The Schwarz alternating method as a means for concurrent multi-scale coupling of conventional and data-driven models

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Multi-scale methods are essential for the understanding and prediction of the behavior of engineering systems when a small-scale event will eventually determine the performance of the entire system. In this talk, I will discuss a novel recently-proposed [1,2] approach that enables concurrent multi-scale coupling in finite deformation quasistatic and dynamic solid mechanics by leveraging the domain decomposition-based Schwarz alternating method. The approach is based on the simple idea that, if the solution to a partial differential equation (PDE) is known in two or more regularly shaped domains comprising a more complex domain, these local solutions can be used to iteratively build a solution for the more complex domain. The proposed methodology has a number of advantages over competing multiscale coupling methods, most notably its concurrent nature, its ability to couple non-conformal meshes with different element topologies and different time integrators with different time steps for dynamic problems all without introducing non-physical artifacts into the solution, and its non-intrusive implementation into existing codes.

In the first part of the talk, I will describe the formulation and theoretical properties of the overlapping version of the Schwarz alternating method as a means for concurrent multiscale coupling of conventional high-fidelity finite element models in quasistatic and dynamic solid mechanics. I will show several large-scale numerical examples demonstrating the method's numerical properties and convergence based on its implementation in two massively-parallel HPC codes: Albany/LCM and Sierra/Solid Mechanics.

The second part of the talk will focus on more recent work involving the extension of the Schwarz alternating method to enable coupling of conventional full order models (FOMs) with projection-based reduced order models (ROMs), and projection-based ROMs with each other [3]. The new approach is based on the following ingredients: (i) a decomposition of the physical domain of interest into two or more overlapping or non-overlapping subdomains, (ii) the construction of either a FOM or a projection-based ROM in each of the subdomains, (iii) the definition of appropriate transmission boundary conditions (BCs) which will propagate information between the overlapping subdomains, and (iv) the introduction of a Schwarz iteration process in which the ROM and FOM solutions on each subdomain are successively updated (to convergence) in an iterative and alternating fashion, with information between subdomains propagating through the transmission BCs. After describing the method's formulation and theoretical properties, I will evaluate its performance on a numerical example from the field of solid mechanics. Specifically, I will demonstrate that the method is capable of producing a more accurate solution than the classical single-domain Proper Orthogonal Decomposition (POD)/Galerkin ROM approach, while still achieving a computational savings with respect to a typical FOM discretization.

References

[1] A. Mota, I. Tezaur, C. Alleman. "The alternating Schwarz method for concurrent multi-scale coupling", Comput. Meth. Appl. Mech. Engng. 319 (2017) 19-51.

[2] A. Mota, I. Tezaur, G. Phlipot. "The Schwarz alternating method for dynamic solid mechanics", Int. Meth. Numer. Meth. Engng. 121 (21) (2022) 5036-5071.

[3] J. Barnett, I. Tezaur, A. Mota. "The Schwarz alternating method for the seamless coupling of nonlinear reduced order models and full order models", in *Computer Science Research Institute Summer Proceedings 2022*, S.K. Seritan and J.D. Smith, eds., Technical Report SAND2022-10280R, Sandia National Laboratories, 2022, pp. 31-55. https://arxiv.org/abs/2210.12551