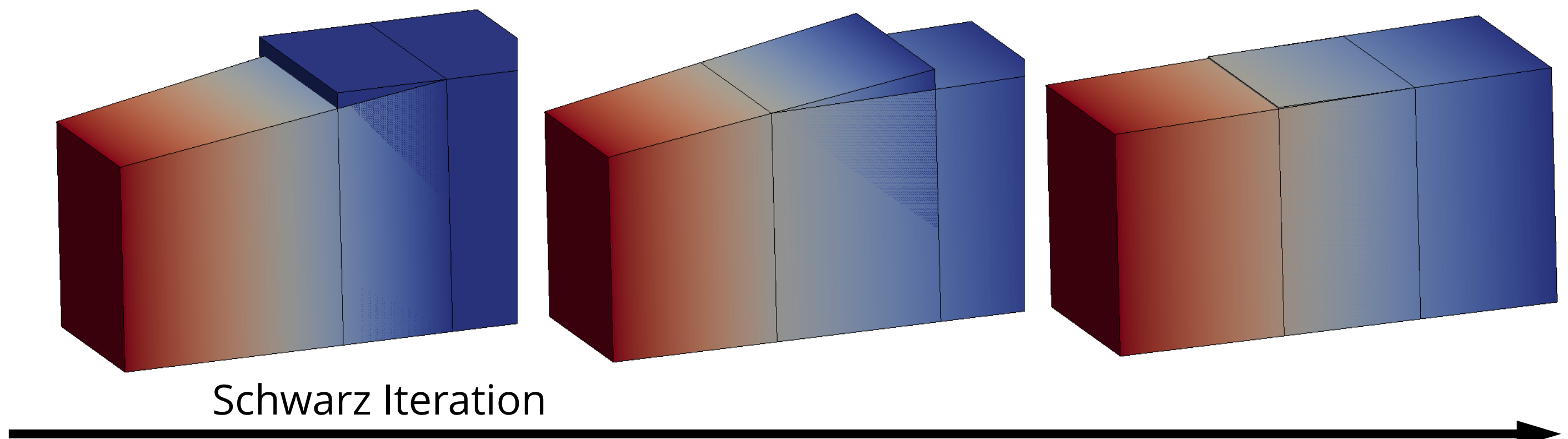
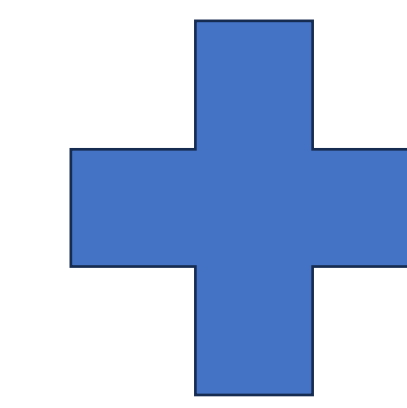
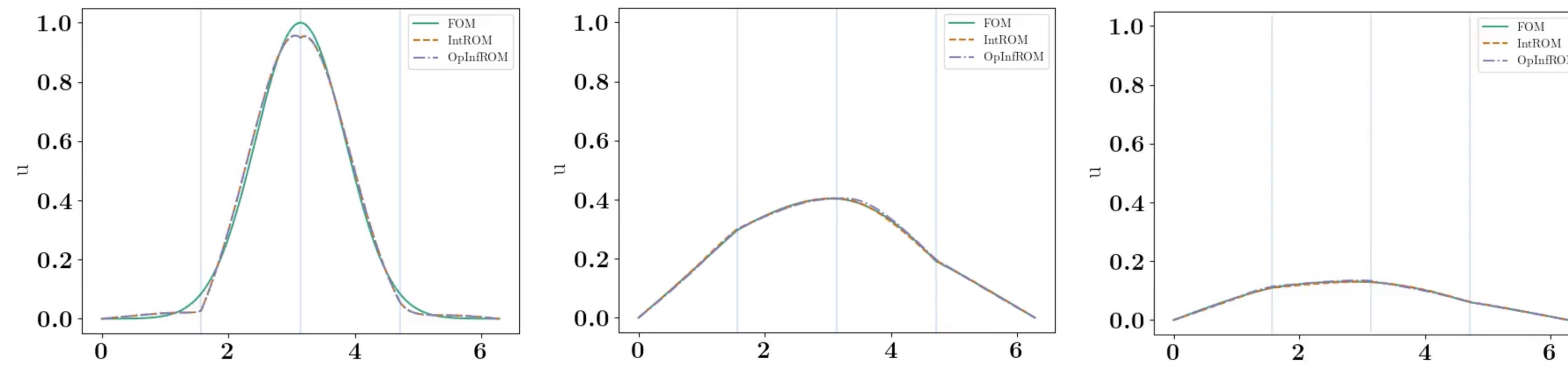




Anthony Gruber, Eric Parish, Alejandro Mota, Chris Wentland, Ian Moore, Irina Tezaur

Enabling Flexible Simulation with Non-Intrusive Model Reduction

Parametric
Operator
Inference
(pOpInf)

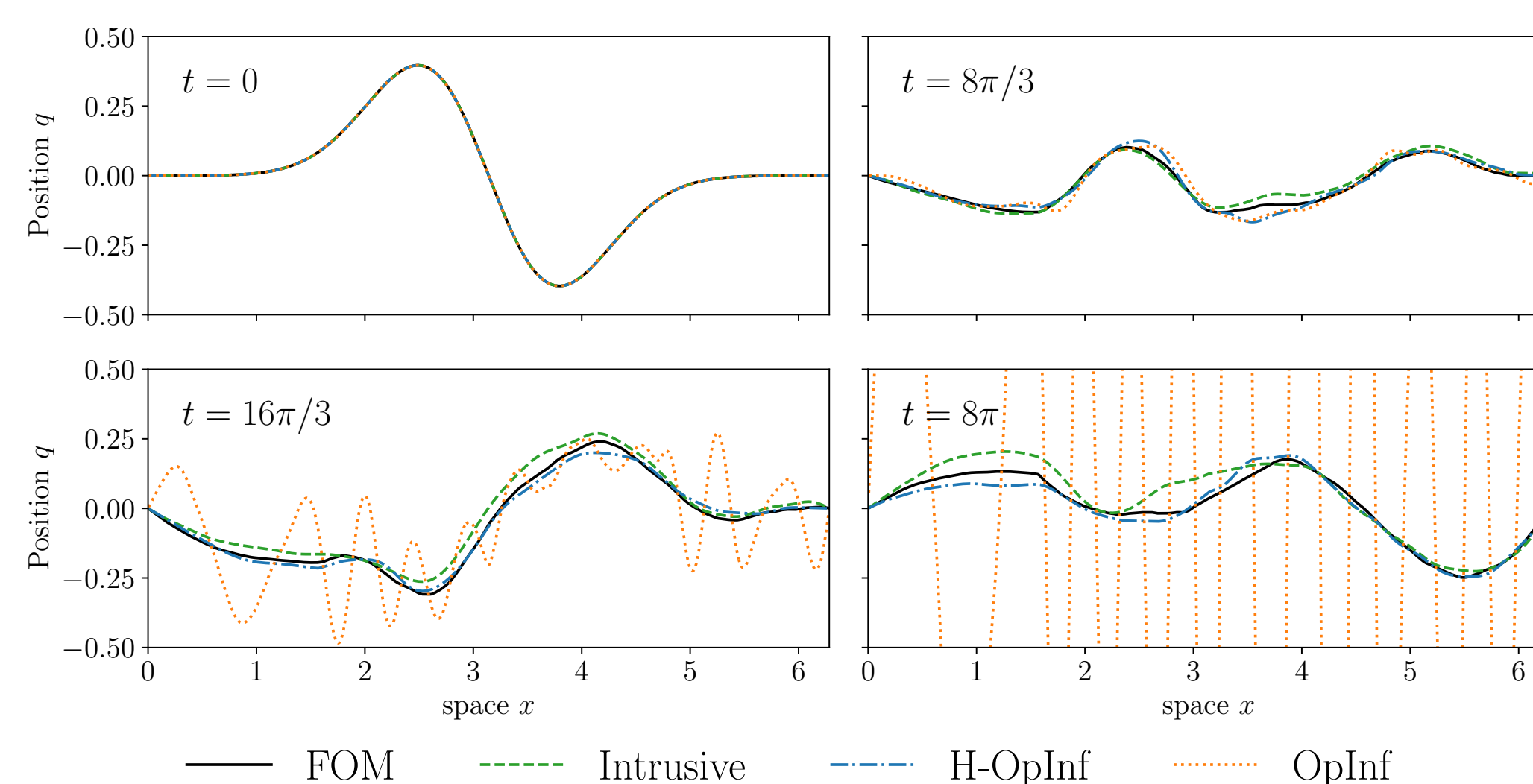


Schwarz
Alternating
Method
(SAM)

pOpInf yields **parametric, structure-preserving** non-intrusive ROMs.

- Linear and affine parametric dependence handled with convex inference
- Hard-constrained symmetry and definiteness properties preserve physics
- Non-convex neural network inference available for increased expressiveness

$$A \mapsto (B \mapsto C) \cong (A \otimes B) \mapsto C \quad \longrightarrow \quad A(\mu) = \mathbf{T}\mu'$$



Model problem (includes solid mechanics):

$$M(\mu)\ddot{x} + K(x, \mu) = f(x, \mu)$$

$$A(x, \mu) + S(x, \mu) \quad A^T = -A, \quad S^T = S$$

Given reduced basis U and snapshots $X_s = X(\mu_s)$, solve

$$\arg \min_{M, A, S, f} \frac{1}{2} \sum_{s=1}^{N_s} \left| \hat{M} D_t^2(\mu_s) \hat{X}_s - U^T M(\mu_s) D_t^2 P_U X_s + \dots \right|^2$$

Hybrid pOpInf + SAM framework for FOM/ROM coupling.

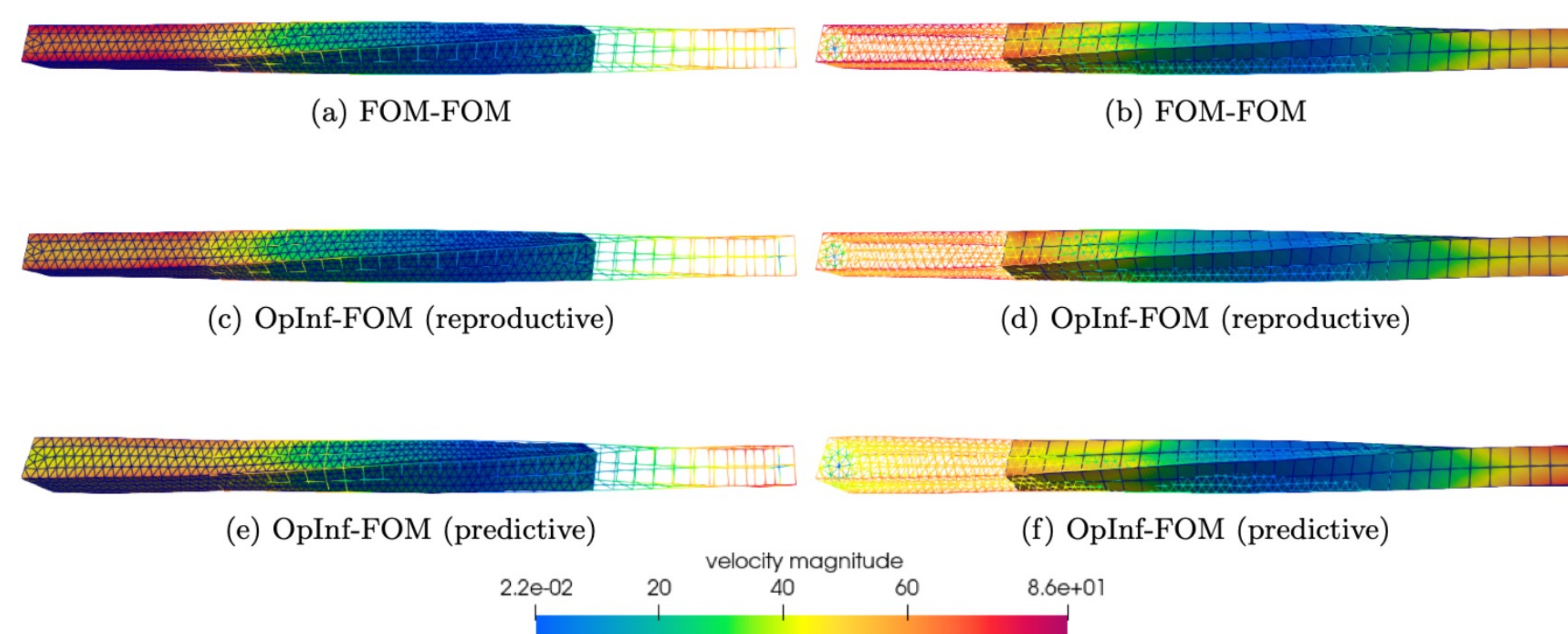
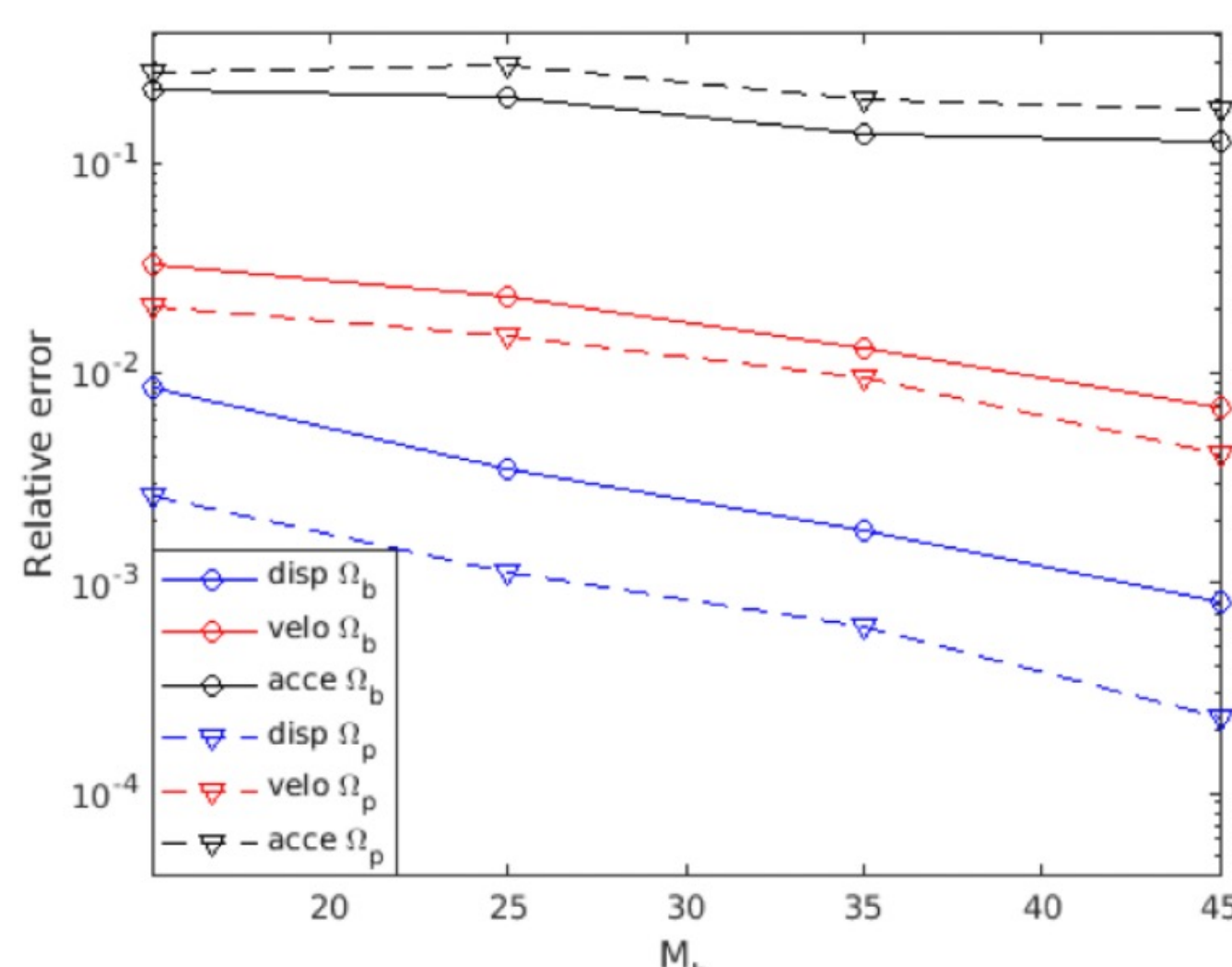
- Plug-and-play inclusion of structured and unstructured terms
- Improved surrogate accuracy through incorporation of FOM

OpInf ROM + Schwarz BCs in Ω_i :

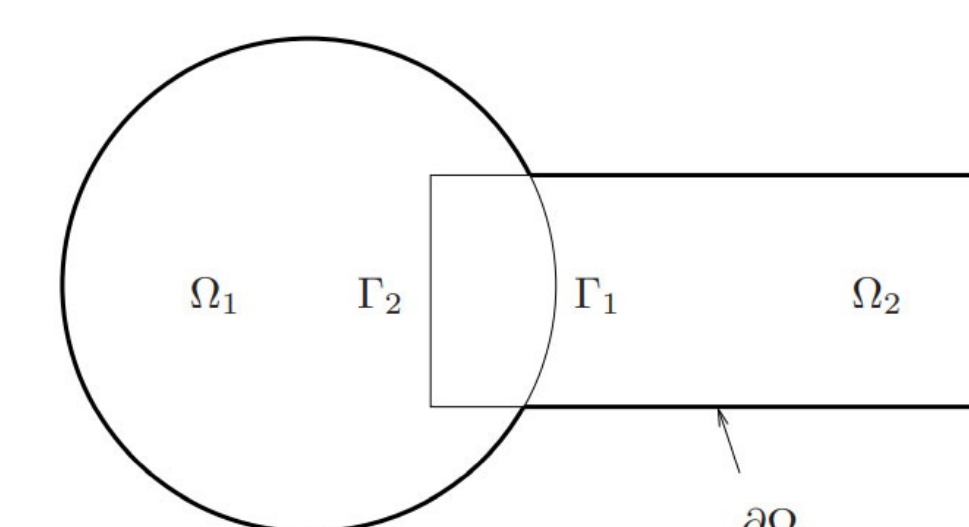
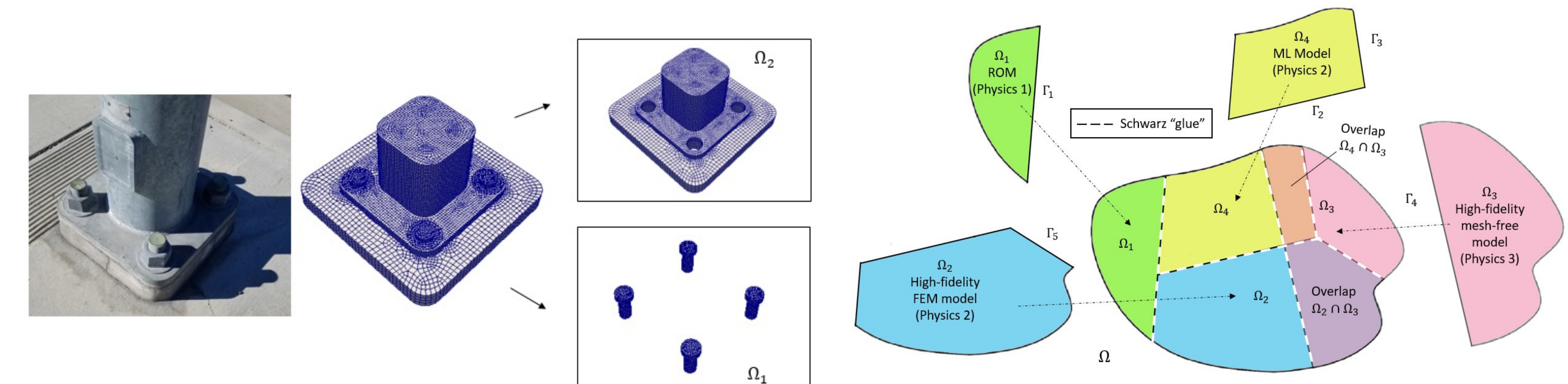
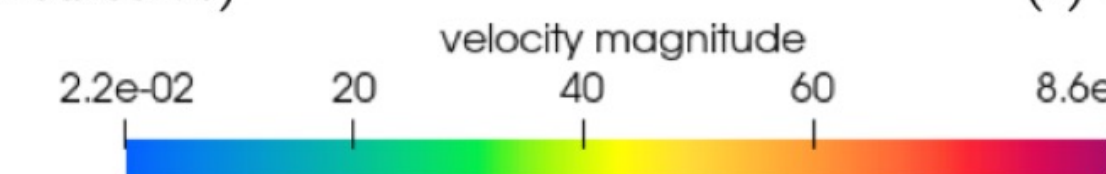
$$\hat{q}_i + \hat{A}_i \hat{q}_i + \hat{H}_i(\hat{q}_i \otimes \hat{q}_i) + \hat{B}_i g_i = 0$$

Motivated by implementation of Dirichlet BCs in FEM

Schwarz Dirichlet BC term

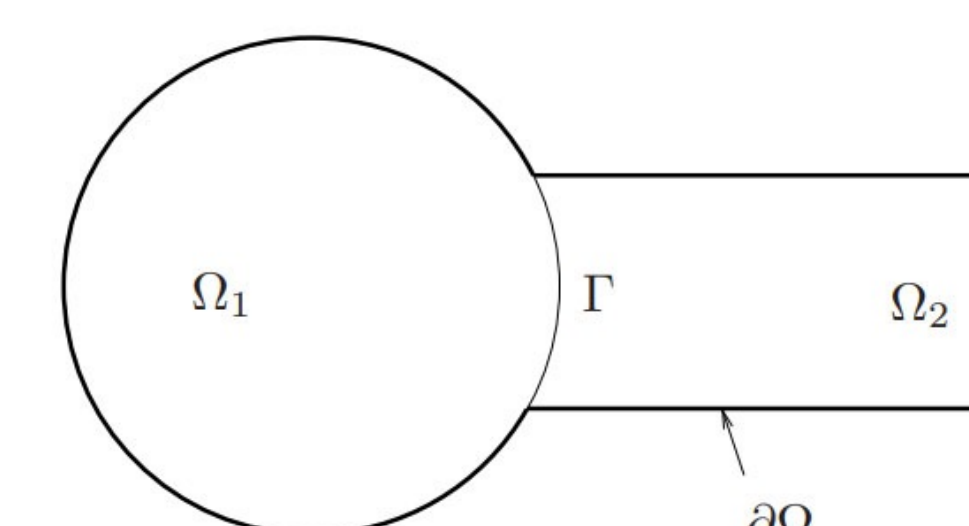


(e) OpInf-FOM (predictive) (f) OpInf-FOM (predictive)



$$\begin{cases} N(u_i^{n+1}) = f & \text{in } \Omega_i \\ u_i^{n+1} = g & \text{on } \partial\Omega_i \setminus \Gamma_i \\ u_i^{n+1} = \delta_{i1} u_2^n + \delta_{i2} u_1^{n+1} & \text{on } \Gamma_i \end{cases}$$

Overlapping SAM:
Dirichlet/Dirichlet BCs.



$$\begin{cases} N(u_i^{n+1}) = f & \text{in } \Omega_i \\ u_i^{n+1} = g & \text{on } \partial\Omega_i \setminus \Gamma \\ u_1^{n+1} = \lambda^{n+1} & \text{on } \Gamma \\ \partial_n u_2^{n+1} = \partial_n u_1^{n+1} & \text{on } \Gamma \\ \lambda^{n+1} = \theta u_2^n + (1 - \theta) \lambda^n & \text{on } \Gamma \end{cases}$$

Non-Overlapping SAM:
Dirichlet/Neumann or Robin/Robin BCs.

Check out our papers here!



A. Mota, I. Tezaur, C. Alleman,
"The Schwarz alternating method in solid
mechanics",
CMAME 2017.



I. Moore, C. Wentland, A. Gruber, I. Tezaur,
"Domain Decomposition-Based Coupling of
Operator Inference Reduced Order Models via
the Schwarz Alternating Method",
ArXiv 2024



A. Vijaywargia, S. A. McQuarrie, A. Gruber,
"Tensor Parametric Hamiltonian Operator
Inference",
ArXiv 2025.



C. Rodriguez, I. Tezaur, A. Mota, A. Gruber,
E. Parish, C. Wentland,
"Transmission Conditions for the Non-
Overlapping Schwarz Coupling of Full Order
and Operator Inference Models",
ArXiv 2025

Check out our code here!



Norma-OpInf
GUI-equipped Python code for performing
operator inference in Norma.



Norma
Julia testbed for algorithms and ideas for
coupling and multiphysics, primarily in
solid mechanics and heat conduction.