Preconditioned Least-Squares Petrov-Galerkin Reduced Order Models for Fluid and Solid Mechanics Problems

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Despite improved algorithms and the availability of massively parallel computing resources, "high-fidelity" models are, in practice, often too computationally expensive for use in a design or analysis setting, which can preclude real-time and multi-query analyses (e.g., optimization, uncertainty quantification). The situation can be ameliorated through the help of reduced order models (ROMs): models constructed from high-fidelity simulations that retain the essential physics and dynamics of their corresponding full order models (FOMs), but have a much lower computational cost. A promising model reduction methodology that has gained prevalence during the past decade is the Least-Squares Petrov-Galerkin (LSPG) projection method. The LSPG method is a discrete-optimal method that performs projection at the fully discrete level of its corresponding FOM, i.e., after the FOM has been discretized in space and time. While LSPG projection does not necessarily guarantee a priori accuracy and stability for generic complex nonlinear problems (e.g., turbulent compressible flows), it has been computationally demonstrated to generate accurate and stable responses for several classes of problems on which Galerkin projection manifests instabilities.

In this talk, we will describe a new methodology for improving the accuracy of LSPG ROMs for a wide range of applications through the introduction of preconditioning. We will demonstrate how the use of a preconditioner within the LSPG model reduction methodology has the effect of changing the norm in which the LSPG residual minimization problem is solved. Additionally, we will show that the use of an ideal preconditioner (the inverse of the Jacobian matrix in a particular Newton iteration) will lead to an l2-optimal projection of the solution increment onto the reduced basis. By selecting preconditioners that approximate the aforementioned "ideal" preconditioner, it is possible to improve not only the conditioning of the linear problems arising within the LSPG formulation, but also the LSPG stability constants. We will demonstrate the proposed method's performance on several solid mechanics applications implemented within the Albany open-source multi-physics code, and on a compressible flow problem simulated using Sandia's in-house compressible flow solver, SPARC. For the former application, where Galerkin projection is in general preferred for its structure-preservation capabilities, we demonstrate that the addition of preconditioning to the LSPG method can enable the method to achieve accuracy that is comparable to or surpasses the Galerkin method. For the latter application, we demonstrate that preconditioning is critical to obtaining time-predictive ROMs with sufficiently accurate spectral properties (pressure power spectral densities, or PSDs).