this talk, we present a flexible framework for coupling full-order models (FOMs) and reduced-order models (ROMs) using the Schwarz alternating method [3]. By decomposing the computational domain into subdomains and iteratively exchanging interface information, this approach enables the coupling of heterogeneous models and discretizations, retaining high-fidelity resolution where needed while leveraging computationally efficient ROMs elsewhere.

The accuracy and robustness of such hybrid simulations depend critically on the quality of the underlying ROMs, particularly for strongly nonlinear dynamical systems. To address this challenge, we introduce neural network operator inference (NN-OpInf) [1], a non-intrusive model reduction approach that combines the expressive power of neural networks with structure-preserving operator representations. By expressing reduced dynamics as a sum of composable neural network operators with prescribed algebraic structure, NN-OpInf enables the construction of flexible nonlinear ROMs that retain important physical properties of the governing equations. We demonstrate the accuracy and stability of this approach on nonlinear convection-diffusion-reaction problems and finite-deformation solid mechanics.

Finally, we demonstrate the integration of structure-preserving NN-OpInf models within the overlapping Schwarz framework. Numerical results for nonlinear solid mechanics illustrate the advantages of NN-OpInf over conventional polynomial operator inference [2] in coupled simulations, highlighting the potential of combining structure-preserving data-driven ROMs with domain decomposition for efficient and robust multiscale modeling.

Keywords: Schwarz alternating method, domain decomposition, neural network operator inference (NN-OpInf), multi-scale coupling, structure preservation.

Mathematics Subject Classifications (2020): 65M55, 68T07, 65M22.

REFERENCES

- [1] E. Parish, A. Gruber, P. Blonigan, and I. Tezaur, NN-OpInf: An operator inference approach using structure-preserving composable neural networks, arXiv preprint arXiv:2603.08488 (2026).
- [2] I. Tezaur, E. Parish, A. Gruber, I. Moore, C. Wentland, and A. Mota, Hybrid coupling with operator inference and the overlapping Schwarz alternating method, *International Journal for Numerical Methods in Engineering*, 127(17), e70408 (2026).
- [3] A. Mota, I. Tezaur, and G. Philipot, The Schwarz alternating method for transient solid dynamics, *International Journal for Numerical Methods in Engineering*, 123(21), 5036–5071 (2022).

SANDIA NATIONAL LABORATORIES
Email address: `ikalash@sandia.gov`

SANDIA NATIONAL LABORATORIES
Email address: `ejparis@sandia.gov`

SANDIA NATIONAL LABORATORIES
Email address: `adgrube@sandia.gov`

SANDIA NATIONAL LABORATORIES
Email address: `pblonig@sandia.gov`