

# Choosing Corners of Rectangles for Mapped Meshing

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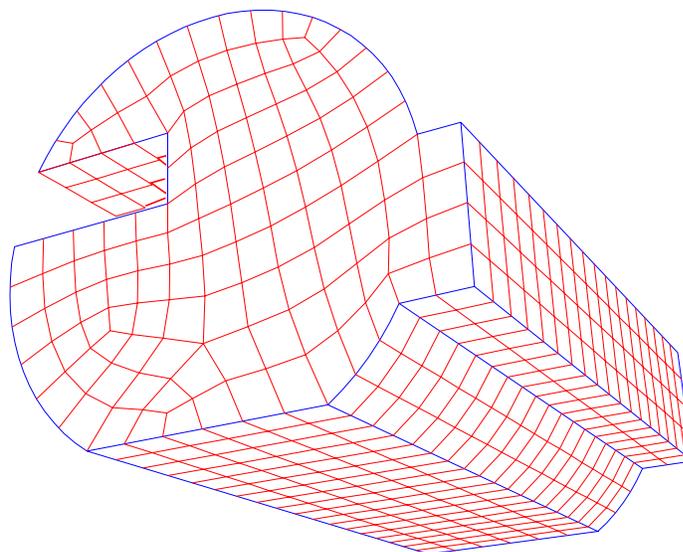
## Structured quadrilateral meshing

### Quads

- better analysis for structural mechanics,
- but, very dependent on shape of quads (not topology)

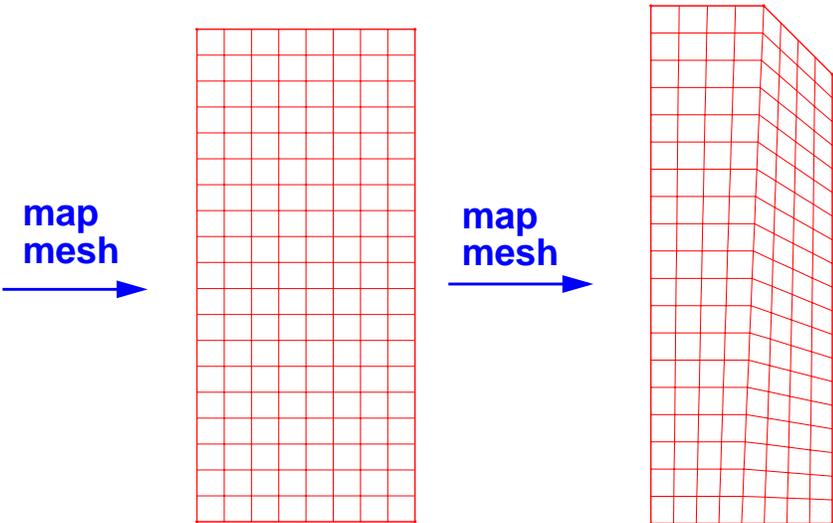
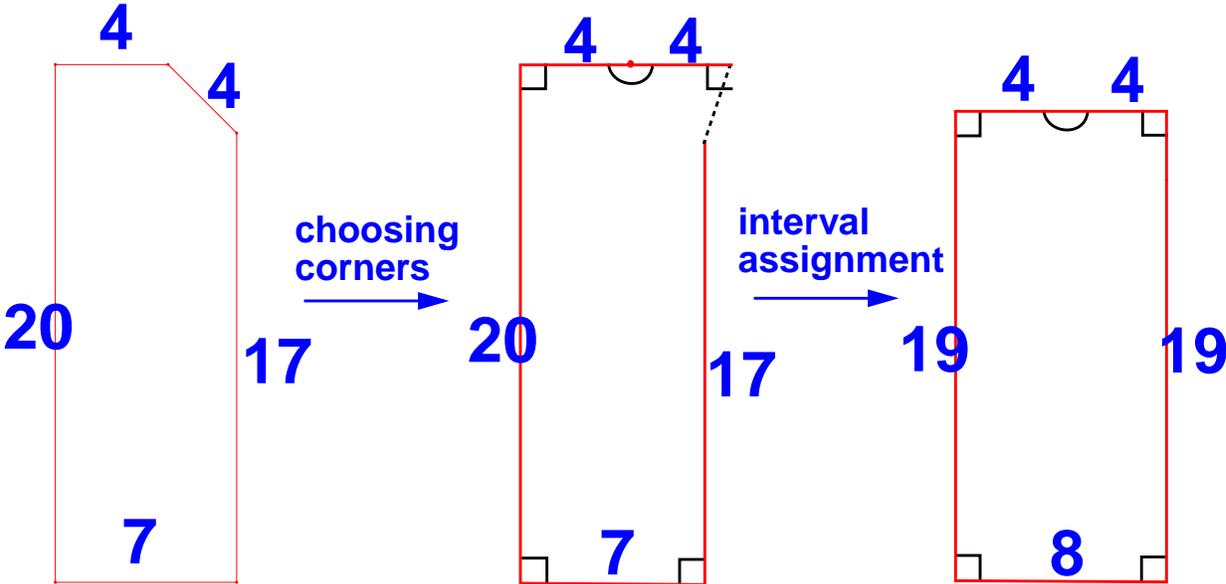
### Structured

- lower memory, faster speed than unstructured
- In 3d, many hex algorithms 2.5 dimensional
- Applications: car crankshafts, tire treads & cross-sections, weapon components



+ twists, turns,  
and branching

# Problem Definition



**Complication: some intervals fixed**

# Problem definition



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## 1. Precise

**Given a surface with prescribed (hard-set) and desired (soft-set) intervals on its curves, for which mapped-meshing is suitable,**

**choose four vertices as corners, such that a good quality mapped-mesh results (feasible).**

## 2. Abstract

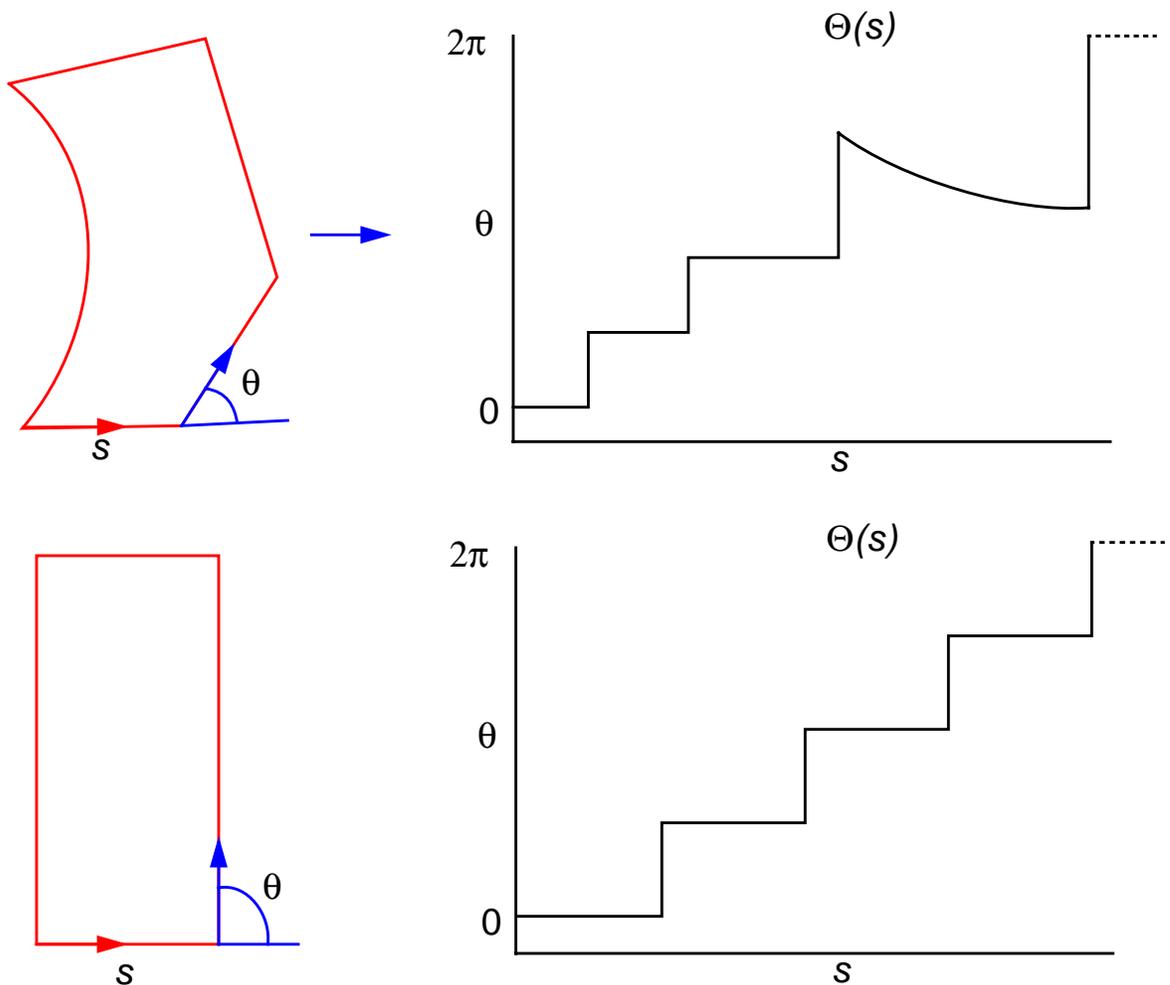
**Given a surface that resembles a rectangle,**

**choose which curves of the surface are on opposite sides.**

**Related: Is mapped-meshing appropriate for a given surface?**

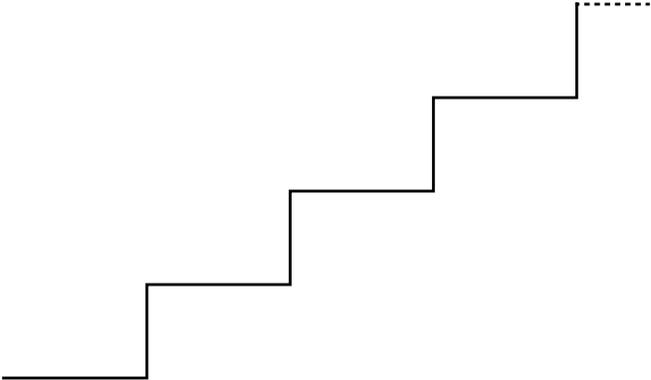
# Previous results

## Metric to determine closeness between two shapes [Arkin et al].



- For all matchings of vertices, angles
  - integrate distance<sup>2</sup> between  $\Theta(s)$ 's,
  - take smallest integral as metric

# overlay



# Previous results

## Related ideas

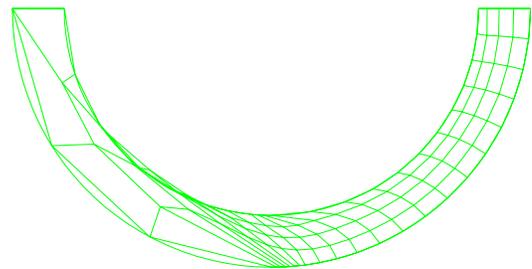
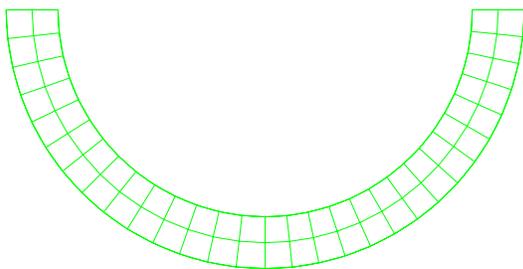
**Hausdorf metric...**

**Smallest enclosing rectangle...**

**Medial axis, skeleton...**

## Our problem

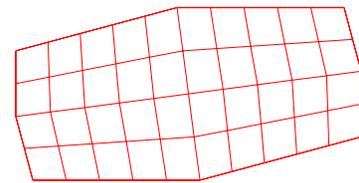
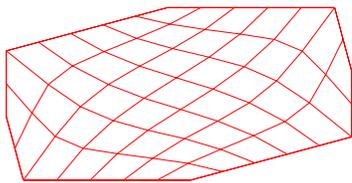
- Indeterminate rectangle height & length
- Input surface curve lengths non-Euclidean
  - some fixed, some flexible. User, previous mesh.
- angles at corners very important
  - curves could be curved, treat as straight



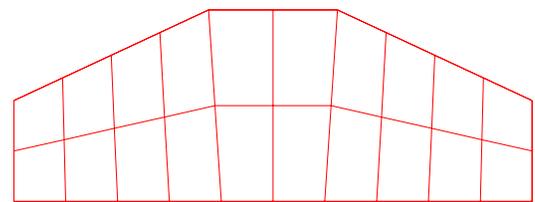
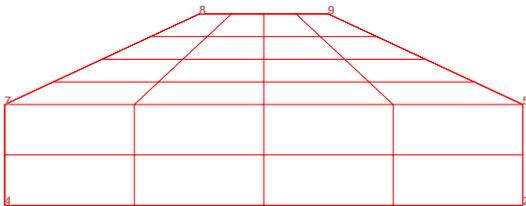
# Corner Function

## 3 Goals (angle & aspect ratio of elements)

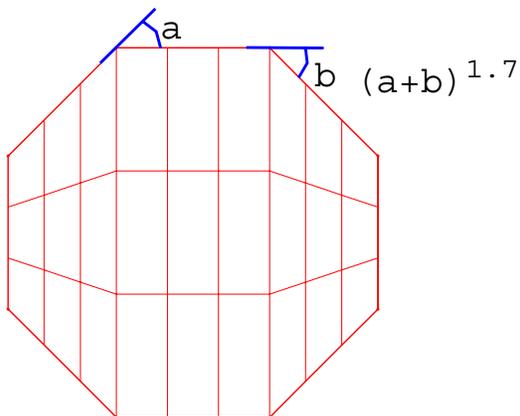
- angles at corners **small** ( $\pi/2$ ), bound away from  $\pi$ 
  - $\text{corner\_angle}^{1.7}$



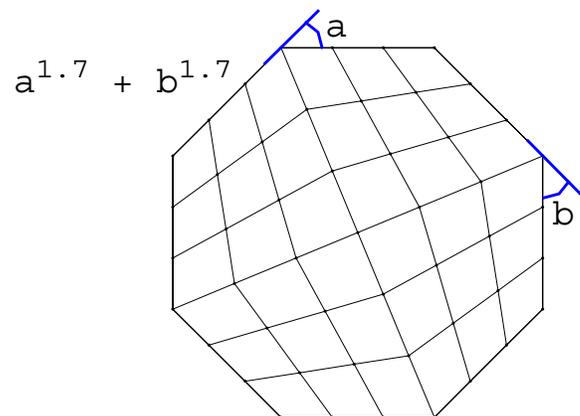
- opposite sides' **intervals equal**, relative change small
  - $\text{opp\_ratio}^{1.7} + \text{new/current}$



- no **turning** between corners, square v.s. elongated
  - $\text{turn}^{1.7}$



v.s.



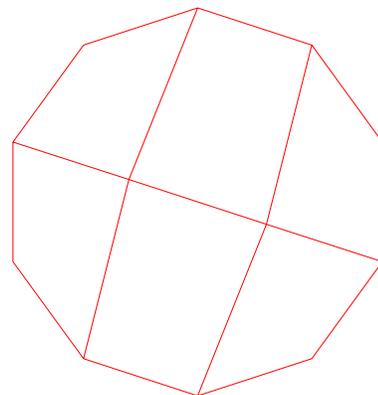
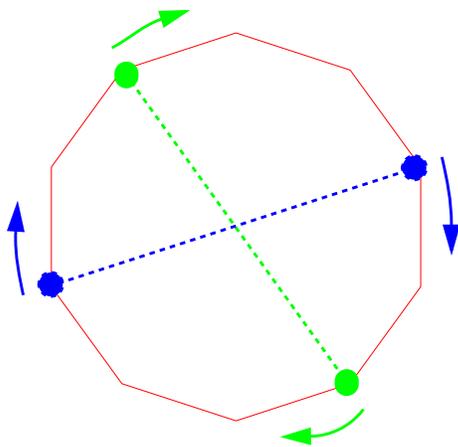
# Corner function

## For a given choice of corners, compute “corner function”

- Gives single value to desirability of given corners.
- 3 goals weighted through practice, examples.
- Doesn't consider side angles (is a rectangle?)

## Heuristic: try all possible combinations of corners. Choose smallest value.

- $O(n^4)$ ,  $n = \#$  (convex) vertices.
  - Shift corner & update value,  $O(1)$
- Done for  $n < 9$ , and for
- Post-curve meshing, corners exactly opposite,  $O(m^2)$ .



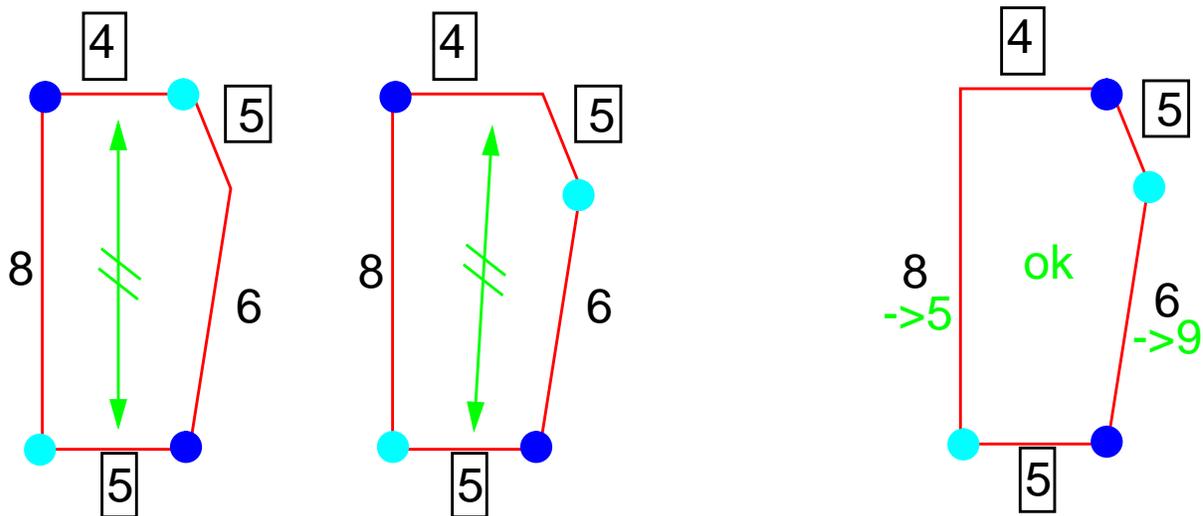
# Faster Heuristic

## Faster heuristic, $O(n^2)$ , for $n > 9$

- pick opposite corners, best 2-corner function
  - different goal on turning ( $\pi/2$ )
  - bisects, consider “best-remaining” corners
- pick remaining 2 corners, best 4-corner function

## Occasional failure

- hard-sets  $\rightarrow$  interval-matching infeasible



- If failure (hard-sets)
  - provably get feasible corners (if any)
  - heuristically shift, singly & in pairs

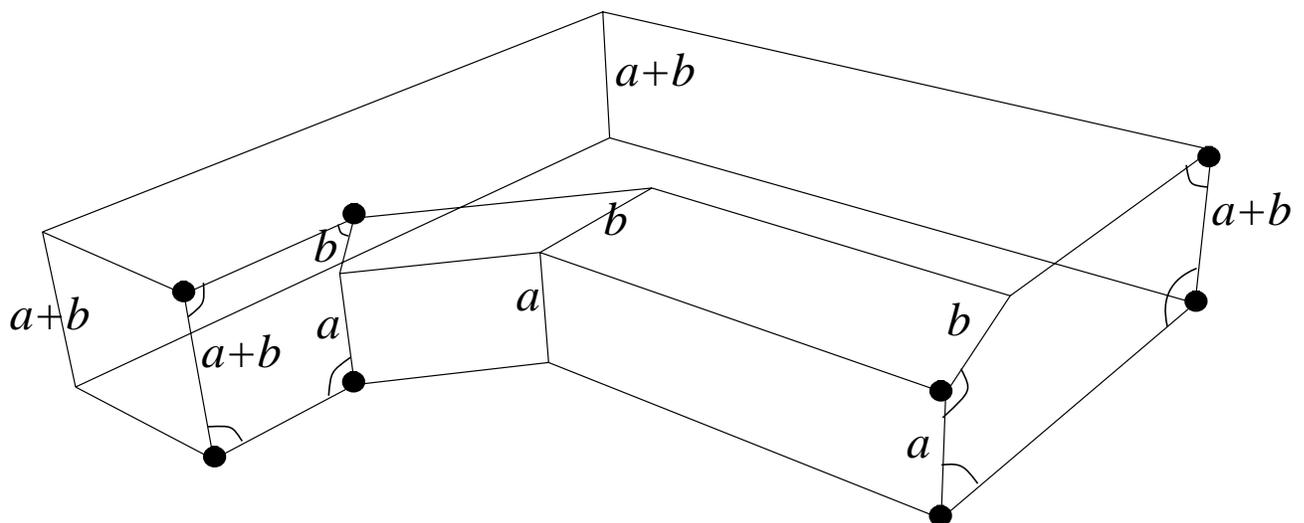
# Infeasibility

## For one surface, from hard-sets

- provable fallbacks

## For collection of surfaces,

- open, NP-complete (Mohring et al)



$$a = a + b, b = 0, \times$$

# Provable algorithm one surface



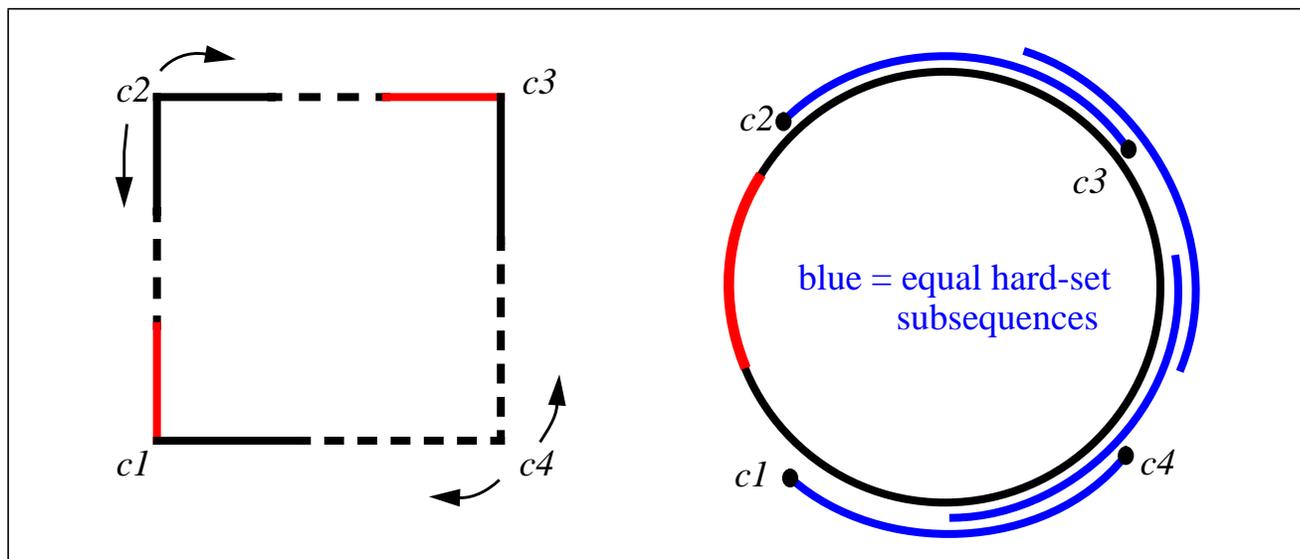
## case 3+ soft-set curves, feasible



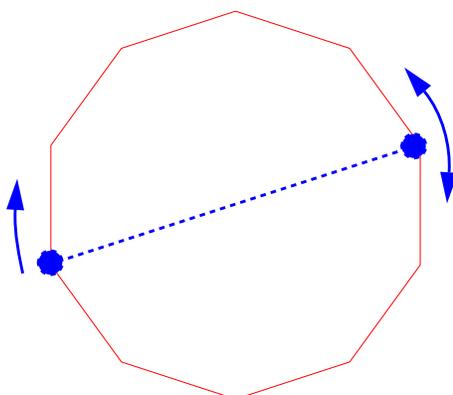
red = soft, variable > 0  
black = hard, fixed

case 2 soft-set curves

case 1 soft-set curve



## case 0 soft-set curves



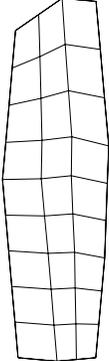
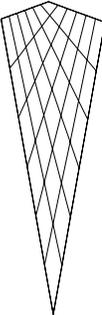
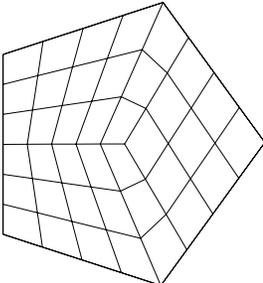
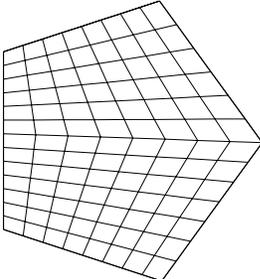
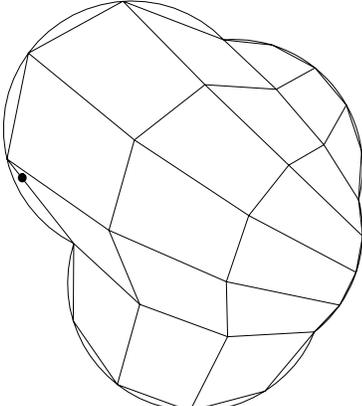
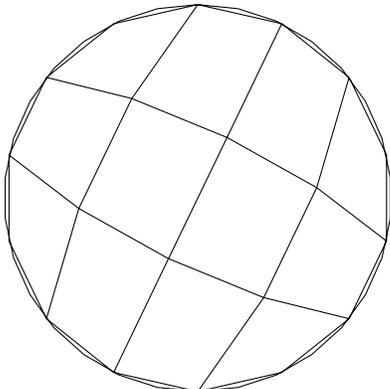
any two pairs that exactly  
divide

like post-curve meshing  
not-quite lock -step

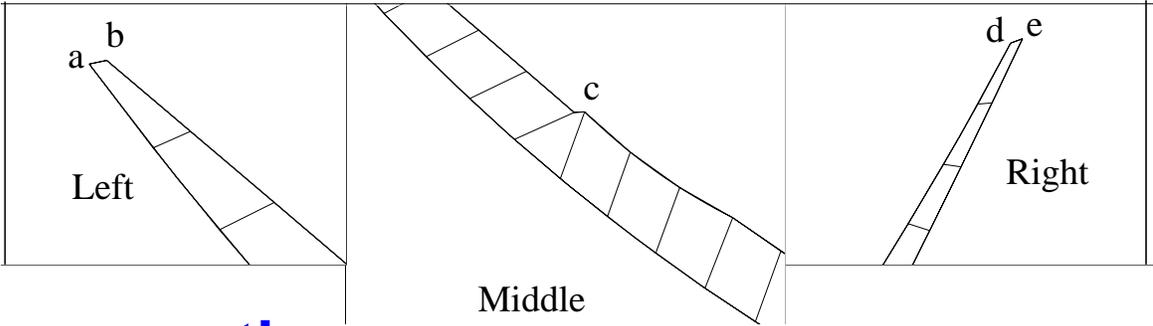
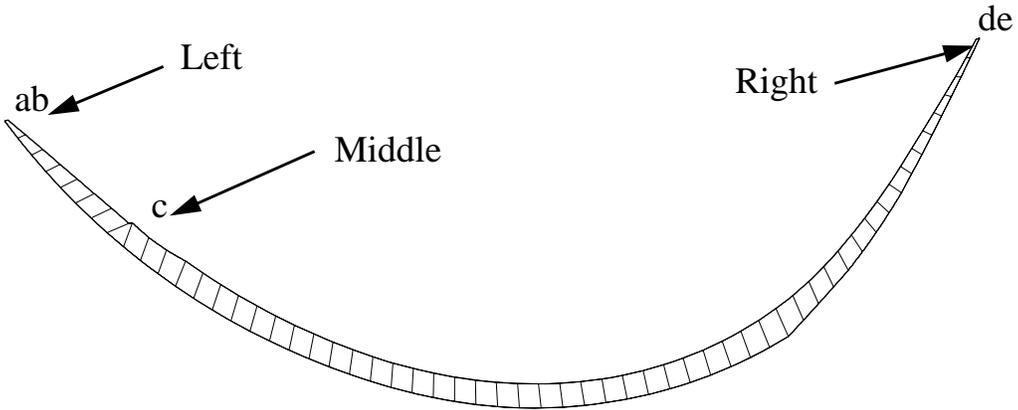
but heuristic always finds

# Examples

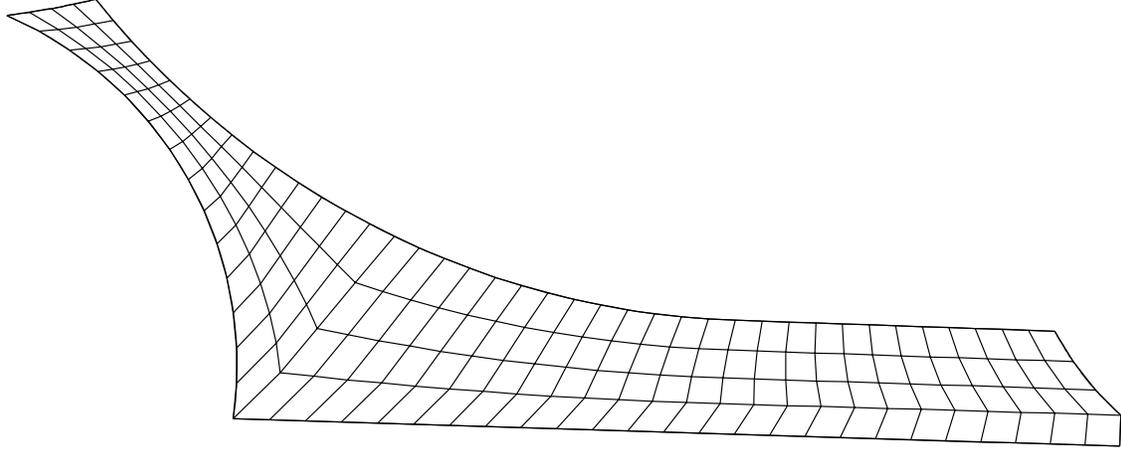
## Choosing corners after curve-meshing needed.



# GoodYear Tire Examples

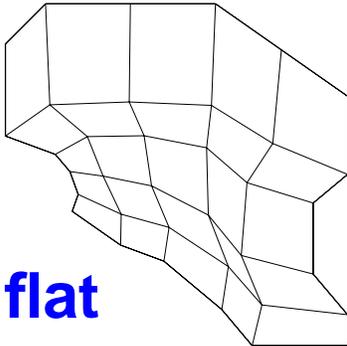
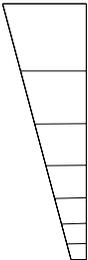
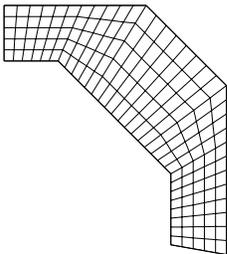
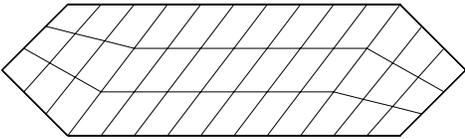
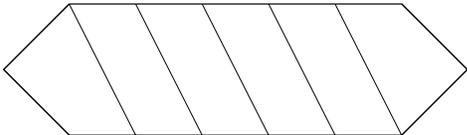


## Cross section

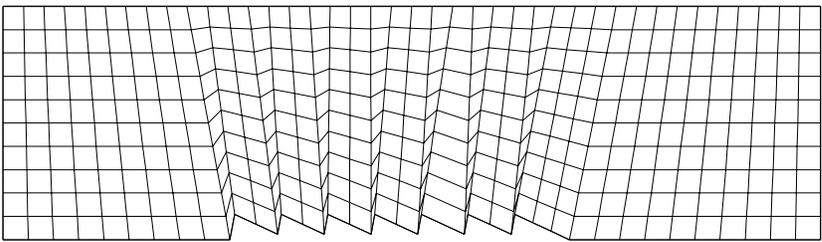
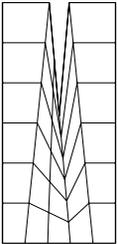


## Tread

# Examples



**flat**



**reflex  
bad,  
align**

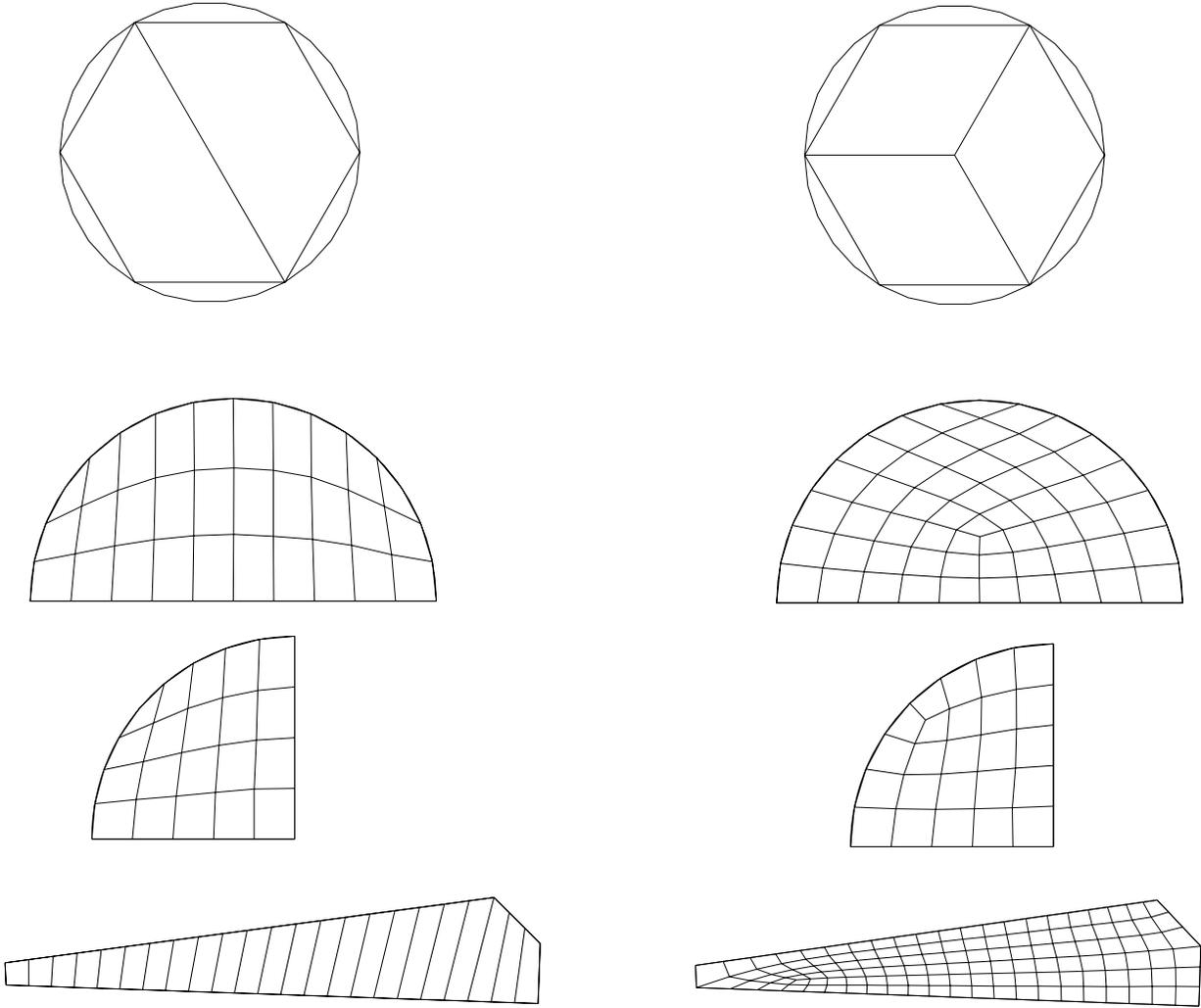
# Examples, Rectangle & Triangle



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## Rectangle

## Triangle



### Triangle intervals less constrained:

- sum-even, + triangle inequality

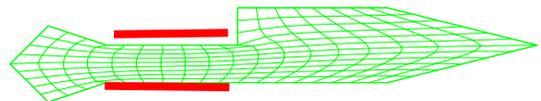
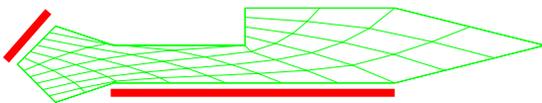
# Open Problems

## Is the polygon close to a rectangle?

- White, add reflex angle considerations

## Choosing “corners” for rectilinear shapes

- Vavasis & Driscoll, CRDT
- Long skinny features important. Skeleton?



## Chose corners and intervals together

- Mohring et al.
  - NP-complete for some set of primitives
  - Network flow instead of LP formulation
- Volume meshing constraints

# Conclusions

**Heuristic,  $O(n^2)$ , hard-sets difficult**

**Provable algorithms, heuristic shift**

**Extended to triangles (not immediate)**

**Problems with coarse meshes of curves**

- if  $n < 4$ , assume corner mid-curve,
- else always treat curves as straight

