

# **A Minimal Subspace Rotation approach for obtaining stable and accurate low-order projection-based reduced order models for nonlinear compressible flow**

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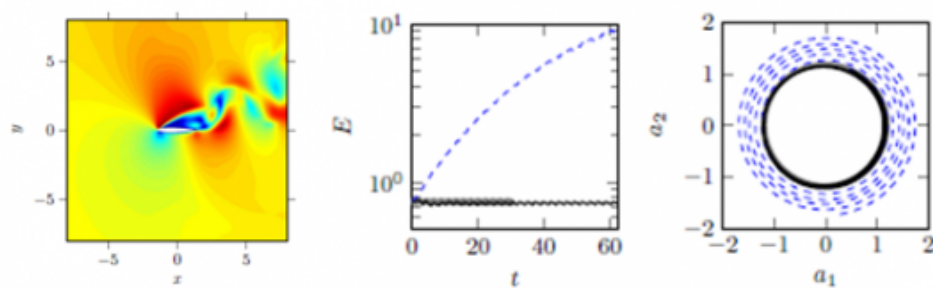
## **ABSTRACT**

Reduced order modeling is a promising tool for bridging the gap between high-fidelity and real-time/multi-query applications such as uncertainty quantification (UQ), optimization and control design. Most existing ROM approaches, e.g., proper orthogonal decomposition (POD), balanced proper orthogonal decomposition (BPOD) and balanced truncation, are based on projection of the governing physics set onto a low-dimensional subspace. This low-dimensional subspace is obtained from a higher-dimensional subspace through basis truncation: the removal of modes that are believed to be unimportant in representing the problem solution. Typically, truncation is based on an energy criterion: modes with low energy are discarded, so that the reduced basis subspace consists of the highest energy modes.

Although truncated modes are negligible from a data compression point of view, it has been argued by a number of authors that the modes are crucial for representing solutions to dynamical flow equations.

In fluid flow applications, the traditional approach involves the addition of empirical energy-absorbing eddy-viscosity terms to the ROM equations. The drawback of this approach is that empirical turbulence models destroy consistency between the Navier-Stokes equations and the ROM. Accurately identifying and matching free coefficients of the turbulence models is another challenge.

This talk presents an alternative approach for stabilizing and enhancing ROMs [1]. The application of interest is computational fluid mechanics, in particular, compressible flow. Instead of adding an empirical turbulence model term to the ROM, the key idea is to derive a transformation of the projection subspace that accounts for modes truncated by a typical reduced basis approach, e.g., POD. The proposed approach can be formulated mathematically as a trace minimization problem on the Stiefel manifold. Because only the projection subspace is modified, consistency between the ROM and the Navier-Stokes equations is retained. The reproductive as well as predictive capabilities of the method are evaluated on several compressible flow problems, including a problem involving laminar flow over an airfoil with a high angle of attack, and a channel-driven cavity flow problem. Results for the high angle of attack airfoil are summarized in the figure (left: solution snapshot, middle: evolution of modal energy, right: phase plot of the first and second temporal basis; dark gray line – DNS, black line – 4 mode stabilized ROM, blue line – 4 mode POD ROM). Recent extensions of the method to problems where the ROM includes gappy-POD for hyper-reduction will be described, and evaluated on some numerical examples involving high-Reynolds number flow over an open cavity.



## References

- [1] M. Balajewicz, I. Tezaur, E. Dowell. "Minimal subspace rotation on the Stiefel manifold for stabilization and enhancement of projection-based reduced order models for the compressible Navier-Stokes equations", submitted to *J. Comput. Phys.*