The Schwarz alternating method for concurrent multiscale coupling in solid mechanics

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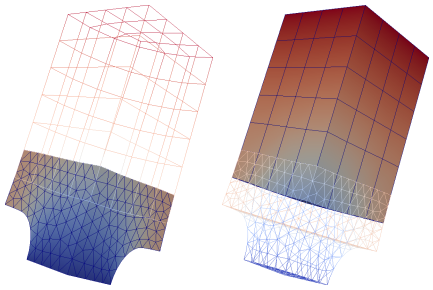
ABSTRACT

The Schwarz alternating method is based on a simple idea: if the solution to a partial differential equation (PDE) is known in two or more regularly shaped domains comprising a more complex domain, these local solutions can be used to iteratively build a solution for the more complex domain. This talk presents some recent novel developments in advancing the Schwarz alternating method, the earliest known domain decomposition method, as a means for concurrent multiscale coupling in finite deformation solid mechanics [2]. The proposed approach has a number of advantages over competing multiscale coupling methods, most notably its concurrent nature, its ability to couple non-conformal meshes with different element topologies (Figure 1), and its non-intrusive implementation into existing codes. Concurrent multiscale methods are essential for understanding and predicting the behavior of engineering systems in which a small scale failure determines the performance of the entire system.

This talk will be organized as follows. First, we will describe the variational formulation of the Schwarz alternating method as a PDE solution technique in the specific context of a finite-deformation inelastic solid mechanics problem. We will then discuss the method's convergence properties; in particular, we will show that the method's convergence rate is geometric provided each of the subdomain problems is well-posed and the overlap size is non-zero. Next, we will show that the use of a Newton-type method for the solution of the resultant nonlinear system leads to two kinds of block linearized systems, depending on the treatment of the Dirichlet boundary conditions. The first kind is a symmetric block-diagonal linear system in which each diagonal block is the tangent stiffness of each subdomain, so that the coupling is only through the right-hand side. The second kind is a nonsymmetric block system with off-diagonal coupling terms. We will present several variants of the Schwarz alternating method that we have developed for the first kind of linear system, including one in which the Schwarz and Newton iterations are combined into a single scheme. An upshot of this version of the method is that it lends itself to a minimally intrusive implementation into existing finite element codes. We will describe our implementation of the Schwarz alternating method in Albany, an open-source multiphysics research platform. Finally, we will demonstrate the accuracy, convergence and scalability of the proposed Schwarz variants on several solid mechanics examples.



(a) Domain 1 (b) Domain 2

REFERENCES

[1] A. Mota, I. Tezaur, C. Alleman. "The alternating Schwarz method for concurrent multiscale coupling", submitted to *Comput. Meth. Appl. Mech. Engng.*