## Poster: Primal-Dual Mesh Optimization with Mathematical Foundations

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### Abstract


We investigate mathematics and algorithms for primal-dual meshing. In the last two years we made progress analyzing primal-dual metrics, improving quality by resampling positions, producing dual Voronoi crust polyhedral meshes, and scaling sampling algorithms to high dimensions.

In primal-dual meshing, the goal is a single configuration with good quality for both the primal and dual. We analyzed the traditional Hodge-Optimized Triangulation (HOT) energy metric, which bounds the discretization error of the diagonalized Hodge-star operator in Discrete Exterior Calculus (DEC) PDE formulations. HOT-energy evaluates quality acceptably, but behaves poorly as an optimization objective. It has bad local minima with values much larger than the global optimum, and optimizing it inverts and collapses elements. We designed $\overline{\mathrm{HOT}}$ to overcome these shortcomings. It is unitless and scale invariant. $\overline{\mathrm{HOT}}$ behaves well under optimization, while retaining the essential features of bounding the discretization error. (HOT's limitations will motivate discrete changes, e.g., for nearly-collapsed elements we may collapse an edge.)

Near-term work is actually optimizing and resampling for $\overline{\mathrm{HOT}}$. Constrained resampling pops us out of bad minima, and optimization quickly converges near desirable minima. We will tightly couple position and weight improvement steps, which is challenging due to their different units and scales. Through computational experiments, we will quantify how well our new metric improves actual simulation error.

We recently synergistically partnered with Darren Engwirda on meshing ocean models for the Energy Exascale Earth System Model (E3SM) climate project. We will perform the same analysis and metric design for HOT, but for Memetic Optimized Discretizations (MOD) in E3SM's element formulations.

Related to resampling, we published "Spoke-Darts," a blue-noise sampling algorithm. It scales well to high dimensions by using line samples around an implicit advancing-front. We prove probabilistic guarantees that the output is well-saturated.


### Project: Primal-Dual Mesh Optimization with Mathematical Foundations