The Optimal Attack: Application to a Power Grid System

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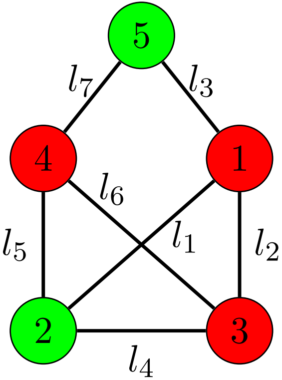
A 5 node power grid network



$$\mathcal{G} = \{2, 5\} \qquad \mathcal{G}' = \{1, 3, 4\} \qquad \mathcal{E} = \{l_1, l_2, \cdots, l_7\}$$

$\mathcal{G}$ is the set of generator buses
$\mathcal{G}'$ is the set of non-generator buses
$\mathcal{E}$ is the set of all lines.

Cascading failure in a power grid network

- Cascading events can occur due to increasing load and additional fluctuations in the high voltage power grid.
- We consider the transient dynamics in order to gain insight into the propagation of cascading failures.
- To model the cascading failure a capacity criterion is set on each line. The capacity criterion can be violated transiently and can lead to additional overloads on other lines. This may eventually lead to a cascading failure [1].

[1] Schäfer B, Witthaut D, Timme M, Latora V. Dynamically induced cascading failures in power grids. Nature communications. 2018 May 17;9(1):1975.

Dynamics of a power grid network

The transient behavior of the system can be determined by the swing equation,

$$\dot{\theta}_i = \omega_i \tag{1a}$$

$$I_i \dot{\omega}_i = P_i - \gamma_i \omega_i + \sum_{j=1}^{N} K_{ij} l_{ij}(t) \sin(\theta_j - \theta_i), \tag{1b}$$

where θ_i is the mechanical rotor angle and ω_i is the angular frequency of node i.

Here $l_{ij}(t)$ is *status* of line (i, j) at time t,

$$l_{ij}(t) = \begin{cases} 1, & \text{if } |F_{ij}(t)| \le C_{ij} \\ 0 & \text{if } \exists \tau \le t \text{ when } |F_{ij}|(\tau) > C_{ij}, \end{cases} \tag{2}$$

Here F_{ij} is the flow on the line (i, j) with coupling K_{ij} at time t,

$$F_{ij}(t) = K_{ij} \sin(\theta_j - \theta_i) \tag{3}$$

and C_{ij} is the capacity of the line (i, j).

Dynamics of a power grid network

When $l_{ij} = 1$, the line l_{ij} is active, and when $l_{ij} = 0$, the line l_{ij} is inactive. Note that here switches are one way. If a line is inactivated, it will remain inactive for the rest of the time.

If node i is a generator bus then $P_i > 0$, and if i is a non-generator, then $P_i < 0$.

In an ideal situation (at an operating point), all $l_{ij} = 1$. Also, in an ideal situation (at an operating point),

$$\sum_i P_i = 0 \tag{4}$$

Attack on a power grid network

- We want to design an optimal attack strategy for which the attacker can fail a line or a set of lines to induce a large cascading failure in a power grid.

- The attacker's goal is to minimize the total number of surviving lines at the end of a certain period of time. (The time duration is short as we are interested in the effects of the transient dynamics due to the violation of the overload criterion).

Problem

We consider a spatio-temporal optimization problem, for which the goal is to find the choice of the most vulnerable line(s), and the particular temporal sequence, i.e., the times at which these are attacked over a given time period.

Problem

Find the set $\mathcal{L} = \{l_{(k)} \in \mathcal{E}, k = 1, 2, \cdots, m\}$ and a set of times $\mathcal{T} = \{t_k \in \mathbb{R}, 0 \leq t_k \leq t_f, k = 1, 2, \cdots, m\}$, $|\mathcal{L}| = m$ and $|\mathcal{T}| = m$ that

$$\min \quad \sum_{ij} l_{ij}(t_f) \tag{5}$$

subject o:

$$\begin{aligned} \dot{\theta}_i &= \omega_i, \\ I_i \dot{\omega}_i &= P_i - \gamma_i \omega_i + \sum_{j=1}^{N} K_{ij} l_{ij}(t) \sin(\theta_j - \theta_i) \end{aligned} \tag{6}$$

$$F_{ij}(t) = K_{ij} \sin(\theta_j - \theta_i) \tag{7}$$

$$l_{ij}(t) = \begin{cases} 1, & \text{if } F_{ij}(t) \leq C_{ij} \\ 0 & \text{if } \exists \tau \leq t \text{ at which } |F_{ij}|(\tau) > C_{ij}, \end{cases} \tag{8}$$

and $\forall l_{(k)} \in \mathcal{L}$

$$l_{(k)}(t) = 0 \text{ for } t \geq t_k, t_k \in \mathcal{T} \tag{9}$$

.

Example Problem: Find the most vulnerable line

Here $|\mathcal{L}| = m = 1$. We set $t_1 = 1(sec)$. We want to find the line $l_{(1)}$ which minimizes the number of surviving lines at time $t_f = 6(sec)$. An exhaustive search (we can do it as there are only 7 lines) gives the following results:

Table: Total number of surviving lines at time $t_f = 6$ (sec) when a line $l_{(1)}$ fails at $t_1 = 1$(sec)

Line $l_{(1)}$	Total number of surviving lines at time t_f
l_1	1
l_2	6
l_3	5
l_4	2
l_5	1
l_6	6
l_7	5

Simulated Annealing Method

The goal is to find a point in the variable space at which a real valued energy function (or cost function) E is minimized.

Simulated annealing is a minimization technique which can provide a good (not necessarily the best) result avoiding local minima; it is based on the idea of taking a random walk through the space at successively lower temperatures, where the probability of taking a step is given by a Boltzmann distribution.

Simulated Annealing Method

During the annealing process over the temperature range from $T_{\max}$ to $T_{\min}$, each new solution $\mathbf{x}_{i+1}$ is accepted with a temperature-dependent probability P_T given by

$$P_T = \begin{cases} 1, & \text{if } E(\mathbf{x}_{i+1}) \leq E(\mathbf{x}_i) \\ e^{-\frac{E(\mathbf{x}_{i+1}) - E(\mathbf{x}_i)}{kT_i}} & \text{if } E(\mathbf{x}_{i+1}) > E(\mathbf{x}_i) \end{cases} \tag{10}$$

Initially for higher temperatures a higher probability of acceptance of new solutions is allowed to explore a wide region of the search space, thereby escaping from local minima is possible. However, as the temperature is reduced, the probability of acceptance of unfavourable solutions is reduced.

Simulated Annealing Method

We need to define

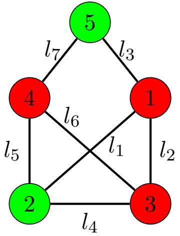
- A cost function (E). This cost function is defined by equations (5) - (9).
- A step function (S) which will determine how to move from $\mathbf{x}_i$ to $\mathbf{x}_{i+1}$ in the variable space and a measure function M to determine the distance between two points $\mathbf{x}_i$ and $\mathbf{x}_{i+1}$.
- A temperature scheduling function/cooling schedule. This will determine how fast the temperature will decrease from $T_{\max}$ to $T_{\min}$.

Simulated Annealing Method

Step function S: A step function will choose a step from $\mathbf{x}_i$ to $\mathbf{x}_{i+1}$, where $\mathbf{x}_{i+1} \in \{\text{neigh}(\mathbf{x}_i)\}$.

Measure a step M: To measure the distance from a step $\mathbf{x}_i$ to $\mathbf{x}_{i+1}$ we define M : $\text{dist}(\mathbf{x}_i, \mathbf{x}_{i+1}) =$ Minimum number of nodes crossed along the shortest path from line $\mathbf{x}_i$ to line $\mathbf{x}_{i+1}$

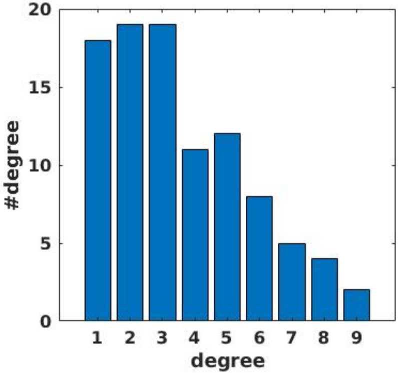
In one step the simulated annealing will only consider replacing a line with one of the neighboring lines. For example, if $\mathbf{x}_i = l_2$ then $\mathbf{x}_{i+1} \in \text{neigh}(l_2) = \{l_1, l_3, l_4, l_6\}$, where $\text{dist}(l_2, l_4) = 1$

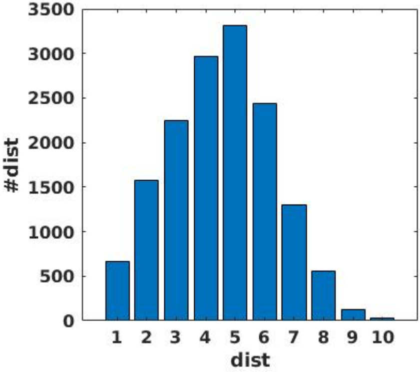


$\mathcal{G} = \{2, 5\}$ $\mathcal{G}' = \{1, 3, 4\}$ $\mathcal{E} = \{l_1, l_2, \cdots, l_7\}$

The Spanish High Voltage Power Grid

The Spanish High Voltage power grid network which consists of total 175 lines and 98 buses. The maximum distance between two lines is 10 and the maximum degree of a node is 9.





Time of failure Propagation Stopped

How we can choose the final time t_f for the optimization problem?

We choose the Spanish High Voltage power grid (consisting of total 175 lines and 98 buses). We randomly choose two lines and fail them at $t_1 = 1$ (sec) and $t_2 = 2$ (sec). We run the ode simulation (Eq. (6)) for the final time $t_f = 100$ (sec) and find the total number of surviving lines at time $t_f = 100$ (sec).

Time of failure Propagation Stopped

Table: Total number of surviving lines at time $t_f = 100$ (sec) when the lines $l_{(1)}$ and $l_{(2)}$ fail at $t_1 = 1$(sec) and $t_2 = 2$ (sec), respectively.

$\{l_{(1)}, l_{(2)}\}$	dist (max dist = 10)	degree (max deg = 9)	No. of surviving lines at time t_f	Time of last failure seen (sec)
$\{2, 10\}$	2	{{2,3},{2,7}}	173	2
$\{59, 23\}$	4	{{2,7},{2,3}}	173	2
$\{160, 143\}$	2	{{1,3},{2,3}}	173	2
$\{67, 131\}$	7	{{2,3},{1,3}}	158	8.5
$\{111, 18\}$	5	{{2,3},{5,8}}	173	2
$\{49, 96\}$	4	{{5,8},{2,3}}	173	2
$\{28, 170\}$	7	{{3,5},{2,7}}	173	2
$\{168, 85\}$	4	{{2,7},{3,5}}	173	2
$\{141, 21\}$	6	{{1,3},{4,7}}	173	2

Simulated Annealing Method

We assume t_1, i.e., the time of the first attack, to be known. We also set $t_f = 20$(sec). We consider the following cases:

- Case 1: $m = 1$. We set $t_1 = 1$ (sec). The variable space contains one variable $l_{(1)}$. We seek to find the line $l_{(1)}$ that maximizes the total number of line failures.
  - For the five node network: We see degeneracy of the solution. We find two solutions $l_{(1)} = l_1$ and $l_{(1)} = l_5$ for two different initial guesses out of many different runs. But the total cost $E^* = \sum_{ij} l_{ij}(t_f) = 1$ for both the solutions.
  - For the Spanish High Voltage power grid (consisting of total 175 lines and 98 buses). We find that $l_{(1)} = l_{57}$ for which the cost $E^* = \sum_{ij} l_{ij}(t_f) = 154$. Here the degree of the line l_{57} is $\{3, 2\}$.

Simulated Annealing Method

- Case 2: $m = 2$. We assume t_1 and t_2 are known, $t_1 = 1$(sec), $t_2 = 2$(sec). The variable space contains two variables $l_{(1)}$ and $l_{(2)}$. To take a step on the variable space, we randomly choose one line from the two and replace the line with one of its neighboring lines (chosen at random) and keep the other line fixed. The goal is to find two lines and their sequence for which the attack will cause the maximum number of line failures.
  - For the Spanish High Voltage power grid (consisting of total 175 lines and 98 buses), we see degeneracy of the optimal solutions. We find the following solutions for which $E^* = \sum_{ij} l_{ij}(t_f) = 151$:
    $\{l_{(1)}, l_{(2)}\} = \{l_{65}, l_{53}\}$, $\text{dist}(l_{65}, l_{53}) = 4$, $\text{degree}(l_{65}) = \{3, 5\}$ and $\text{degree}(l_{53}) = \{5, 8\}$.
    $\{l_{(1)}, l_{(2)}\} = \{l_{65}, l_{122}\}$, $\text{dist}(l_{65}, l_{122}) = 3$, $\text{degree}(l_{65}) = \{3, 5\}$ and $\text{degree}(l_{122}) = \{1, 3\}$.

Simulated Annealing Method

- *The sequence does matter.* For example, the sequence $\{l_{(1)}, l_{(2)}\} = \{l_{65}, l_{53}\}$ causes $E^* = \sum_{ij} l_{ij}(t_f) = 151$, but the sequence $\{l_{(1)}, l_{(2)}\} = \{l_{53}, l_{65}\}$ causes $E = \sum_{ij} l_{ij}(t_f) = 153$.

Simulated Annealing Method

Table: The minimum number of surviving lines E^* obtained from the simulated annealing process for fixed $t_1 = 1$ and t_2. The lines $\{l_{(1)}, l_{(2)}\}$, their distance and the degree of each line are listed.

$\{t_1, t_2\}$	E^*	$\{l_{(1)}, l_{(2)}\}$	dist (max dist = 10)	degree (max deg = 9)
$\{1, 1\}$	153	$\{65, 41\}$	3	{{3,5},{5,5}}
$\{1, 1.5\}$	150	$\{65, 53\}$	4	{{3,5},{5,8}}
$\{1, 2\}$	151	$\{65, 53\}$	4	{{3,5},{5,8}}
$\{1, 2.5\}$	153	$\{92, 83\}$	5	{{2,2},{5,8}}
$\{1, 3\}$	153	$\{65, 90\}$	5	{{3,5},{2,7}}
$\{1, 3.5\}$	151	$\{121, 33\}$	5	{{4,7},{5,8}}
$\{1, 4\}$	150	$\{121, 90\}$	5	{{4,7},{2,7}}

Simulated Annealing Method

- Case 3: $m = 2$. We assume t_1 is assigned, $t_1 = 1$(sec). The variable space contains three variables $l_{(1)}$, $l_{(2)}$ and t_2. To take a step on the variable space, we randomly choose one variable out of the three. If the chosen variable is time t_2, we take a step in time and keep the other two variables $l_{(1)}$ and $l_{(2)}$ fixed: we randomly move either forward or backward in time with maximum step size 0.5. If the chosen variable is a line, we replace that line with one of its neighboring lines and keep the other two variables fixed.

Simulated Annealing Method

- For the Spanish High Voltage power grid (consisting of total 175 lines and 98 buses), we find the solution for which $E^* = \sum_{ij} l_{ij}(t_f) = 150$: $\{l_{(1)}, l_{(2)}\} = \{l_{65}, l_{53}\}$, $\{t_1, t_2\} = \{0, 1.5\}$.
  Also $\text{dist}(l_{65}, l_{53}) = 4$, $\text{degree}(l_{65}) = \{3, 5\}$ and $\text{degree}(l_{53}) = \{5, 8\}$.

- *the schedule of attacks matters*. For example, $\{l_{(1)}, l_{(2)}\} = \{l_{65}, l_{53}\}$, $\{t_1, t_2\} = \{1, 3\}$, $E = \sum_{ij} l_{ij}(t_f) = 153$.