SAND2009-5095C

Accurate Prediction of Dynamic Fracture with Peridynamics

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Multiscale Dynamic Material Modeling

Sandia National Laboratories

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Fracture Mechanics Theory and Dynamic Fracture

- Onset of crack growth can be accurately predicted
- Crack growth speed and direction *cannot!*

Wanted: A successful method for simulating Dynamic Fracture

- Such a method must be able to reproduce the characteristic phenomena of Dynamic Fracture.
- All current simulation methods severely fail this test of efficacy
  - The governing PDEs break down at cracks.

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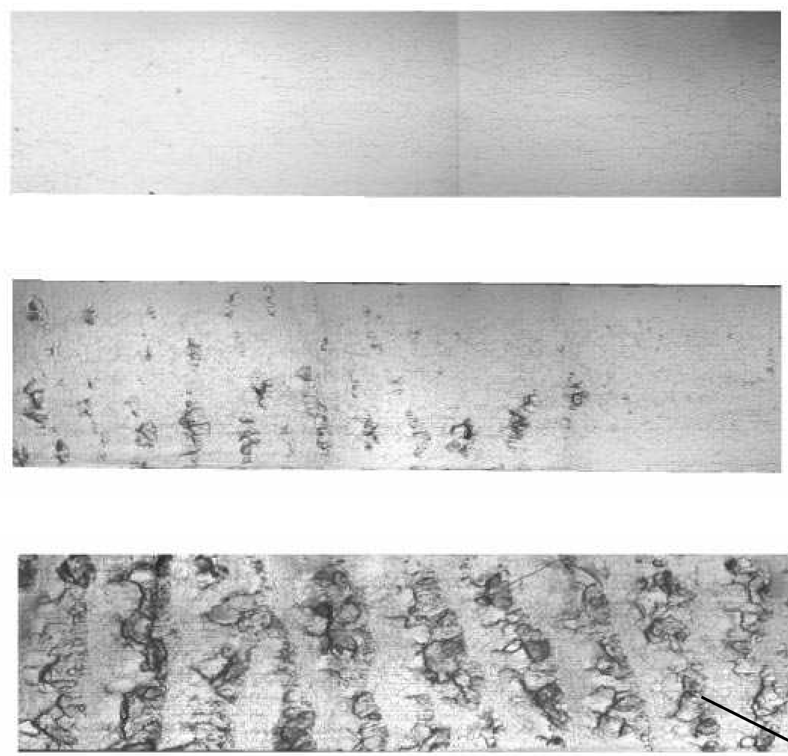
Characteristic Phenomena of Dynamic Fracture:

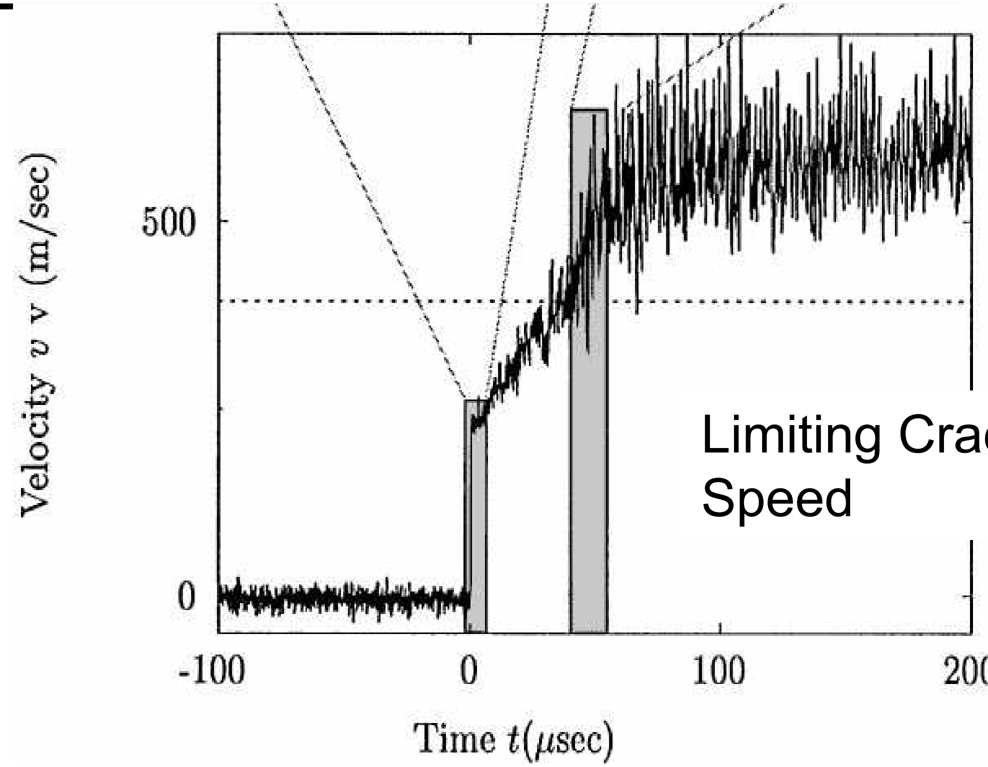
1. The Mirror, Mist, Hackle sequence of textures on the fracture surface;
2. A steady, limiting crack speed ;
3. The transition from stable to unstable crack growth;
4. Crack branching;
5. Fragment size distribution;
6. The specific angle of cracking following impact of a notched plate (the Kaltoff-Winkler experiment);
7. The multiple, unstable cracking modes of fiber-reinforced composites;
8. Membrane bursting;
9. Unstable peeling and tearing of thin sheets.

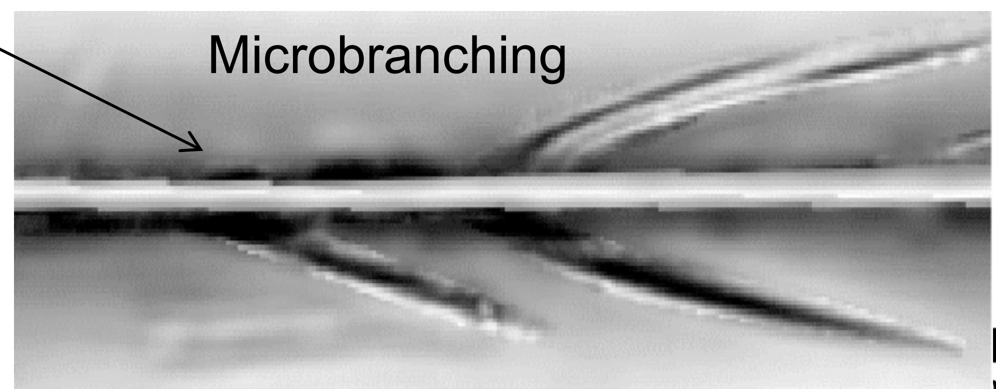
1, 2. **Dynamic fracture in PMMA:**

J. Fineberg & M. Marder, Phys Rpts 313 (1999) 1-108



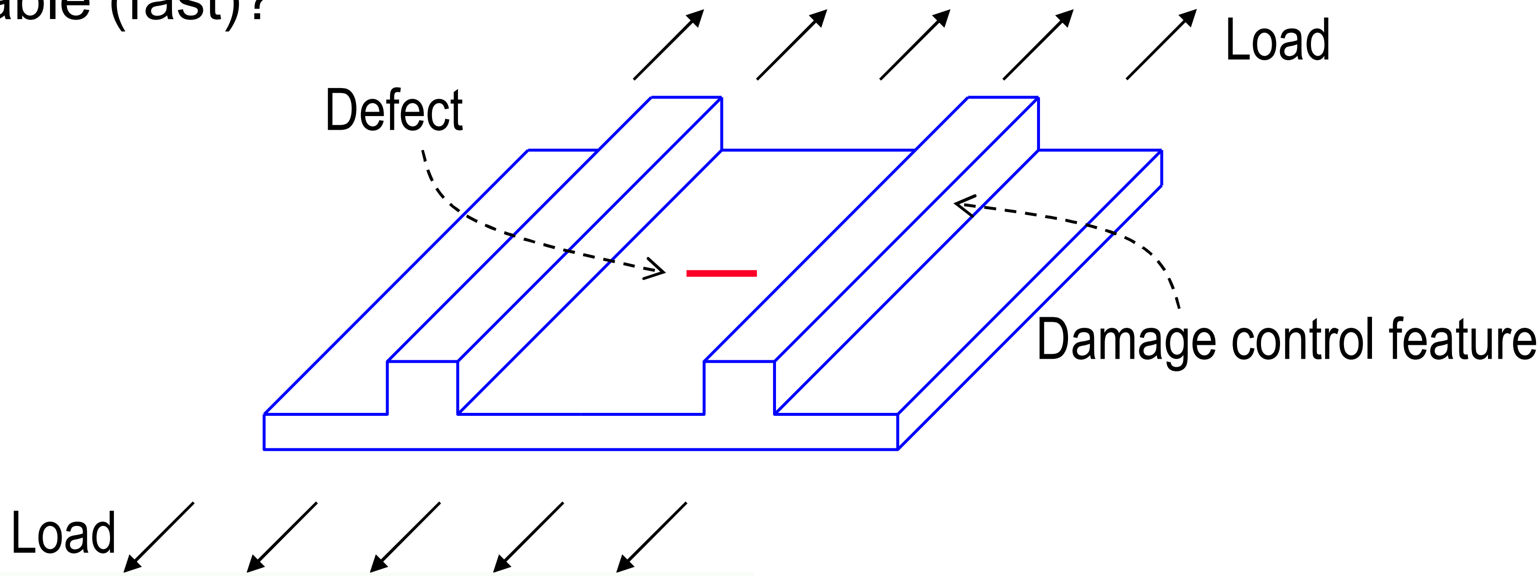
Mirror-mist-hackle transition

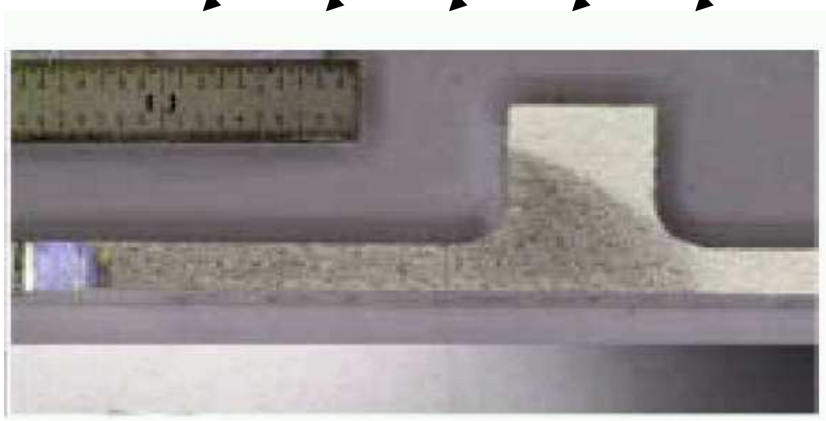




3. Transition from Stable to Unstable Crack Growth

• At what load and what length does crack growth change from stable (slow) to unstable (fast)?





Crack just before transition to Unstable (photo courtesy Boeing)

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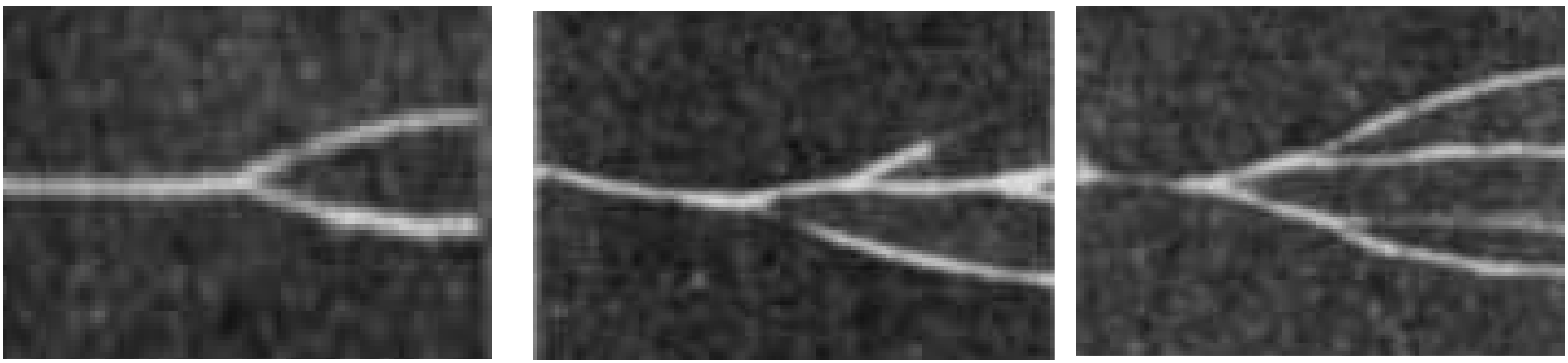
4. **Crack Branching**

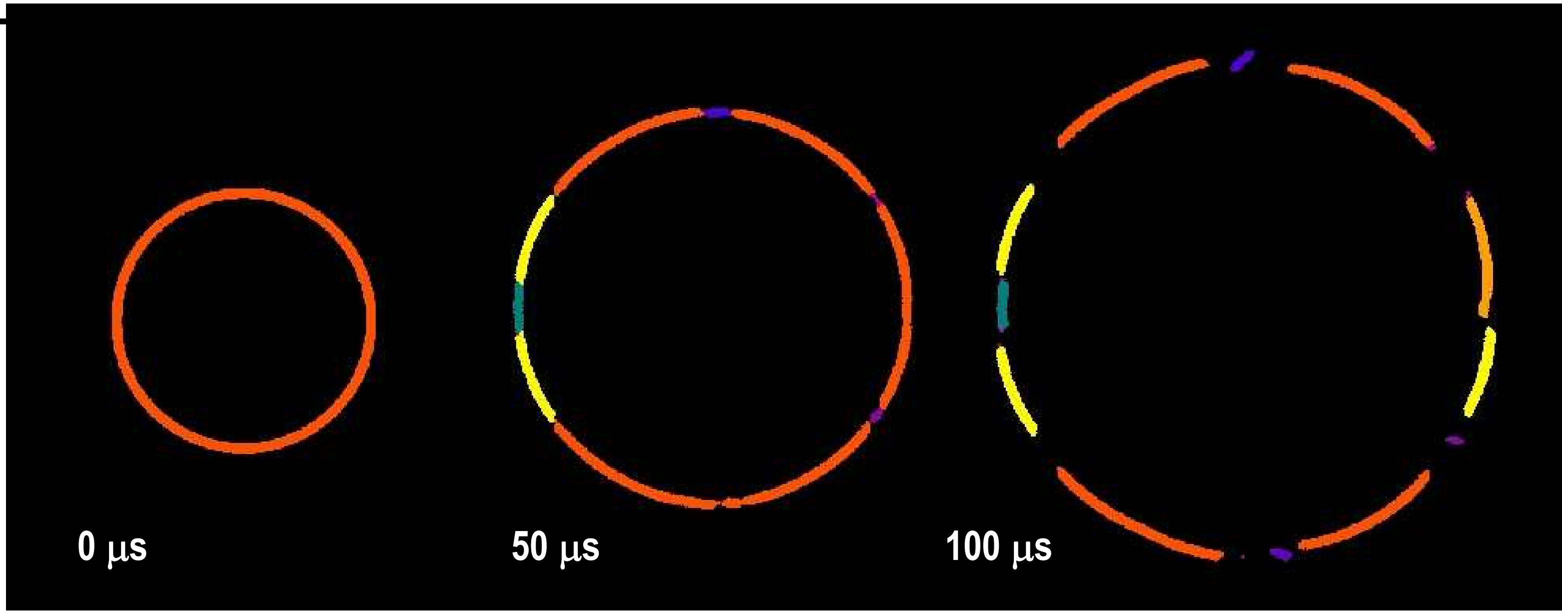
At high levels of stress intensity factor (e.g., at a round notch) a crack propagates at higher stress levels and sequential crack branching can occur.



Crack branching in notched specimens of soda-lime glass, pulled vertically (specimens 3’” x 1” x 0.05”; load stress increasing left to right)

S F. Bowden, J. Brunton, J. Field, and A. Heyes, *Controlled fracture of brittle solids and the interuption of electrical current*, Nature (1967) 216, 42, pp.38-42.

5. Fragmentation: Ductile Aluminum Ring*



1100-0 Al ring has initial radial velocity of 200m/s.

- 32mm diameter, 1mm x 1mm cross-section.
- Rapid acceleration by elecromagnetic pulse.
- Recovered 11-13 fragments per shot over range of initial velocities 182 – 220 m/s.

*D. E. Grady & D. A. Benson, *Experimental Mechanics* 23 (1983) 393-400

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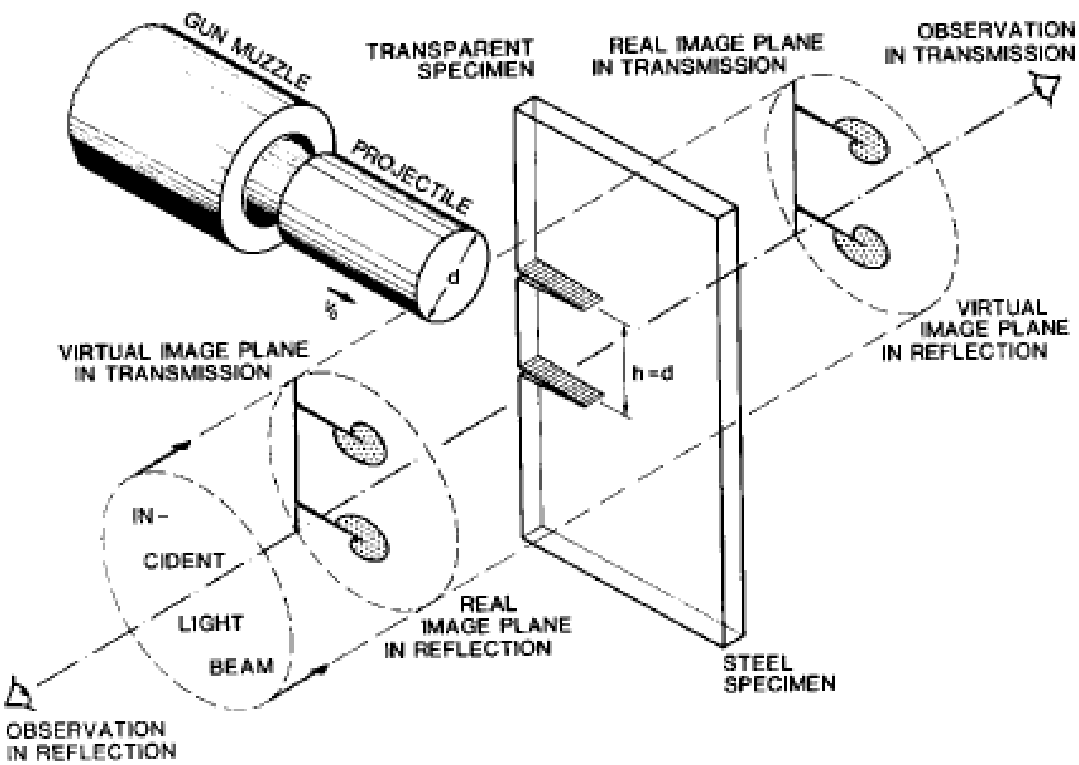
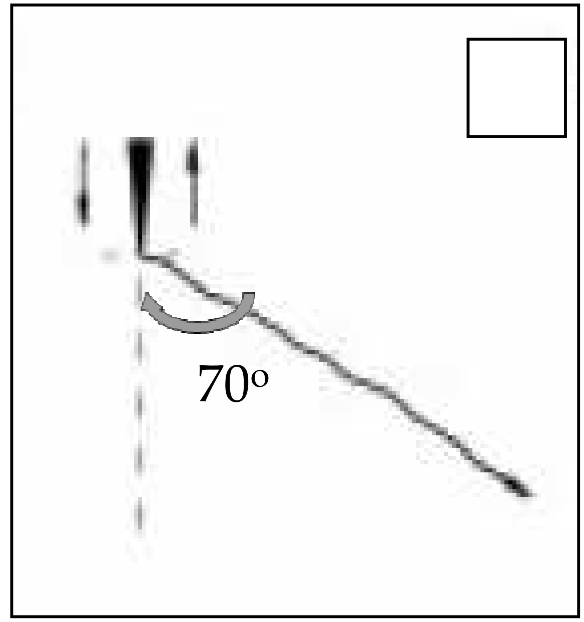
6. **Dynamic fracture in a hard steel plate:** Kalthoff-Winkler Experiment

Dynamic fracture in maraging steel (Kalthoff & Winkler, 1988)

• Mode-II loading at notch tips results in mode-I cracks at 70° angle.





Experimental Results

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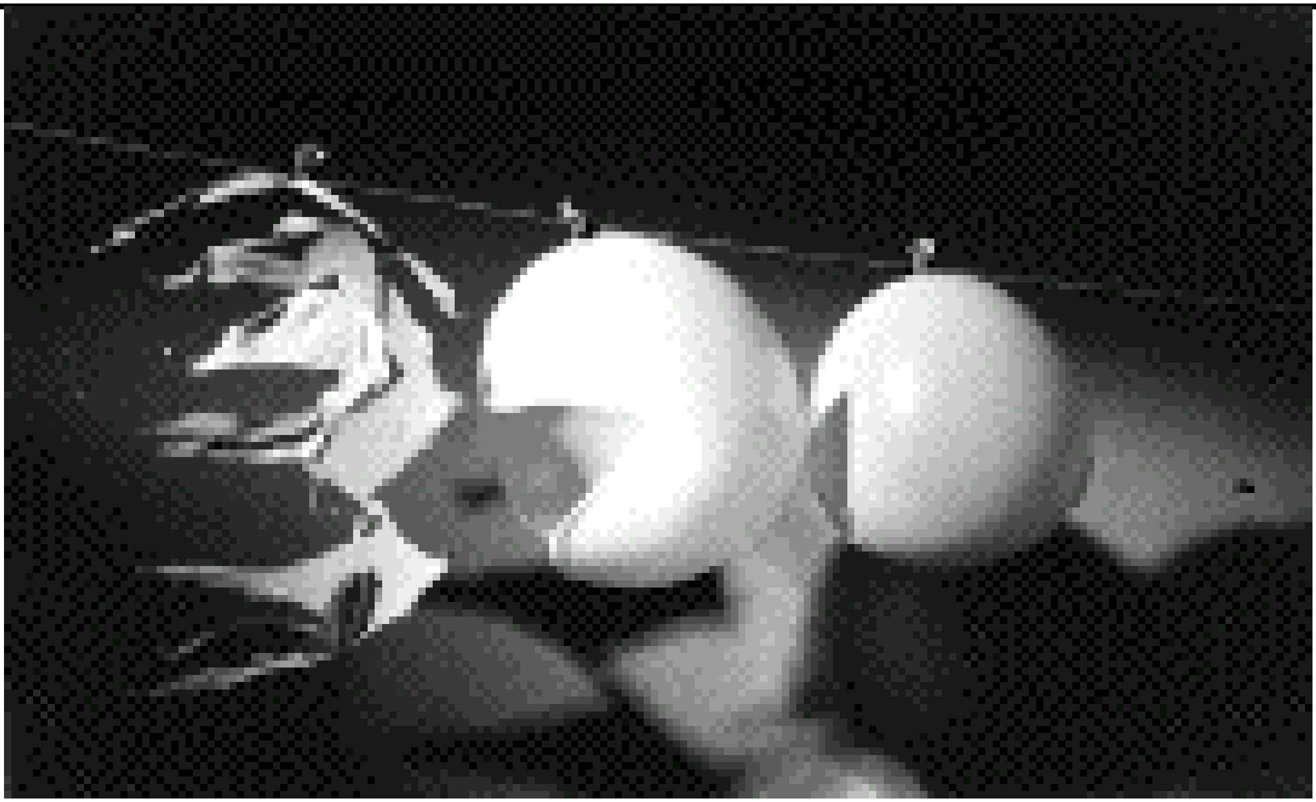
7. Splitting and fracture mode changes in Fiber-reinforced Composites

The distribution of fiber directions between plies strongly influences the way cracks grow.

Typical crack growth in a notched laminate (photo courtesy Boeing)

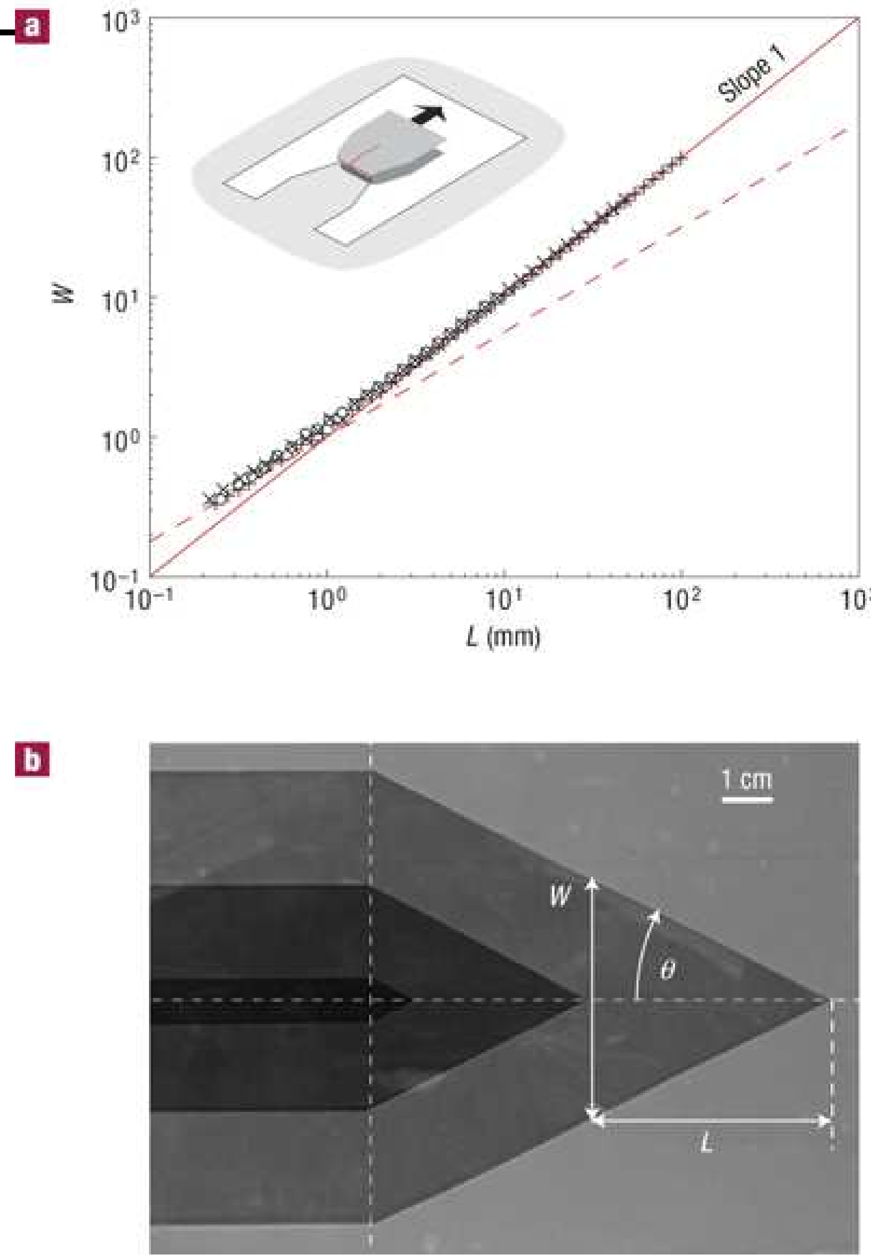
8. **Dynamic fracture in membranes**



Bursting of pressurized membrane by a bullet:
High speed photography of H. Edgerton (MIT collection)
(http://mit.edu/6.933/www/Fall2000/edgerton/edgerton.ppt). Time increases right to left.

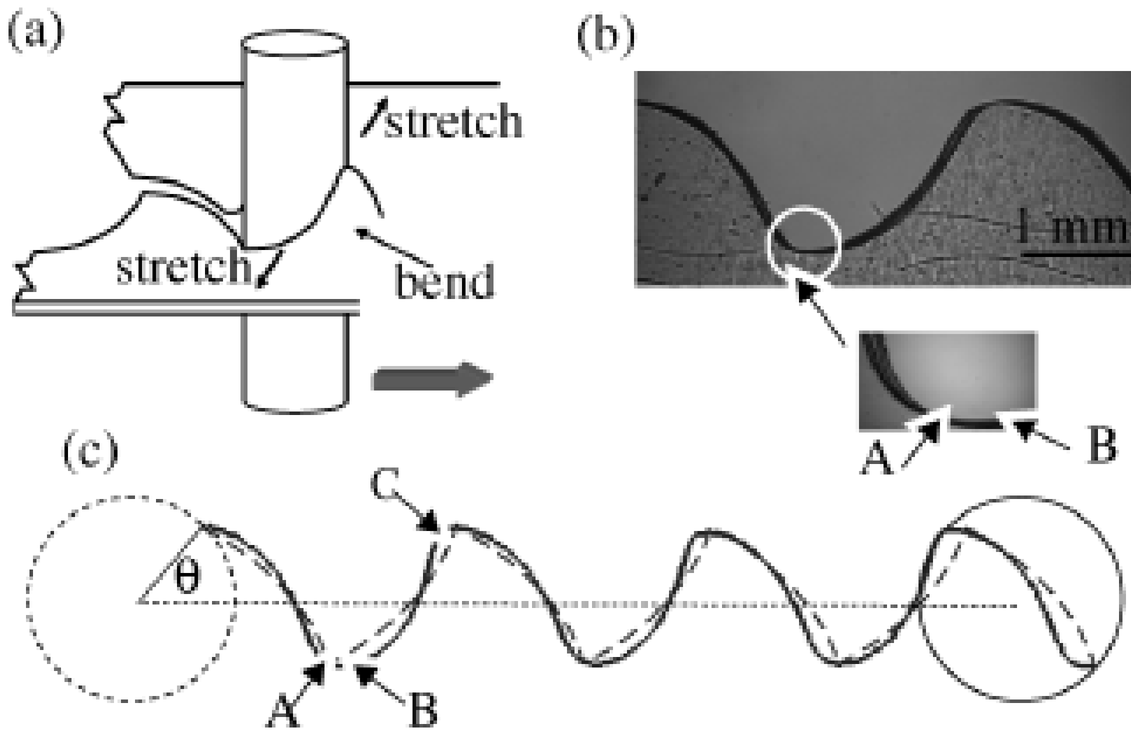
9a. **Peeling and Tearing**

Pealing of adhesive film from rigid substrate exhibits characteristic tearing behavior.



E. Hamm et al,
Nature Materials 7 (2008) 386-390

9b. **Instability in the slow tearing of an elastic membrane**



Ghatak & Mahadevan, *Physical Review Letters* 91 (2003) 215507-1–2155-7-4

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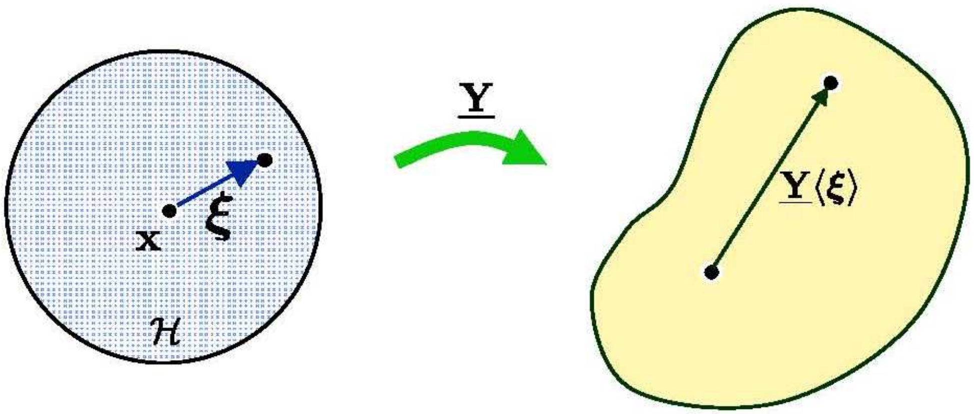
Starting point for peridynamics

Strain energy at $\mathbf{x}$ depends collectively on the deformation of the family of $\mathbf{x}$.



Undeformed family of $\mathbf{x}$

Deformed family of $\mathbf{x}$

Standard:	Peridynamic:
$\hat{W}\left(\dfrac{\partial \hat{\mathbf{y}}}{\partial \mathbf{x}}\right)$	$\hat{W}(\underline{\mathbf{Y}})$

The deformation state is the function that maps each bond $\boldsymbol{\xi}$ into its deformed image $\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle$.

Force state is the work conjugate to the deformation state

- Suppose we perturb the deformed bond $\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle$ by a virtual displacement $\boldsymbol{\epsilon}$. The resulting change in $W(\mathbf{x})$ is

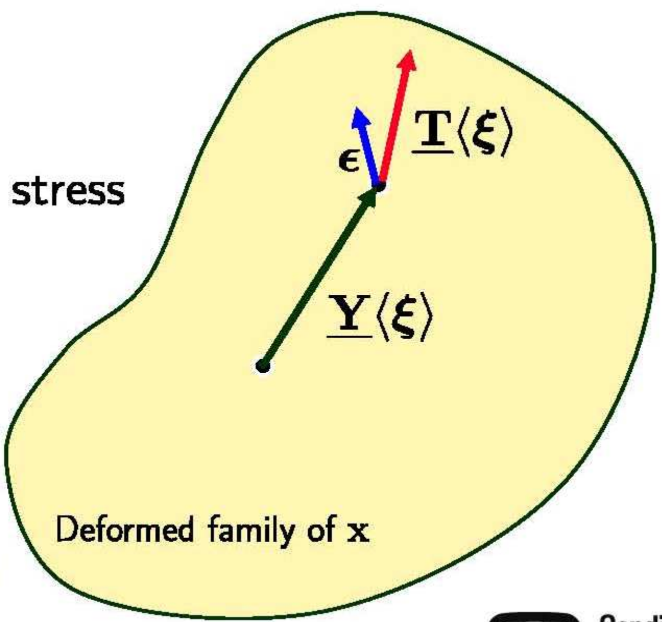
$$\Delta W = \underline{\mathbf{T}}\langle\boldsymbol{\xi}\rangle \cdot \boldsymbol{\epsilon}$$

  where $\underline{\mathbf{T}}\langle\boldsymbol{\xi}\rangle$ is a vector.

- The "force state" $\underline{\mathbf{T}}$ is the work conjugate to $\underline{\mathbf{Y}}$:

$$\dot{W} = \underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} = \int_{\mathcal{H}} \underline{\mathbf{T}}\langle\boldsymbol{\xi}\rangle \cdot \dot{\underline{\mathbf{Y}}}\langle\boldsymbol{\xi}\rangle \, dV_{\boldsymbol{\xi}}$$

- $\underline{\mathbf{T}}$ is the Frechet derivative of $W(\underline{\mathbf{Y}})$ – analogous to a stress tensor.



Displace just one bond $\boldsymbol{\xi}$

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Peridynamic equilibrium equation

- Total potential energy in $\mathcal{B}$:

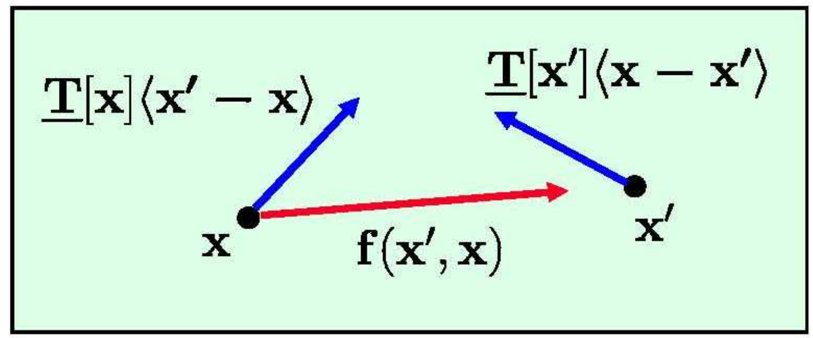
$$\Phi = \int_{\mathcal{B}} (W(\underline{\mathbf{Y}}) - \mathbf{b} \cdot \mathbf{u}) \, dV_{\mathbf{x}}$$

- Take first variation. Euler-Lagrange equation is

$$\int_{\mathcal{H}} \Big(\underline{\mathbf{T}}[\mathbf{x}]\langle \mathbf{x}' - \mathbf{x} \rangle - \underline{\mathbf{T}}[\mathbf{x}']\langle \mathbf{x} - \mathbf{x}' \rangle \Big) \, dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x}) = \mathbf{0}.$$

- Write this in terms of the "bond force" :

$$\int_{\mathcal{H}} \mathbf{f}(\mathbf{x}', \mathbf{x}) \, dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x}) = \mathbf{0}.$$



- where the bond force is defined by

$$\mathbf{f}(\mathbf{x}', \mathbf{x}) = \underline{\mathbf{T}}[\mathbf{x}]\langle \mathbf{x}' - \mathbf{x} \rangle - \underline{\mathbf{T}}[\mathbf{x}']\langle \mathbf{x} - \mathbf{x}' \rangle$$

Peridynamic equation of motion

- Equilibrium equation:

$$\int_{\mathcal{H}} \mathbf{f}(\mathbf{x}', \mathbf{x})\, dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x}) = \mathbf{0}.$$

- where

$$\mathbf{f}(\mathbf{x}', \mathbf{x}) = \underline{\mathbf{T}}[\mathbf{x}]\langle \mathbf{x}' - \mathbf{x} \rangle - \underline{\mathbf{T}}[\mathbf{x}']\langle \mathbf{x} - \mathbf{x}' \rangle$$

- Now use d'Alembert's principle to get the equation of motion:

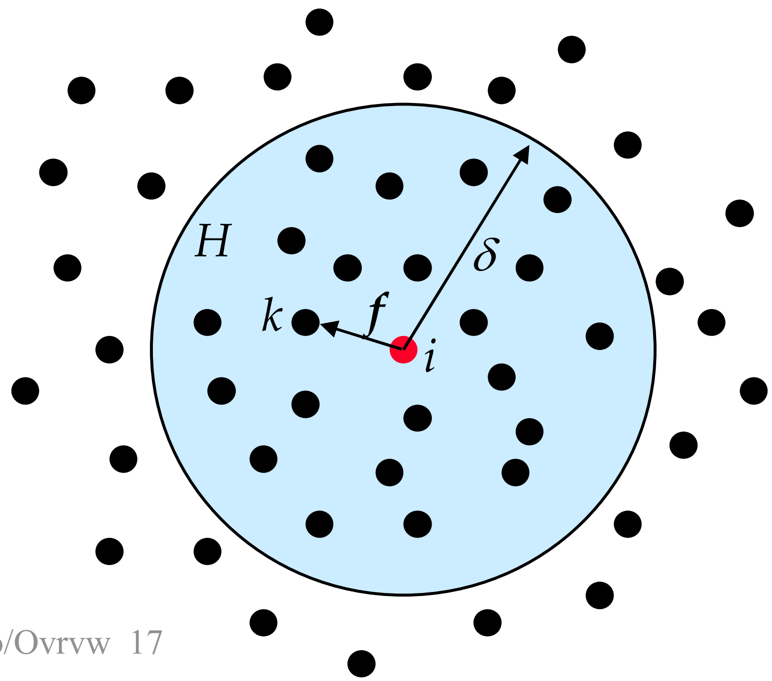
$$\rho(\mathbf{x})\ddot{\mathbf{y}}(\mathbf{x}, t) = \int_{\mathcal{H}} \mathbf{f}(\mathbf{x}', \mathbf{x}, t)\, dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x}, t)$$

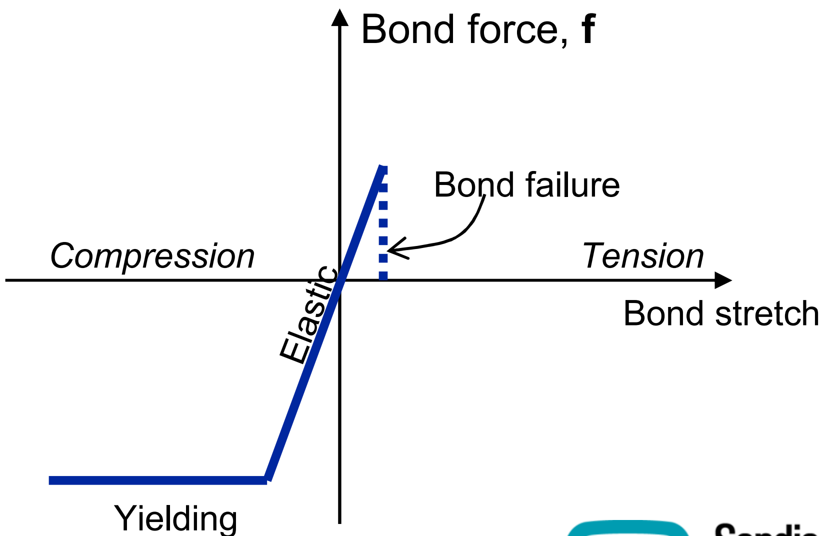
Numerical method and material model incorporate damage at the "bond" level

- Finite sum replaces integral: method is meshless and Lagrangian.
- Force parameters come from measurable elastic-plastic and fracture data.
- Simulate explicit time integration dynamics or static equilibrium; extension to implicit dynamics is underway.

$$\rho \ddot{\boldsymbol{u}}_i^n = \sum_{k \in H} \boldsymbol{f}(\boldsymbol{u}_k^n - \boldsymbol{u}_i^n, \boldsymbol{x}_k - \boldsymbol{x}_i)\Delta V_i + \boldsymbol{b}(\boldsymbol{x}_i, t)$$

All material-specific behavior is contained in the force density, *f*.





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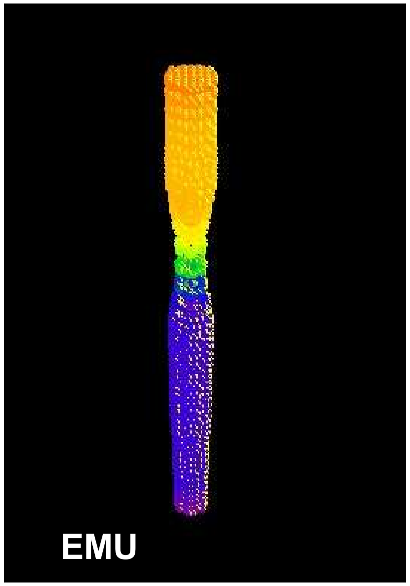
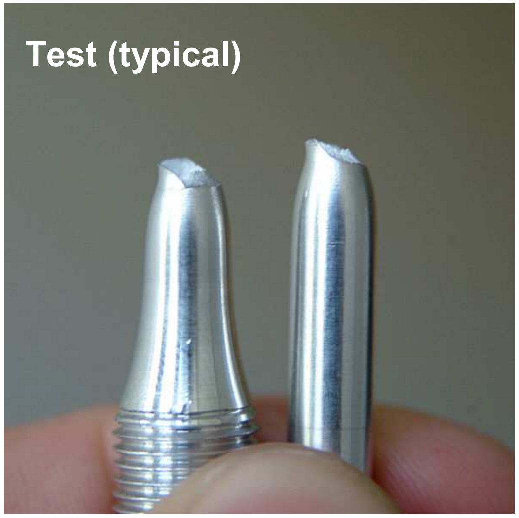
Peridynamic Can Use Diverse Constitutive Models

- **Peridynamics can model large-strain, rate-dependent, strain-hardening plasticity.**





Necking in tension

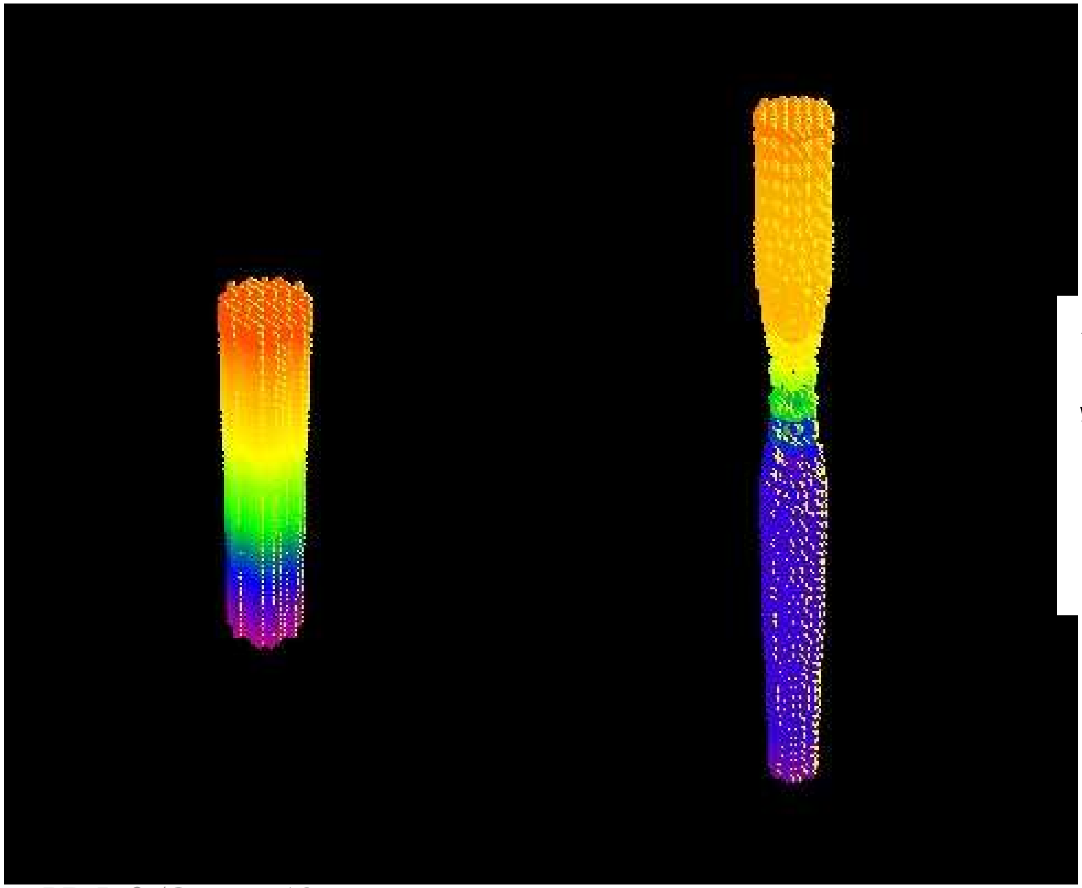
Taylor impact test
Dynamic compression
(EMU calculation by J. Foster)

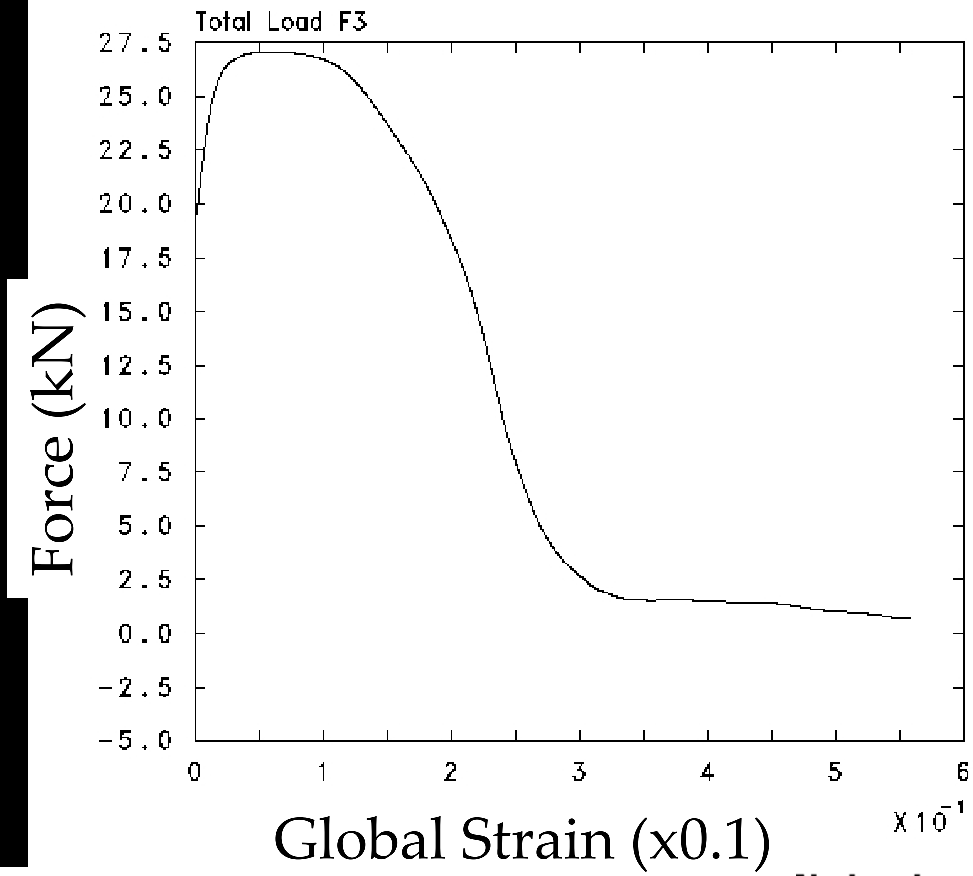
*C. E. Anderson Jr, A. E. Nicholls, I. S. Chocron, and R. A. Ryckman. Taylor anvil impact. In AIP Conference Proceedings, volume 845, page 1367. AIP, 2006

Necking in a 6061-Aluminum Bar

Simulation with state-based peridynamic implementation of large-deformation, strain-hardening, rate-dependent material model.

- Force-strain plot shows stable response with decreasing load.





Fracture Phenomena *Emerge* from a Peridynamics Simulation

Peridynamics is a history-dependent theory in which crack initiation and growth, and all associated phenomena, *emerge spontaneously*, in an *unguided* fashion, simply from the choice of system geometry, ICs, BCs, and the constitutive model.

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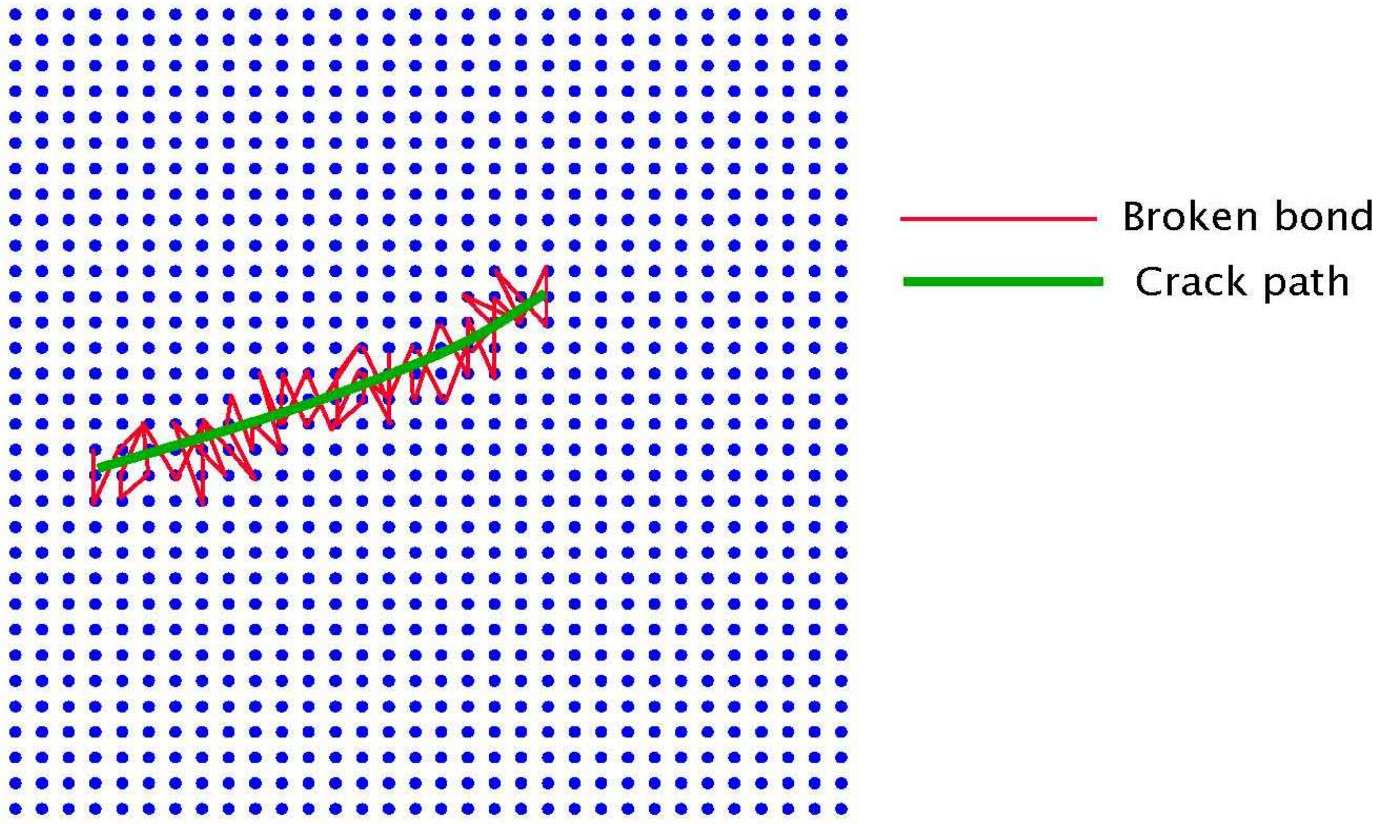
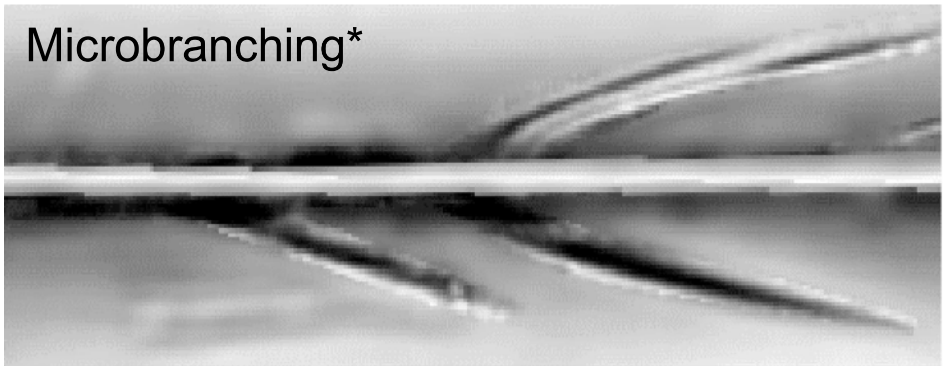
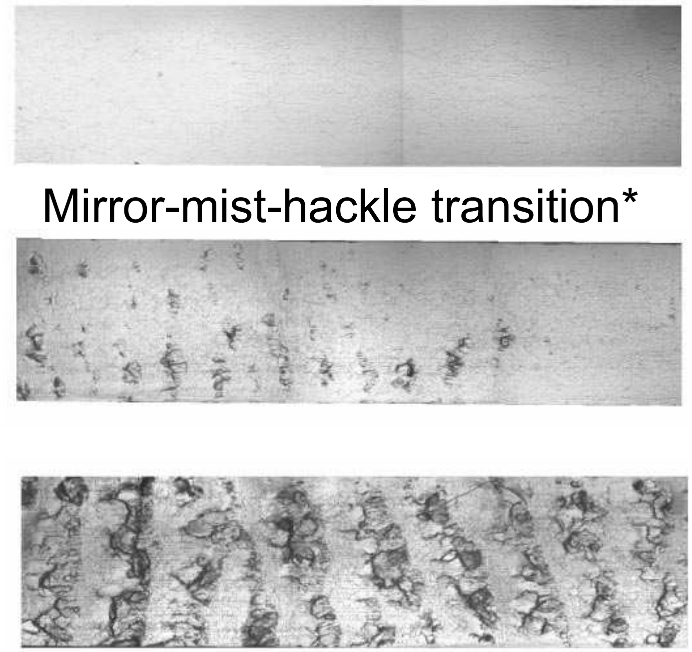
Bond breakage forms cracks “autonomously”

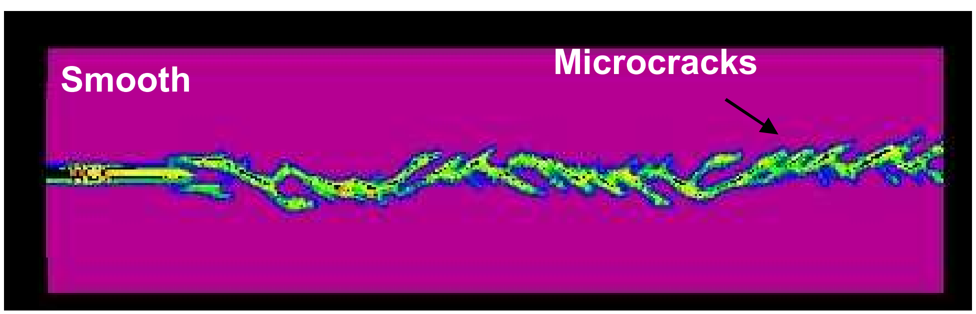


When a bond breaks, its load is shifted to its neighbors, leading to progressive failure.

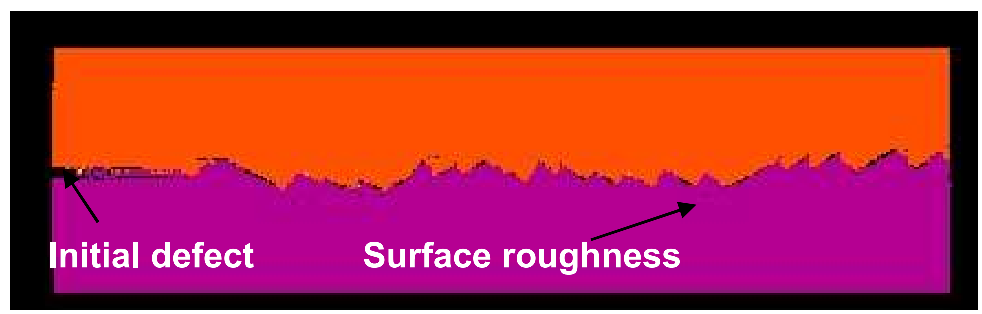
1. Dynamic fracture in PMMA: Damage features







Peridynamics damage

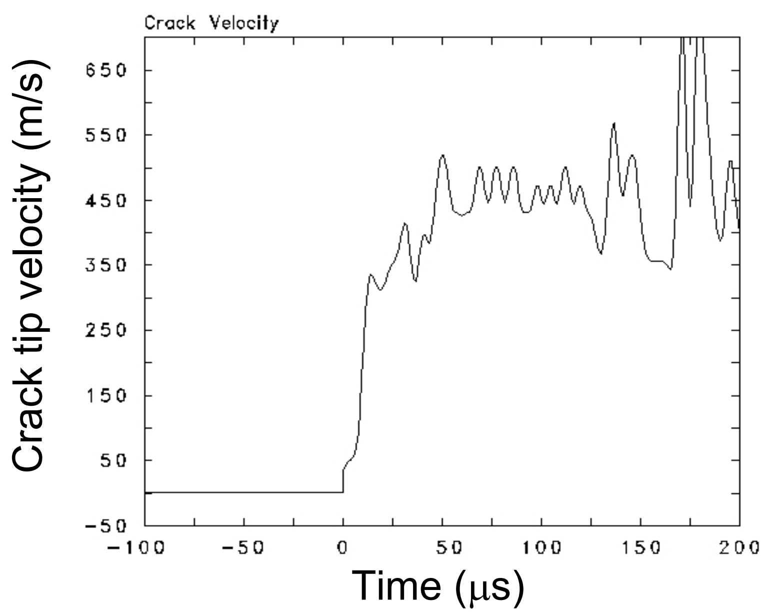


Peridynamics crack surfaces

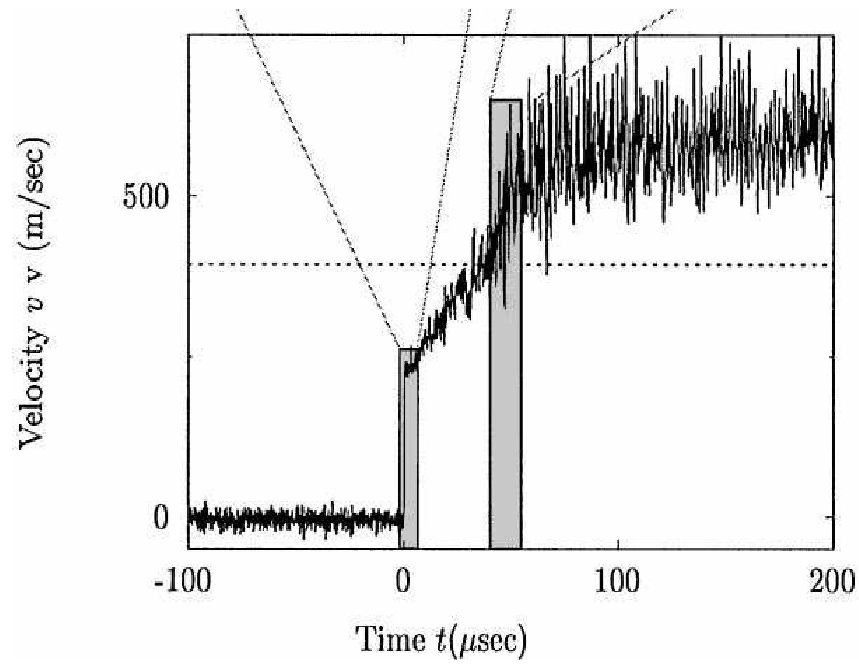
* J. Fineberg & M. Marder, *Physics Reports* 313 (1999) 1-108

2. Dynamic fracture in PMMA: Crack tip velocity

- Crack velocity increases to a critical value, then oscillates.



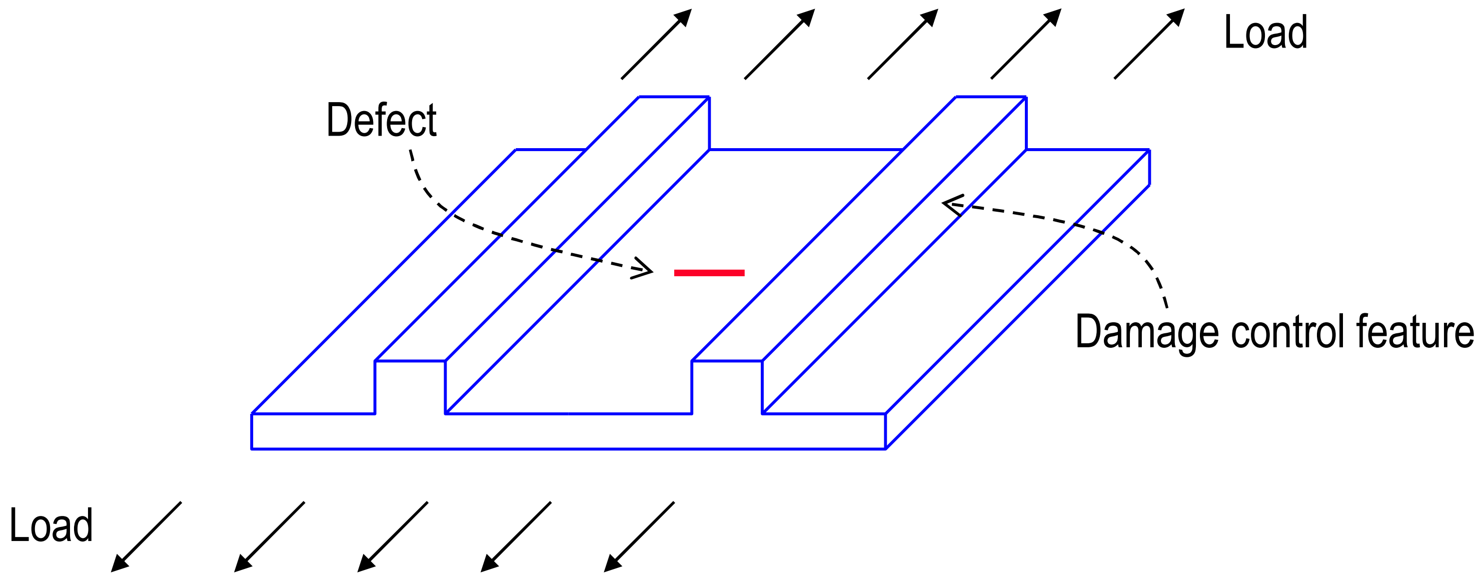
Peridynamics

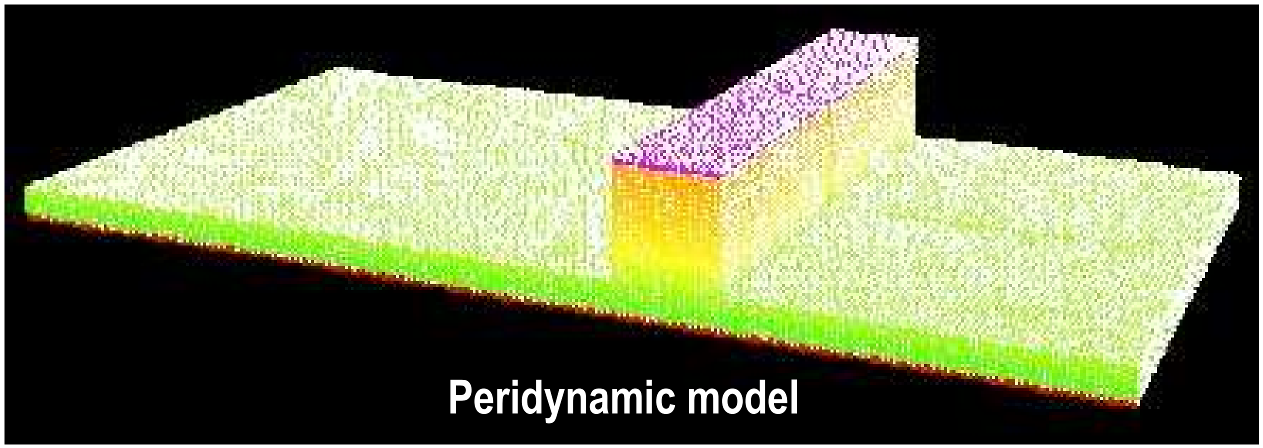


Experiment*

* J. Fineberg & M. Marder, *Physics Reports* 313 (1999) 1-108

3. Peridynamics model of Ribbed Plate





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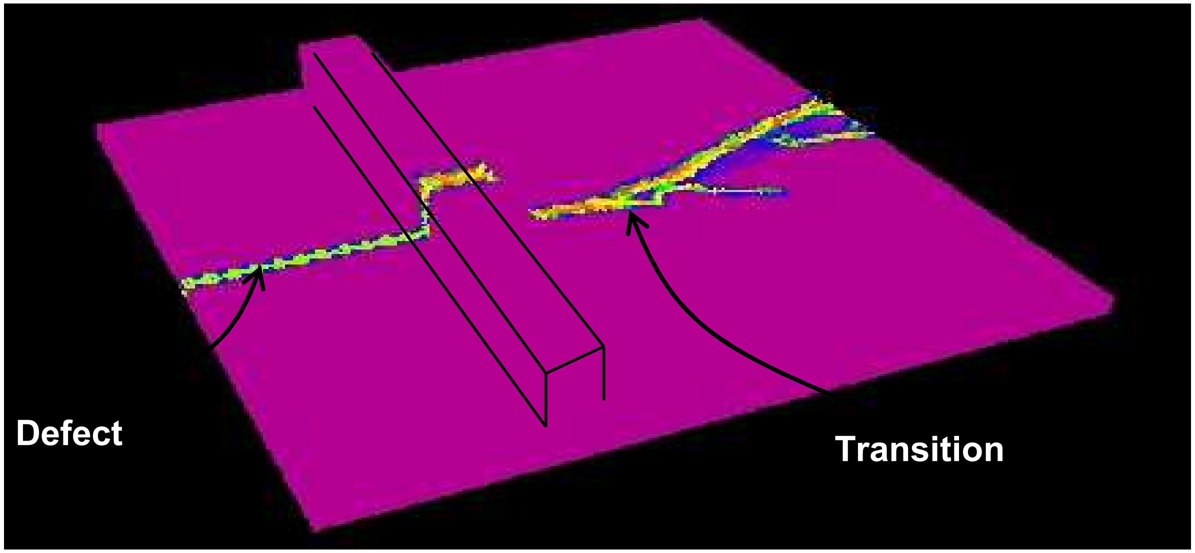
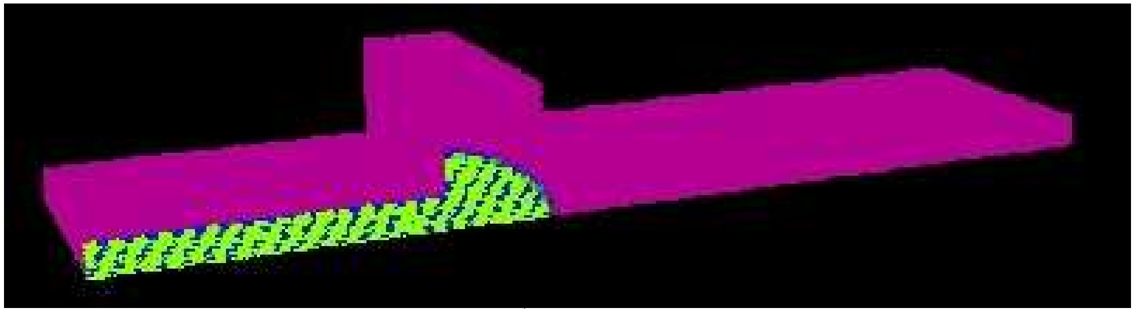
3. Simulation of damage control feature and crack instability



Crack trajectory after instability



Crack just before transition to unstable (photo courtesy Boeing)

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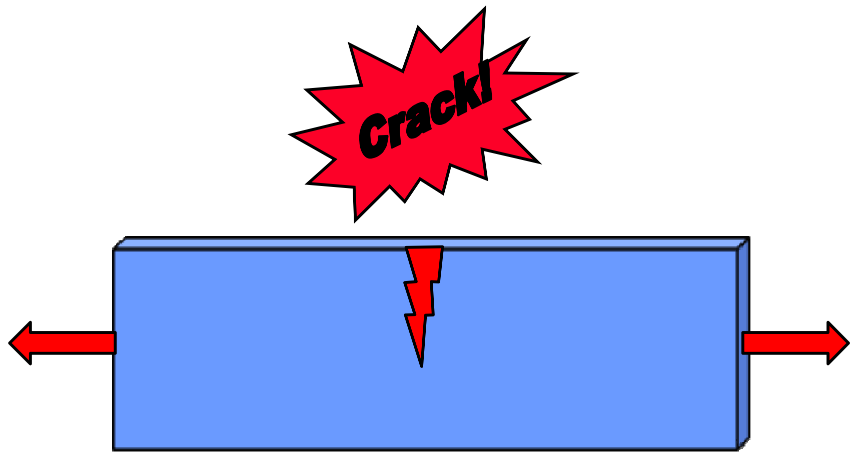
4. Dynamic Brittle Fracture in Glass

- Soda-lime glass plate
  - Dimensions: 3" x 1" x 0.05"
  - Density: 2.44 g/cm^3
  - Elastic Modulus: 79.0 GPa
- Notch at top; apply tension



- Discretization (finest)
  - Mesh spacing: 35 microns
  - Approx. 82 million particles
  - Time: 50 microseconds (20k timesteps)
  - 6 hours on 65k cores using DAWN (LLNL): IBM BG/P System
  - 500 teraflops; 147,456 cores
- Simulation by M. Parks (SNL), F. Bobaru & Y. Ha (Nebraska)

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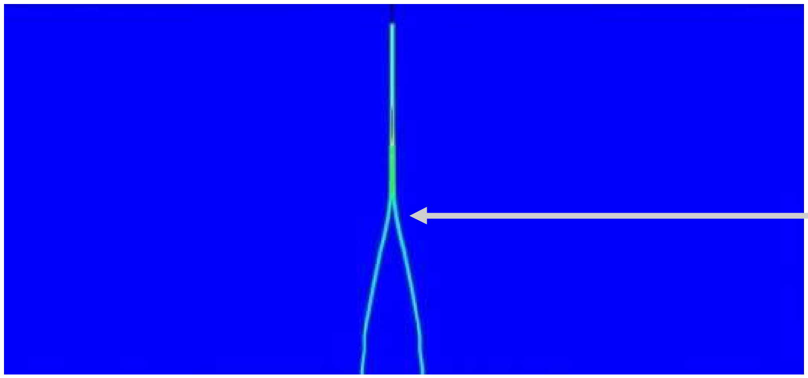
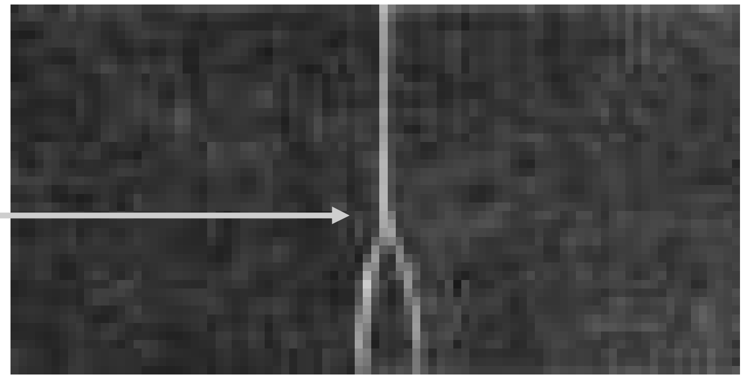
4. Crack Branching in Brittle in Glass



Peridynamics Simulation



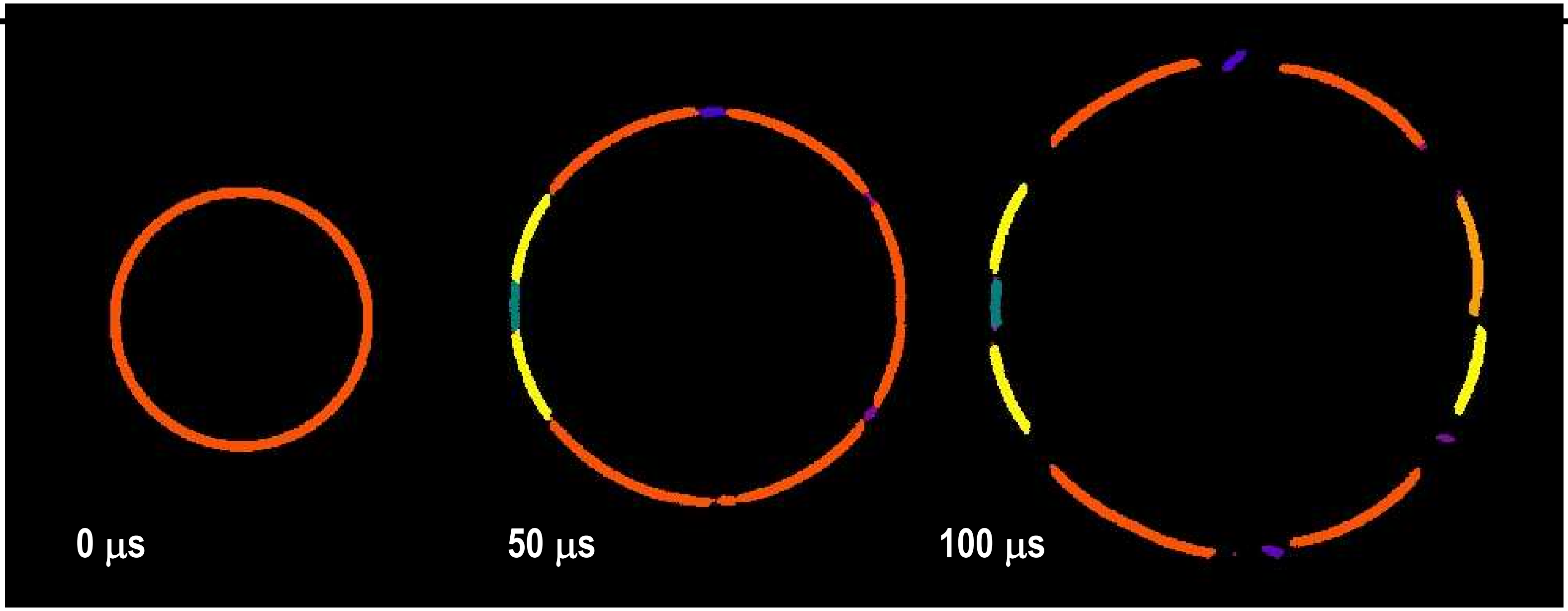
Physical Experiment*

Crack branching ***emerges*** in the Peridynamics simulation of the experimental specimen and loading conditions:

- Branching is qualitatively identical to experiment;
- Onset of simulated branching is earlier than experiment.

*S F. Bowden, J. Brunton, J. Field, and A. Heyes, *Controlled fracture of brittle solids and the interuption of electrical current*, Nature (1967) 216, 42, pp.38-42.

5a. **Fragmentation of a Ductile Aluminum Ring***



Experiment *:

- Recovered 11-13 fragments per shot over range of initial velocities 182 – 220 m/s.

Peridynamics model produces 12 fragments.

*D. E. Grady & D. A. Benson, *Experimental Mechanics* 23 (1983) 393-400

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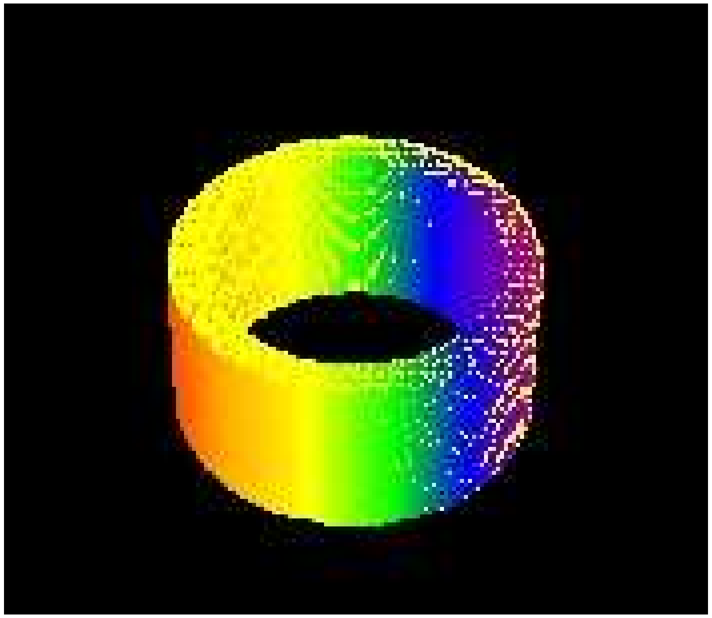
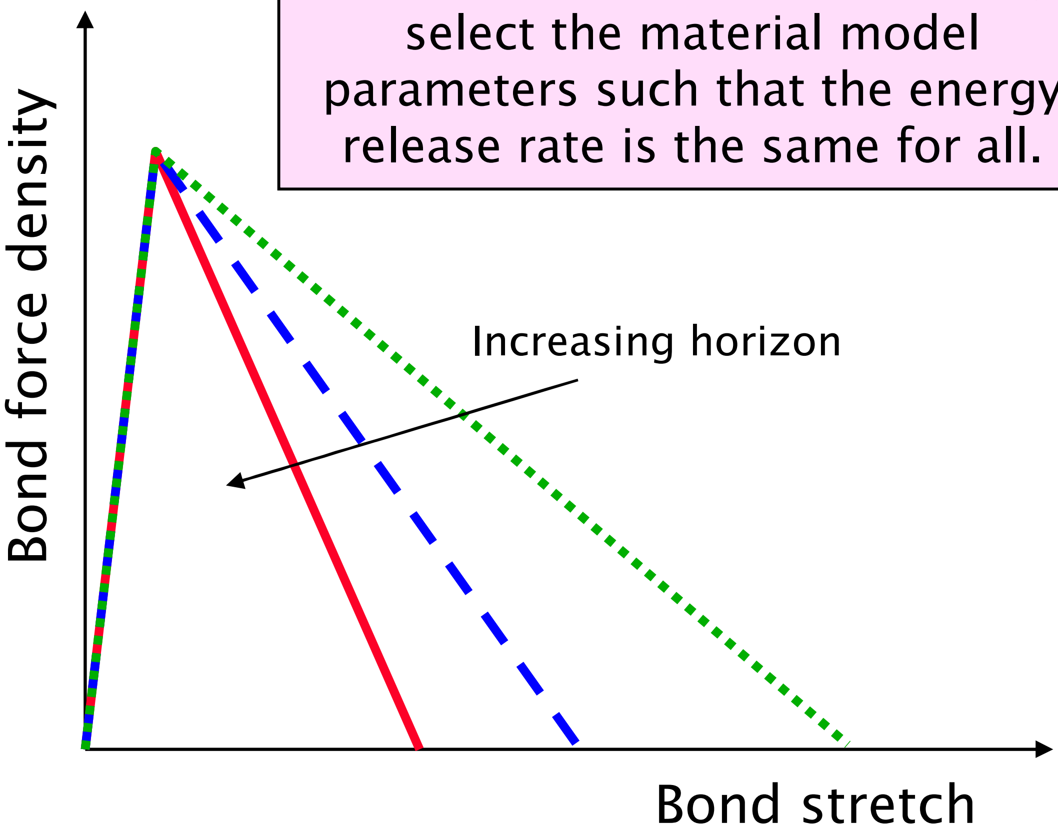
5b. Convergence in fragmentation: Expanding 3D annulus



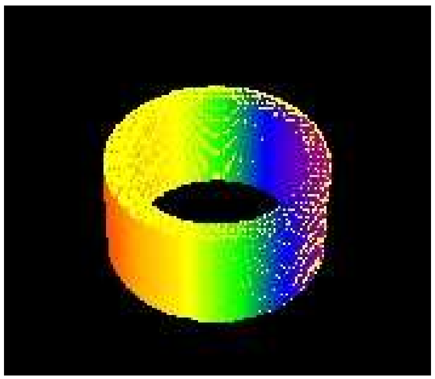


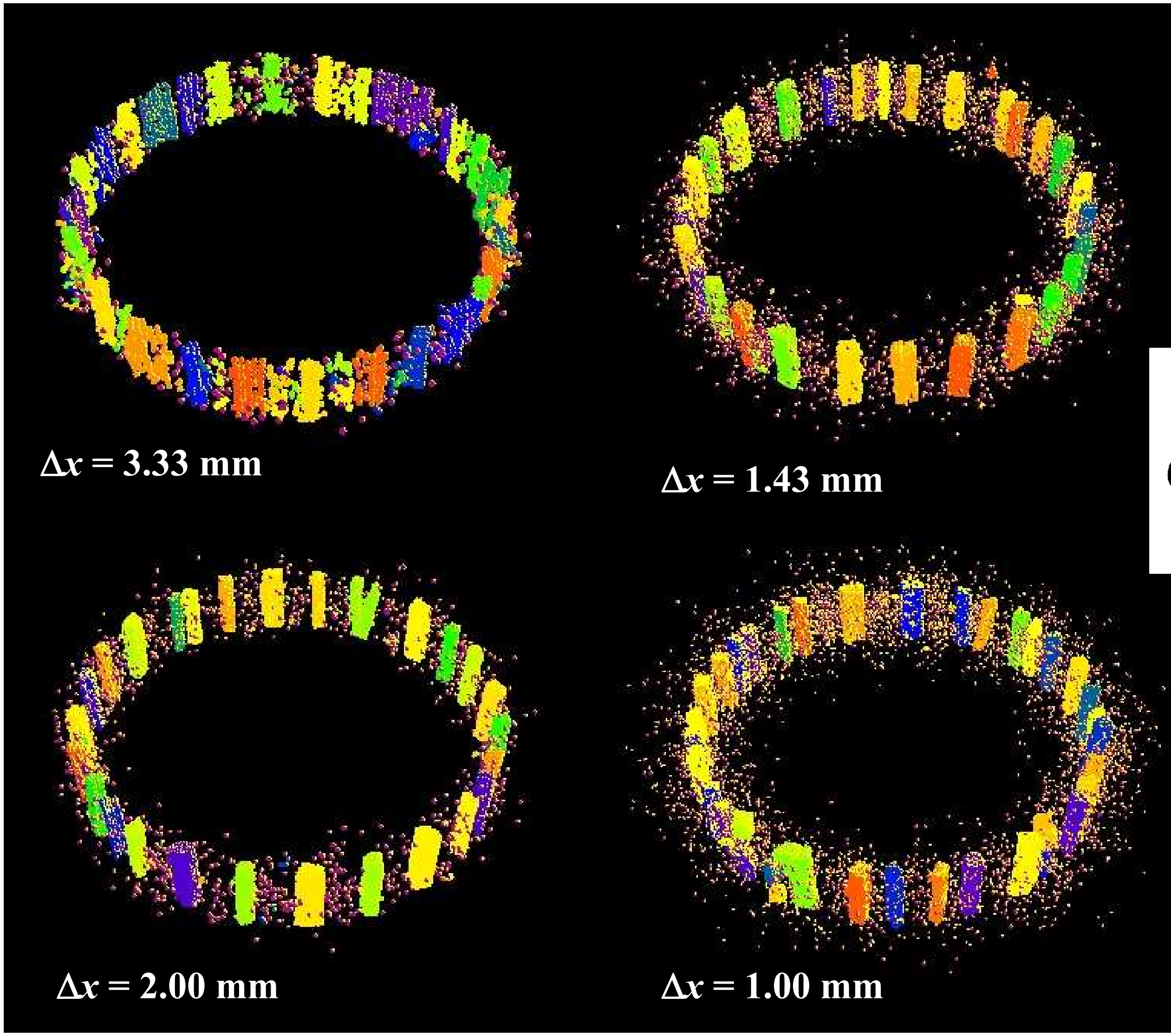
For each choice of the horizon, select the material model parameters such that the energy release rate is the same for all.

- Outer radius = 100 mm
- Thickness = 10 mm
- Radial velocity = 600 m/s

- Grid contains 1% random perturbations to act as seed.

5b. Convergence in fragmentation: Same problem with 4 different grid spacings





$$\delta = 3\Delta x$$

Horizon = 3x grid spacing

Colors are for visualization

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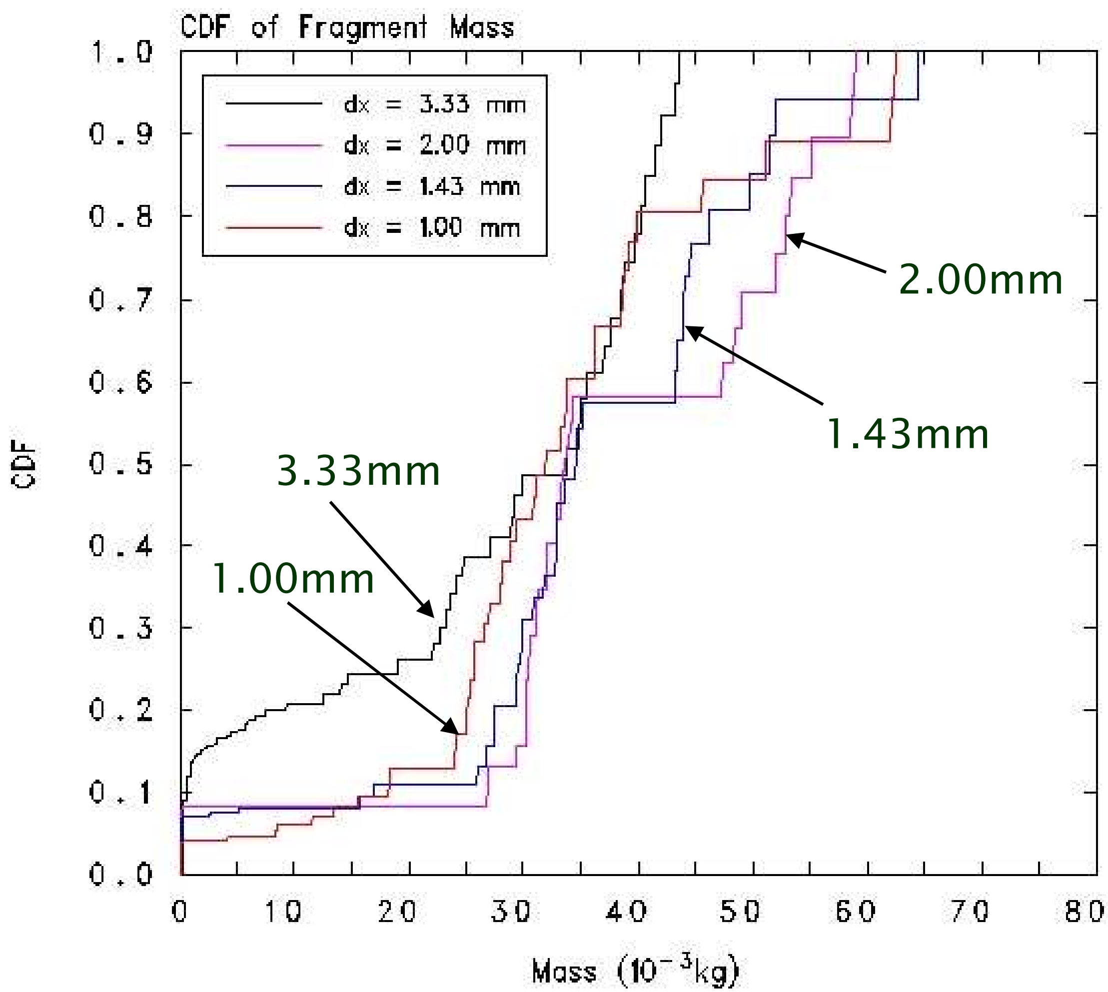
5b. **Convergence in fragmentation: Fragment mass distribution**

Cumulative distribution function
for 4 grid spacings

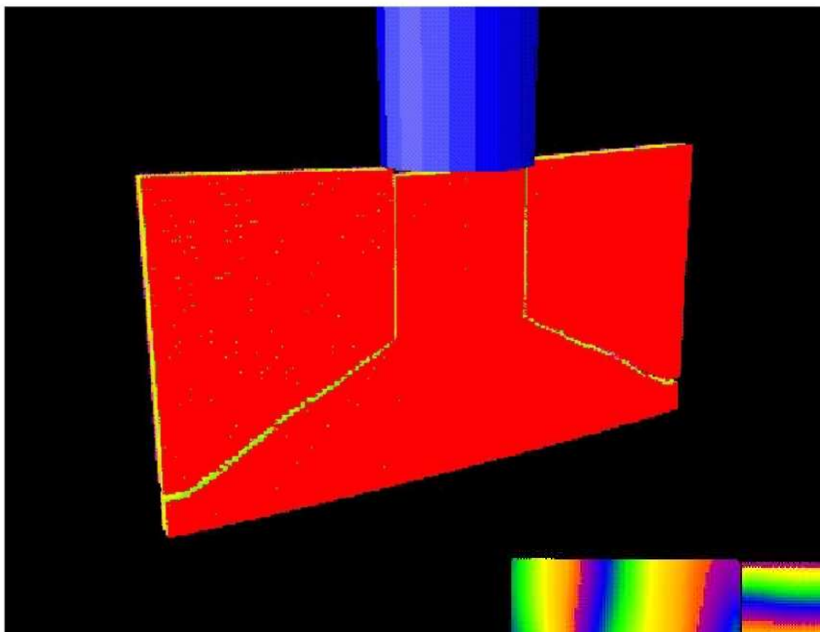


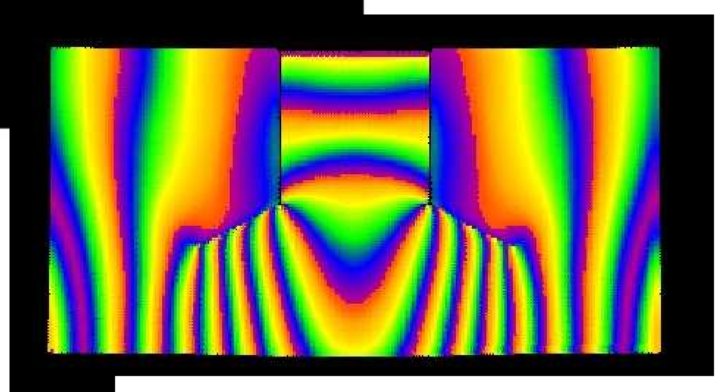
Δx (mm)	Mean fragment mass (g)
3.33	27.1
2.00	37.8
1.43	35.9
1.00	33.5

Solution appears essentially converged

6. **Dynamic fracture in a hard steel plate**

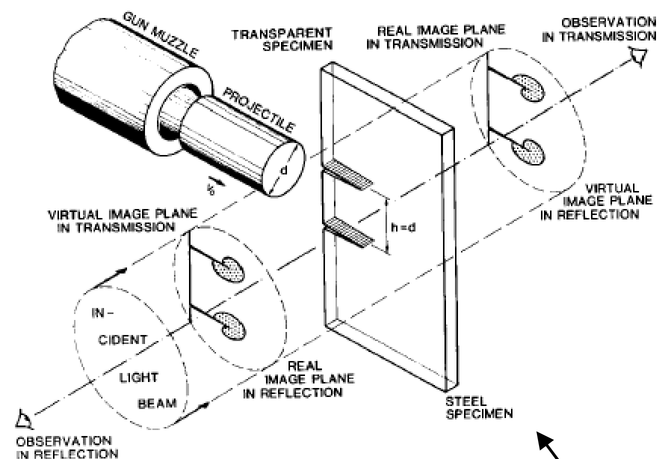
Dynamic fracture in maraging steel (Kalthoff & Winkler, 1988)
- 3D Peridynamics model reproduces the 70° crack angle.

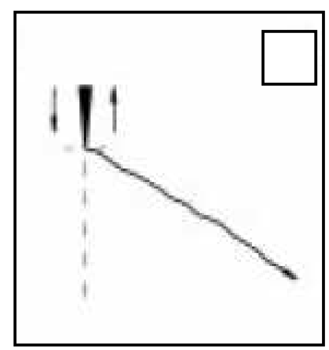




Peridynamics

Vertical displacement contours





Experiment

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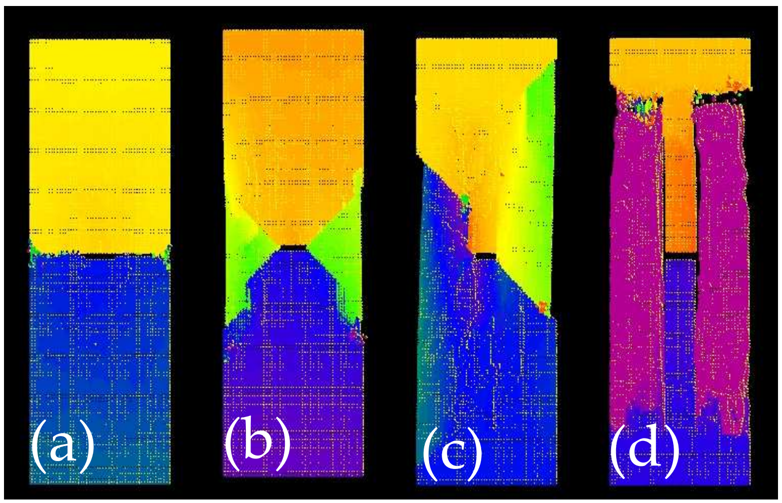
7. Splitting and fracture mode changes in Fiber-reinforced Composites

- Distribution of fiber directions between plies strongly influences the way cracks grow.

Crack growth in a notched laminate bar



a) Quasi-isotropic;
b) Additional +/- 45°;
c) Extra +/- 45°;
d) Mostly 0° (along length)

Typical crack growth in a notched laminate (photo courtesy Boeing)

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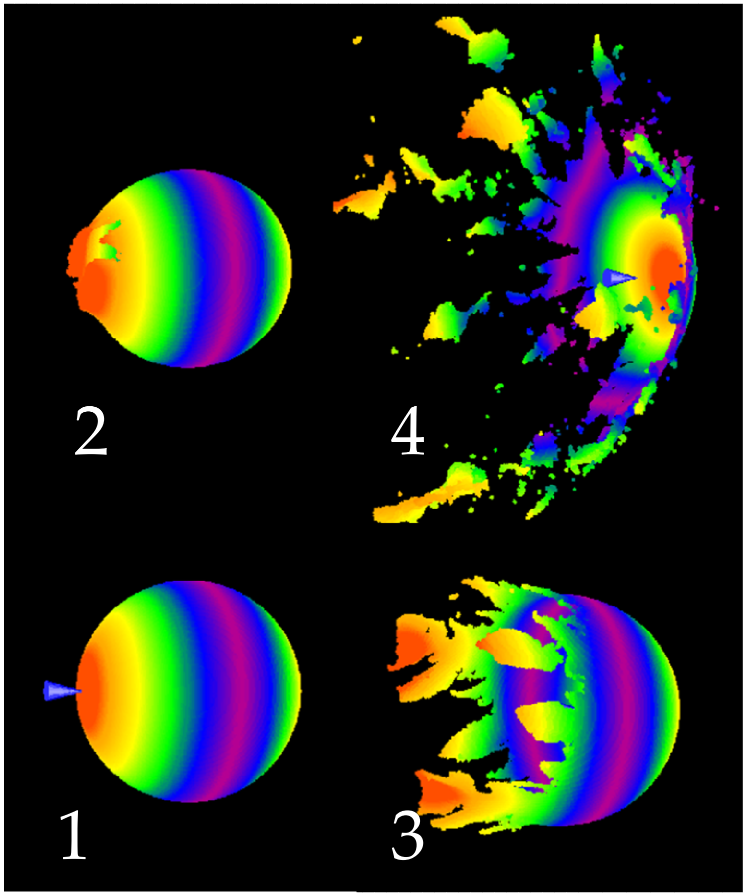
8. Dynamic fracture in membranes



Peridynamics model of a balloon penetrated by a fragment.
(Time sequence numbered.)

Bursting of a balloon by a bullet.
Time increases *right to left*.
High speed photo by H. Edgerton (MIT collection)

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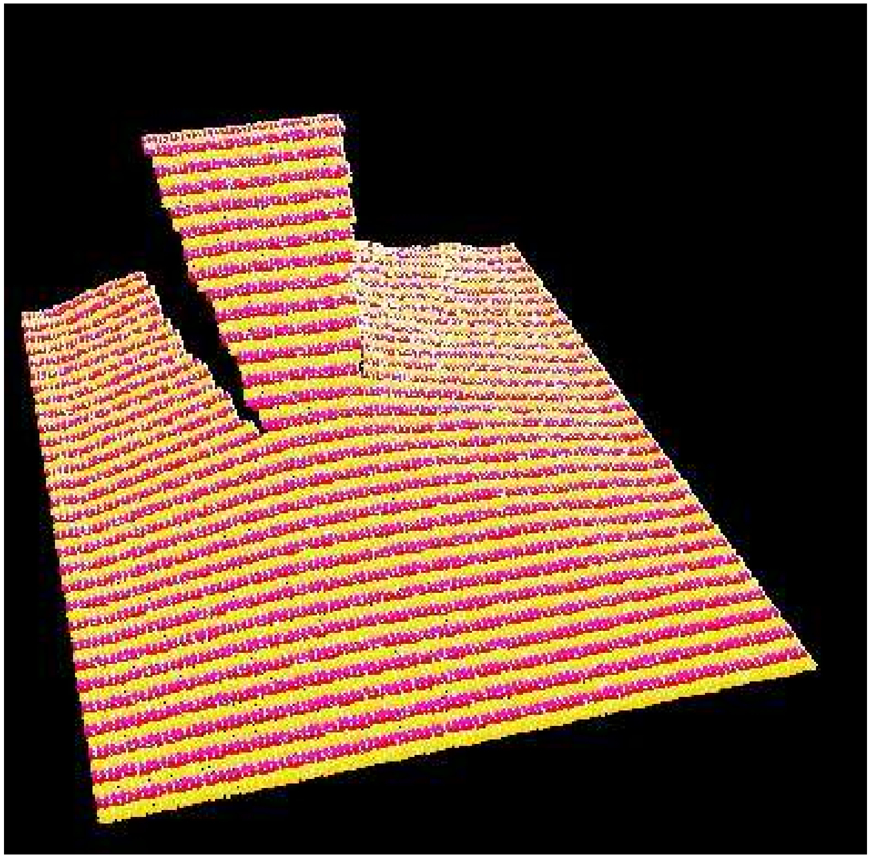
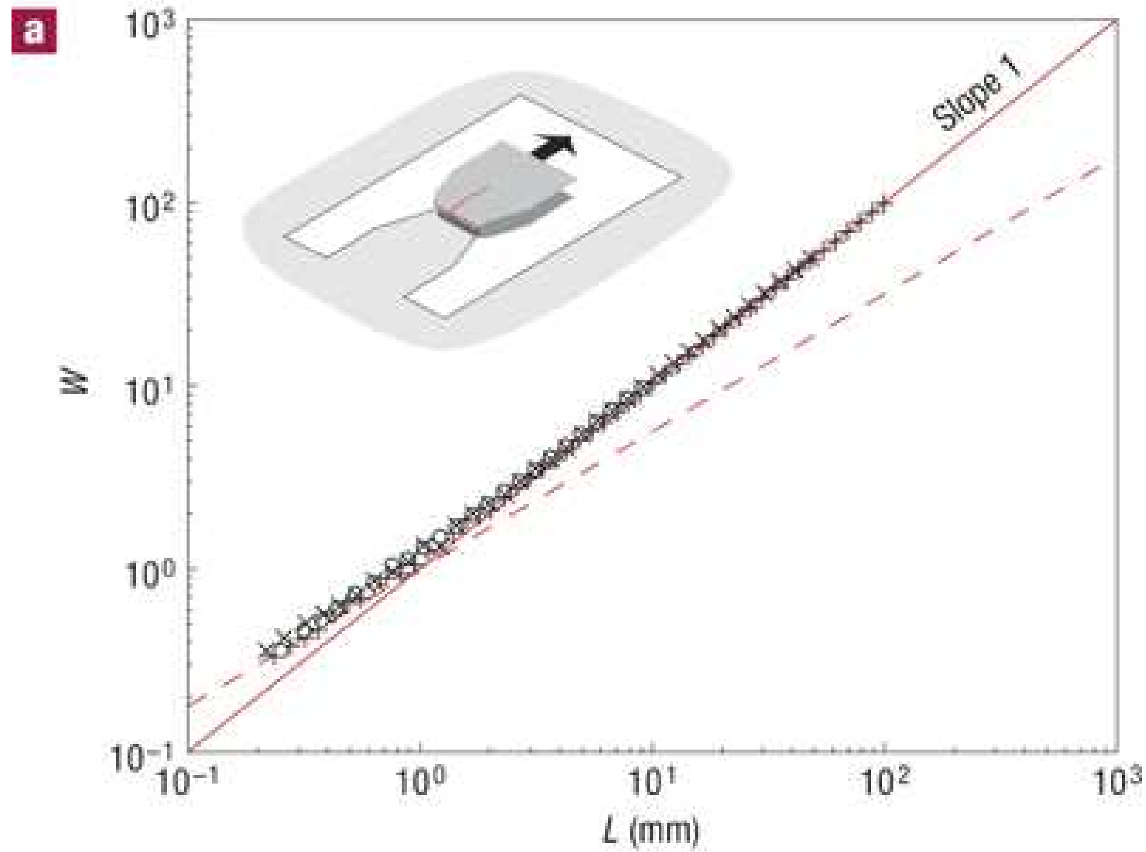
9a. Peeling and tearing



Peridynamics: Unsupported sheet, pinned along 3 edges (Computers and Structures, 2005)



E. Hamm et al, *Nature Materials* 7 (2008) 386-390

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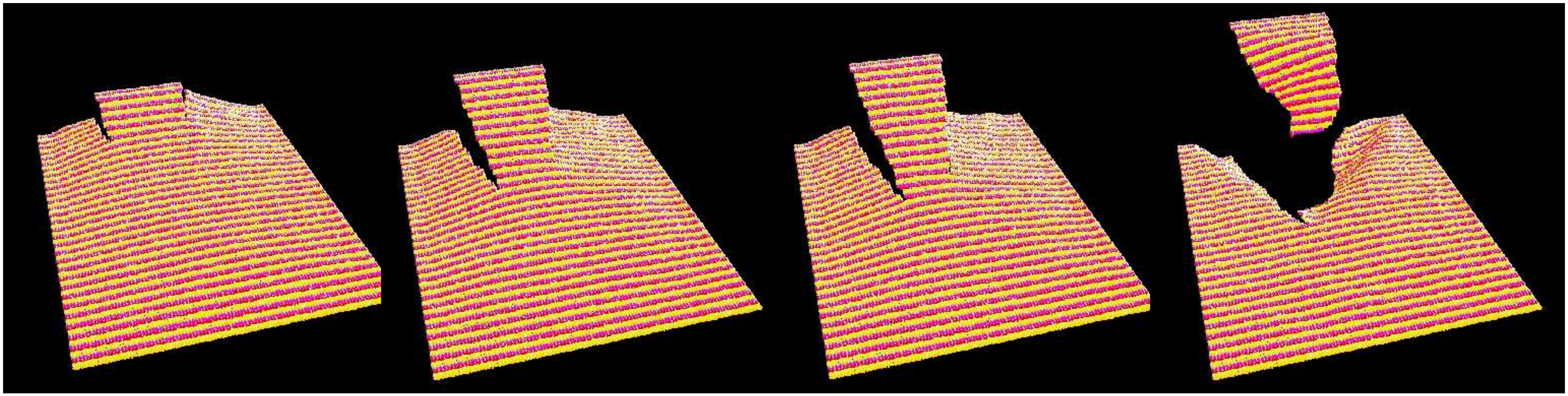
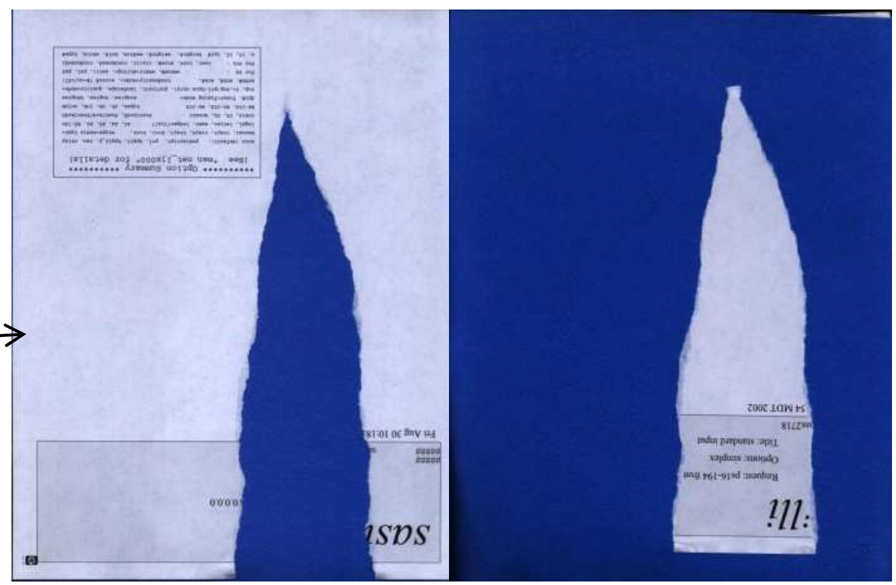
9a. Interaction of 2 cracks: Peeling of a sheet

- Pull upward on part of a free edge – other 3 edges are fixed.



"Experimental data"



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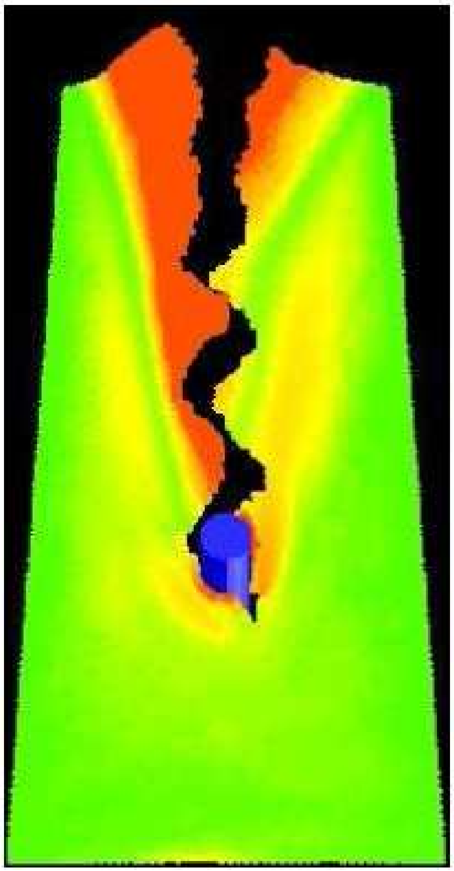
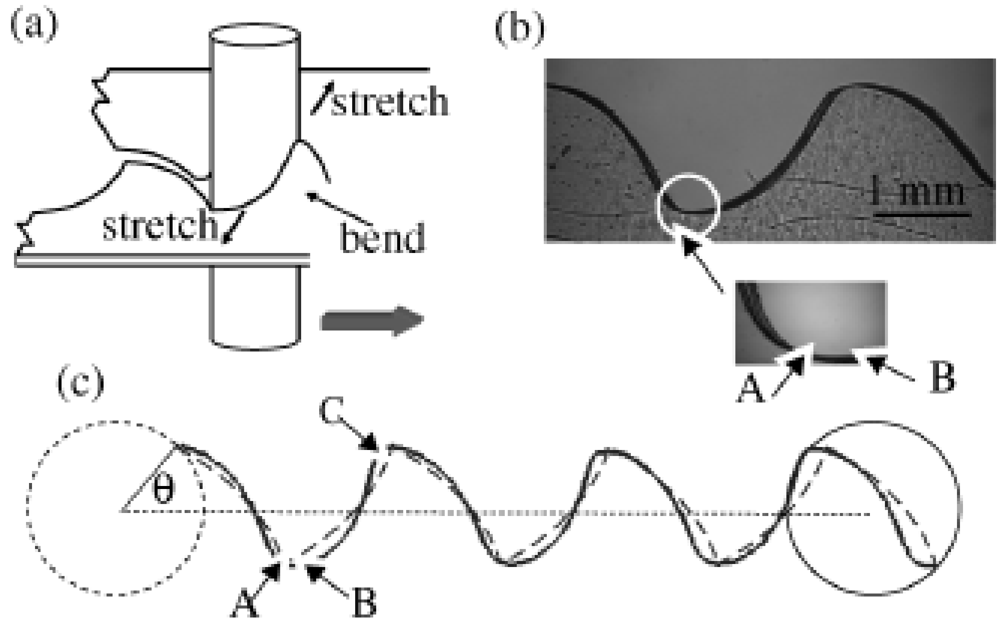
9b. Instability in the slow tearing of an elastic membrane



Peridynamics

Silling & Bobaru, *Int'l Journal Non-Linear Mech.* 40 (2005) 395–409



Ghatak & Mahadevan, *Physical Review Letters* 91 (2003) 215507-1 – 2155-7-4

Conclusion

Fracture phenomena spontaneously emerge in peridynamics simulations simply as a result of the choice of system geometry, ICs, BCs, and material model.

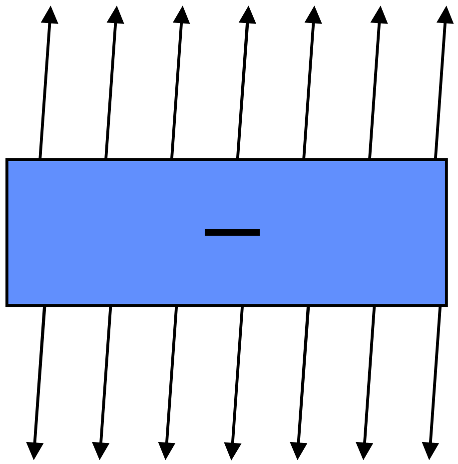
No supplemental kinetic relations are required; cracks grow in an unguided fashion.

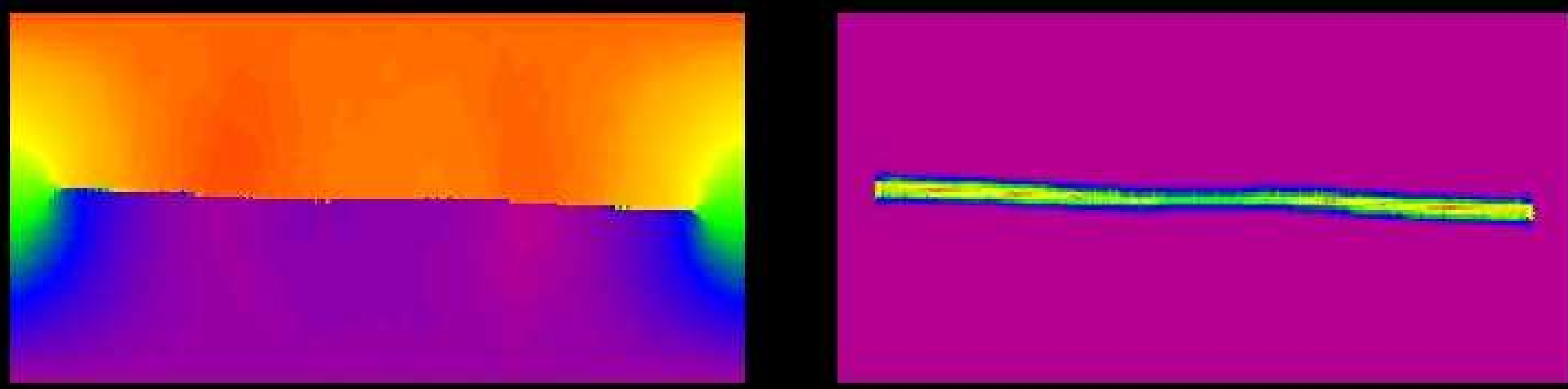
In this sense peridynamics is predicitive of dynamic fracture phenomena.

Crack growth at a shallow angle relative to a grid

- Plate with a pre-existing defect is subjected to prescribed boundary velocities.
- These BC correspond to mostly Mode-I loading with a little Mode-II.

$$\dot{\varepsilon} = (0.25\mathrm{s}^{-1}) \begin{bmatrix} 0 & 0.1 \\ 0 & 1 \end{bmatrix}$$





Contours of vertical displacement   Contours of damage

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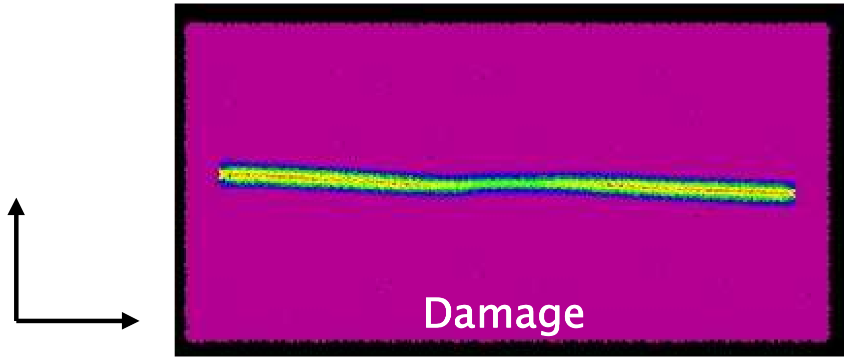
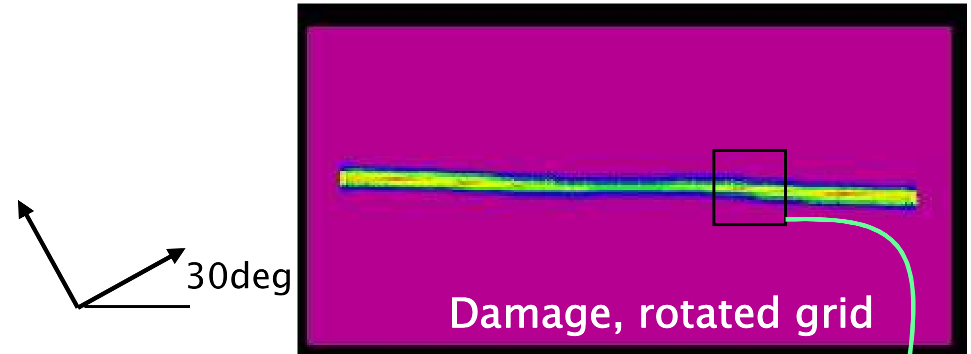
Crack growth direction is nearly independent of the grid orientation



Original grid direction



Rotated grid direction

Network of identical bonds in many directions allows cracks to grow in any direction.