A minimal subspace rotation approach for extreme model reduction in fluid mechanics

Maciej Balajewicz, Aerospace Engineering Dept., University of Illinois, Urbana-Champaign, IL, USA.

Irina Tezaur, Quantitative Modeling & Analysis Dept., Sandia National Laboratories, Livermore, CA, USA.

High-fidelity modeling (HFM) and simulation is critical in many science and engineering applications; however, high-fidelity simulations remain prohibitive for parametric, time-critical and many-query applications (e.g., design optimization, control, uncertainty quantification). Model reduction seeks to circumvent this difficulty and achieve near real-time prediction by approximately solving the state equations of the HFM via projection onto a low-dimensional subspace. This subspace is "learned" offline from a small set of high-fidelity snapshots, and constructed using data compression and truncation: the removal of modes deemed unimportant in representing the problem solution. Truncation is usually based on an energy criterion: modes with low energy are discarded, so that the reduced basis subspace consists of the highest energy modes. In most realistic applications, e.g., nonlinear compressible fluid flow, a basis that captures 99% or more of the snapshot energy is required for a reduced order model (ROM) to accurately reproduce the snapshots from which it was constructed. Capturing 99% of snapshot energy is only possible for toy problems and/or low-fidelity models, however. Moreover, higher order modes are in general unreliable for prediction, so including them in the basis is unlikely to improve the predictive capabilities of a ROM.

This talk presents an *a priori* approach for creating stable and accurate projection-based compressible flow ROMs of extremely low order based on an approach known as minimal subspace rotation. Of particular interest is stability and accuracy of fluid ROMs for long-time simulations. The proposed approach is based on the idea that, although low-energy modes are negligible from a data compression point of view, they are actually crucial for representing solutions to dynamical flow equations. Traditionally, low-dimensional ROMs of fluid flows are stabilized and enhanced using empirical turbulence models. These approaches have several downsides, e.g., they destroy consistency between the ROM and the HFM. In the minimal subspace rotation method, we model the modal truncation *a priori* by "rotating" the projection subspace into a more dissipative regime rather than through the addition of an ad hoc empirical turbulence term to the ROM equations. The minimal rotation is determined in a goal-oriented fashion through the formulation and offline numerical solution of small-scale quadratic matrix program on the Stiefel manifold. The proposed approach can be interpreted as an *a priori* consistent formulation of the eddy-viscosity turbulence modeling approach: because only the projection subspace is modified, consistency between the ROM and the Navier-Stokes equations is retained. The reproductive as well as predictive capabilities of POD/Galerkin ROMs stabilized via the proposed method are evaluated on several nonlinear compressible flow problems of extremely low order (e.g. $O(10)$). Recent extensions of the method to minimum-residual-based nonlinear compressible flow ROMs are described and evaluated numerically.

[1] M. Balajewicz, I. Tezaur, E. Dowell. "Minimal subspace rotation on the Stiefel manifold for stabilization and enhancement of projection-based reduced order models for the compressible Navier-Stokes equations", *J. Comput. Phys*, 2016 (in press)*.