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Schwarz-Based Coupling of Diverse Subdomain-Local Data-Driven Models for Predictive Digital Twins

Irina Tezaur*, Eric Parish[1], Anthony Gruber[1], Chris Wentland[2], Ian Moore[3], Alejandro Diaz[1], George Chumbipuma[3], Trishit Mondal[4]

[*] Sandia National Laboratories, Livermore, CA, USA
e-mail: ikalash@sandia.gov, web page: www.sandia.gov/~ikalash
[1] Sandia National Laboratories, Livermore, CA and Albuquerque, NM, USA
[2] Ford Motor Company, Dearborn, MI, USA
[3] Rice University, Houston, TX, USA
[4] Worcester Polytechnic Institute, Worcester, MA, USA

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ABSTRACT

This talk will describe a novel approach for creating adaptive hybrid models through domain decomposition (DD) and the Schwarz alternating method (SAM) [1], towards enabling predictive digital twins. In this approach, the solution on the full domain is obtained via an iterative process in which a sequence of subdomain-local problems are solved, with information propagating between subdomains through transmission boundary conditions (BCs). The models being coupled can be subdomain-local full order models (FOMs) and/or subdomain-local data-driven reduced order models (ROMs). During the past five years, our group has pioneered the use of the SAM to couple domain decomposition (DD)-based reduced-order models (ROMs) constructed using a variety of data-driven methodologies, including intrusive projection-based ROMs [2], Physics-Informed Neural Networks (PINNs) [3], non-intrusive Operator Inference (OpInf) ROMs [4,7], and, more recently, Kernel ROMs (KROMs) [5] and Numerics-Informed Neural Networks (NINNs) [6]. Numerical examples will illustrate the robustness, convergence behavior, and flexibility of the proposed methodology, demonstrating its potential as a general framework for hybrid modeling. We will also discuss some preliminary work on enabling on-the-fly ROM-FOM switching within SAM, toward the design of a hybrid DD + SAM-based coupling framework.

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