Combining DD with Optimal Control for Improved QIP
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**DQD Effective One-Qubit Model**

\[ \tilde{C}(t) \]

\[ H = H_0 - \vec{\sigma} \cdot \tilde{C}(t) - \vec{\sigma} \cdot \vec{I} + H_e \]

\[ \rightarrow H = \sigma_z C_z(t) + c\sigma_x \]

Different magnetic fields at each dot produce a rotation about the logical $x$-axis

Pulse area: $\theta(t) = \int_0^t \tilde{C}(\tau) d\tau$

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**Optimal Dynamical-Decoupling Pulses**

By searching the space of controls satisfying

1. $\tilde{\eta} \approx 0$

2. $F = \frac{1}{2} \left| \text{Tr} \left[ U(t_\xi) Z^\pi \right] \right| \approx 1$

we improve control fidelities and system robustness for $Z_{\pi/2}$ and $Z_\pi$.

Systematic searching $\Rightarrow$ Optimal control theory

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**Optimal Control Theory for Quantum-Mechanical Objectives**

- Define an objective: $J = \frac{1}{2} \left| \text{Tr} \left[ U(t_\xi) Z^\pi \right] \right|$

- Incorporate constraints:
  - Schrödinger’s equation
  - Experimental limitations of the control field

- Perform variational analysis and optimize iteratively
  - Evolutionary algorithms
  - Gradient-based methods

\[ \frac{\delta F(C(t))}{\delta C(t)} = 0 \]

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**Dynamical-Decoupling Sequences**

\[ \prod_i^{\infty} U_i \approx 1, \quad \text{where } U_i \text{ represent } \pi \text{ and } \pi/2 \text{ rotations and free evolutions.} \]

This is an approximation to $1$ because

- $\{U_i\}$ and $N$ are finite
- Non-unitary evolution is corrected with unitary “time-reversal” operations

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**Dynamical-Decoupling Pulses**

To remove 1st- and 2nd-order errors in $\pi$ and $\pi/2$ $z$-axis rotations when $\vec{I} = \Gamma_x \Rightarrow \tilde{\eta} = 0$:

\[ \begin{align*}
\eta_1 &= \int_0^t \sin[\theta(t)] dt, \\
\eta_2 &= \int_0^t \cos[\theta(t)] dt, \\
\eta_3 &= \int_0^t t \sin[\theta(t)] dt, \\
\eta_4 &= \int_0^t t \cos[\theta(t)] dt, \\
\eta_5 &= \int_0^t \int_0^t \sin[\theta(t_1) - \theta(t_2)] dt_1 dt_2,
\end{align*} \]


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**Level Sets and Tangent Spaces**

After calculating $\frac{\delta F'}{\delta C(t)}$, all gradient directions $\frac{\delta \eta_i}{\delta C(t)}$ are removed:

\[ \frac{\delta F'}{\delta C(t)} \rightarrow \frac{\delta F'}{\delta C(t)} - \sum_i \frac{\delta \eta_i}{\delta C(t)} \]

So $\left( \frac{\delta F'}{\delta C(t)}, \frac{\delta \eta_i}{\delta C(t)} \right) = 0$.

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**Fidelity Response to Control Noise: $Z_{\pi/2}$**

\[ C(t) \rightarrow C(t) + \delta C(t), \text{ where } \delta C \in \mathcal{U}, \]

\[ |\delta C(t)| = \alpha C(t), \text{ and } 0 \leq \alpha \leq 0.2 \]

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**Control with Experimental Constraints**

- Max. voltage = $\sim$3mV
- Step size = $\sim$3\text{V}
- Step time = $\sim$1 ns

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**Current Work and Future Directions**

- Incorporate these optimal $\pi$- and $\pi/2$-pulses for memory and information processing.
- Investigate various noise models (e.g., spin baths).
- Determine how these pulses extend spin-echoes.
- Consider $\epsilon$ as time-dependent during the application of the control field.
- Extend this formalism to arbitrary rotation axes.
- Analyze the control landscape within the region of feasibility, i.e., $\eta = 0$. 