Towards Feedback Control of Compressible Flows Using Galerkin Reduced Order Models

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The open- and closed-loop control of fluid flows has been of interest to the engineering community for several decades. A canonical problem that requires the use of control in system design is the problem of flow past an open cavity. Such a cavity can represent, for example, a weapons bay in a military aircraft that is exposed to a grazing external flow field. In this application, the conditions are such that the cavity can experience large dynamic loads and intense resonant pressure fluctutations on its surfaces, resulting in sound pressure levels that can be in excess of 160 dB. It is of engineering interest to suppress these fluctuations using flow control strategies, as the fluctuations can damage the components of the cavity. The optimal control problems that arise lead to high-dimensional Ricatti equations, the numerical solution of which can present an intractable computational burden in a design or analysis setting.

The ultimate objective of the present research effort is to develop stable and efficient ROMs for real-time control of compressible flow over open cavities using linear control theory. Consider the following linear time-invariant dynamical system

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t), \end{aligned} \tag{1}$$

where $\mathbf{x}(t)$ is the system state vector, $\mathbf{u}(t)$ is a vector of inputs, and $\mathbf{y}(t)$ is the vector of outputs. A system of the form (1) is obtained following the linearization of the full compressible Navier-Stokes equations around a steady base state, and the discretization of this system of PDEs in space. The problem of stabilizing a flow governed by the equations (1) using closed-loop control corresponds mathematically to moving an unstable eigenvalue of $\mathbf{A}$ into the stable half-plane. In the case that a controller for the full system (1) is designed using a ROM for (1), it is crucial that the numerical solutions of the ROM are consistent with the behavior of the solutions to the full model.

The focus of this talk is the construction of POD/Galerkin ROMs that retain the stability properties of the continuous PDEs from which these models are constructed [1]. The proposed model reduction technique differs from the approach taken in many applications in that the Galerkin projection step is applied to the continuous system of PDEs, rather than a semi-discrete representation of these equations. An energy stability analysis reveals that the inner product used to define the Galerkin projection step of the model reduction is intimately tied to the stability of the resulting model. For linearized compressible flow equations, the $L^2(\Omega)$ inner product does not correspond to an energy integral. This means that a ROM constructed using the $L^2(\Omega)$ inner product will not satisfy the energy conservation relation implied by the governing equations, and may exhibit instabilities that are inconsistent with the physics inherent in the governing continuous PDEs. A symmetry transformation leads to the construction of a weighted $L^2(\Omega)$ inner product referred to as the "symmetry inner product". It is demonstrated that the symmetry inner product is the energy inner product for the equations of linearized compressible flow, and hence guaranteed to produce a Galerkin ROM that preserves the stability of the full order system from which it was constructed for *any* reduced basis, provided well-posed boundary conditions are prescribed. Stability of the proposed ROM is demonstrated on a problem involving compressible flow over an open cavity. Future research directions towards the design of Linear Quadratic Gaussian (LQG) closed-loop controllers using the proposed model reduction approach are discussed.

References

[1] M.F. Barone, I. Kalashnikova, D.J. Segalman, H. Thornquist. "Stable Galerkin Reduced Order Models for Linearized Compressible Flow". *J. Comput. Phys.* **288** 1932–1946 (2009).