Sandia National Laboratories

# NON-INTRUSIVE COUPLING OF REDUCED-ORDER AND FULL-ORDER MODELS WITH ADAPTIVE DOMAIN DECOMPOSITION


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## MOTIVATION for ROM-FOM coupling

Reduced-order models [ROMs] achieve significant computational speed-up through projection onto low-dimensional reduced spaces but are only as expressive as their reduced spaces. Meanwhile, full-order models [FOMs] are highly expressive but computationally slow.

**Objective:** Attain computational efficiency at the target accuracy through adaptive, local ROM-FOM coupling.

## RESEARCH GOALS

### ROM-FOM coupling with adaptive subdomains:

- Use ROM for computational speed-up
- Use *local* FOM for under-resolved features
- Retrain ROM on generated FOM data online to improve resolution
- Adjust the FOM subdomain for comp. speed-up

### Non-intrusive ROMs for modular components:

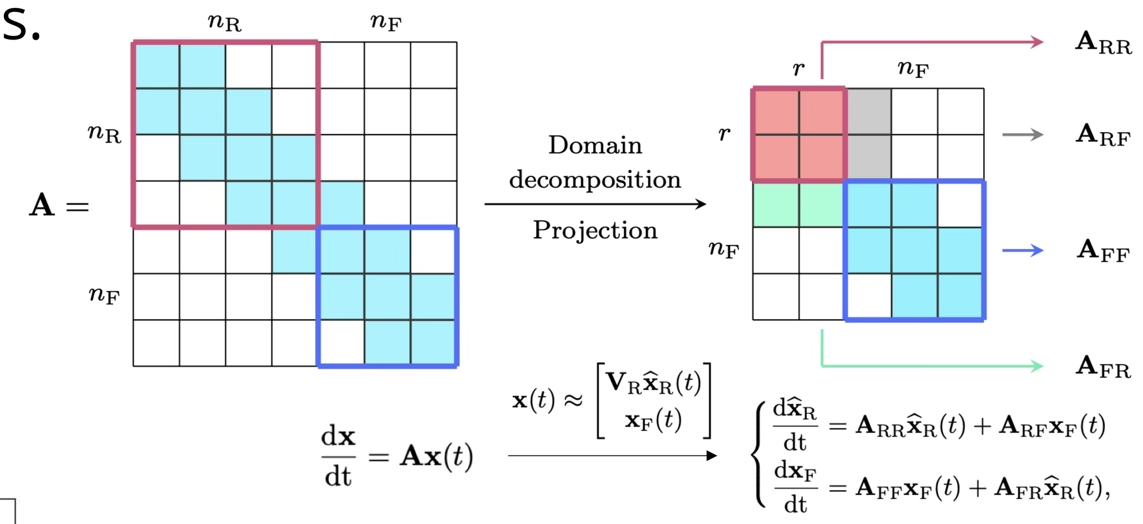
- Offline: Train bubble-ROM for reference components
  - Train on individual or assembled systems
- Online: Assemble complex geometry via ports
- Keep everything non-intrusive
- Provide error guarantees if possible

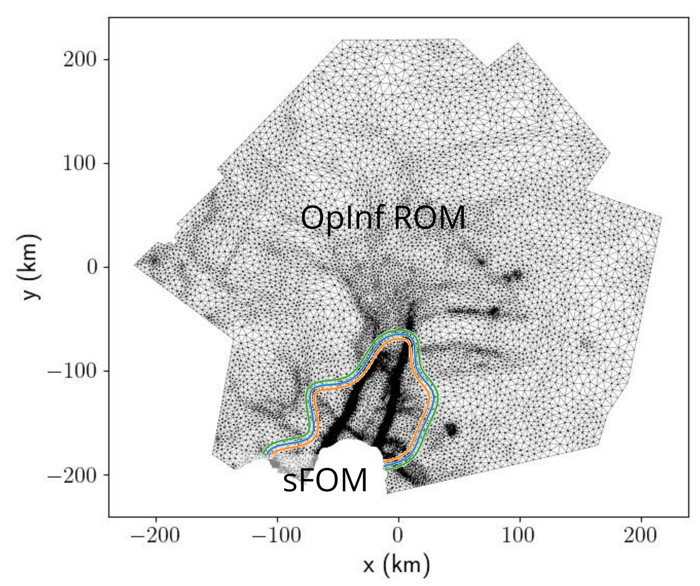
## NON-INTRUSIVE COUPLING OF OPINF AND sFOM MODELS

**Operator Inference** [OpInf, 1] learns a ROM on a chosen low-dimensional subspace by leveraging the structure of the governing equations.

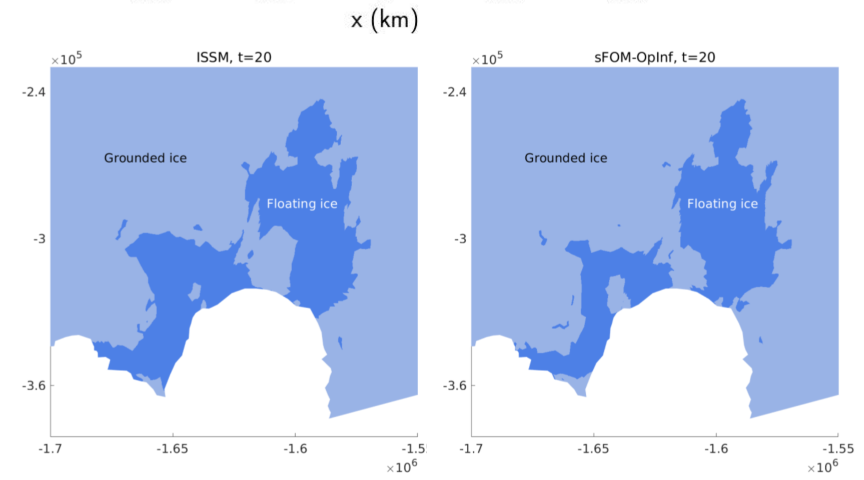
**Sparse FOM inference** [sFOM, 2] imposes a sparsity pattern to learn full-order dynamics.

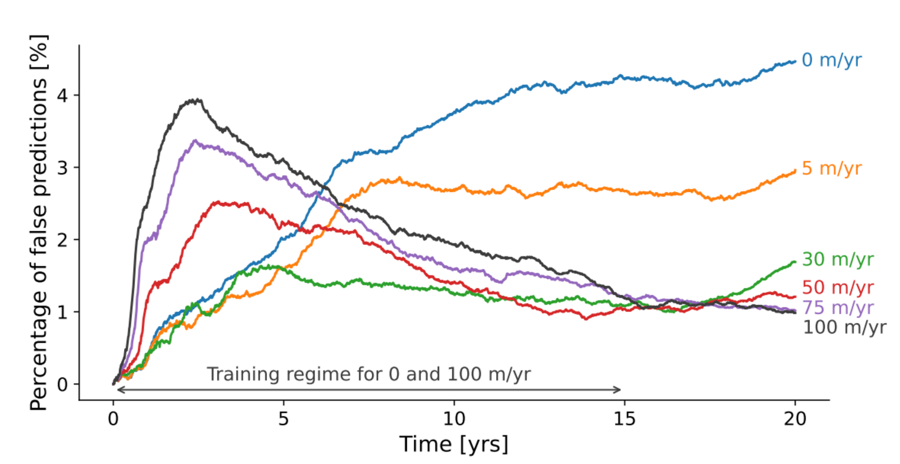
In [3] we combine both approaches to approximate dynamical systems with spatially localized features.




Applied to an ice-thickness model of the Pine Island Glacier in Western Antarctica with varying melt parameters, we achieve below 1% error at 8 × speed-up. Moreover, we accurately predict the grounding line between floating and grounded ice.

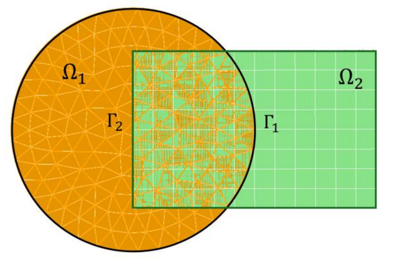



## SCHWARZ ALTERNATING METHOD FOR OPINF COUPLING

### Basic Schwarz algorithm for coupling

- Decompose $\Omega = \Omega_1 \cup \Omega_2$
- Solve PDE by any method in $\Omega_2$ w/ BCs on $\Gamma_2$ from just-obtained $\Omega_1$ solution.
- Solve PDE by any method in $\Omega_1$ w/ BCs on $\Gamma_1$ from just-obtained $\Omega_2$ solution.
- Iterate until convergence




**OpInf ROM + Schwarz BCs in $\Omega_i$:**

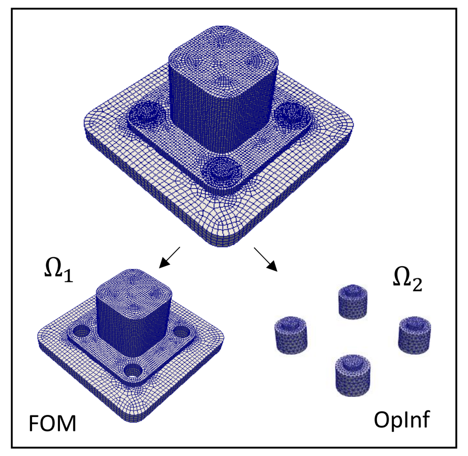
$$\dot{\hat{\boldsymbol{q}}}_i + \hat{\boldsymbol{A}}_i\hat{\boldsymbol{q}}_i + \hat{\boldsymbol{H}}_i(\hat{\boldsymbol{q}}_i \otimes \hat{\boldsymbol{q}}_i) + \tilde{\boldsymbol{B}}_i\boldsymbol{g}_i = \boldsymbol{0}$$

OpInf BC implementation is motivated **Dirichlet BC** treatment in **FEM**

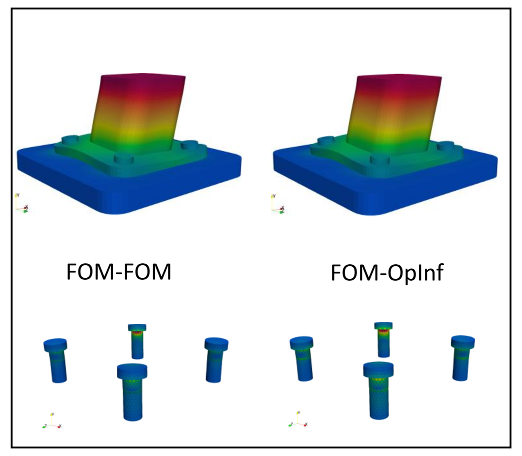
Learned Schwarz Dirichlet BC term

$\hat{\boldsymbol{A}}_i, \hat{\boldsymbol{H}}_i, \tilde{\boldsymbol{B}}_i$: learned operators, $\hat{\boldsymbol{q}}_i$ : OpInf solution

Schwarz enables the seamless coupling of disparate models (FOM, OpInf), meshes, solvers and time-steppers.




Schwarz has achieved meshing simplification & successful FOM-OpInf coupling for complex solid mechanics problems, with speedups up to 106× [4].





## REFERENCES

[1] Peherstorfer, Willcox (2016). Data-driven operator inference for nonintrusive projection-based model reduction.
[2] Gkimisis, Richter, Benner (2024). Adjacency-based, non-intrusive model reduction for vortex-induced vibrations.
[3] Gkimisis, Aretz, Tezzele, Richter, Benner, Willcox (2025). Non-intrusive reduced-order modeling for dynamical systems with spatially localized features.
[4] Tezaur, Parish, Gruber, Moore, Wentland, Mota (2026, submitted). Hybrid coupling with operator inference and the Schwarz alternating method.



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