Multiscale spatial models for representing anthropogenic CO_2 emissions

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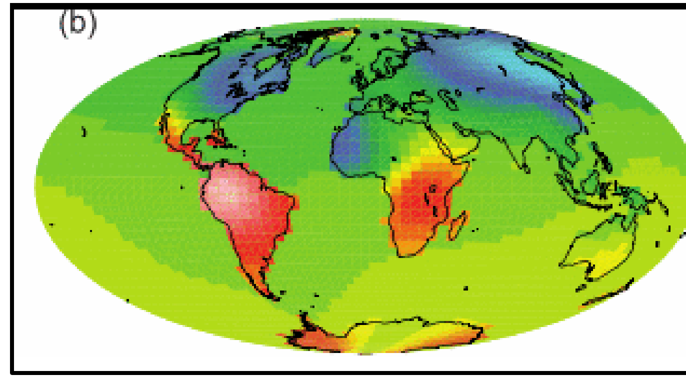
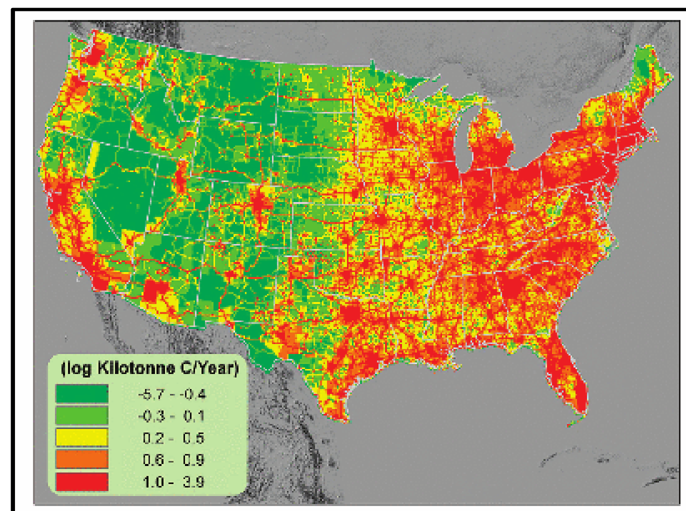
Problem statement

- Aim: To develop spatial models that can capture the spatial variability of anthropogenic (fossil fuel) CO_2 emissions
- Motivation
  - Bottom-up estimates of FF emissions are often compared against top-down estimates (inversions)
  - If one desires some degree of spatial resolution, one needs a spatial model
  - Gaussian Process models will not work
    - Used for biogenic emissions
  - So what's a model for FF emissions?



Biogenic emissions: Mueller et al, *JGR*, 2008



Anthropogenic emissions: Gurney et al, *EST*, 2009

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Properties desired of the spatial model

- We plan to use the spatial model in a statistical inversion
  - To develop PDFs of the inferred FF emissions
  - Will use a method like EnKF
- The spatial model needs to be
  - Low dimensional (few parameters to be inferred from limited data)
  - If not possible, the model should be sparse
  - Joint (prior) PDFs between model parameters would be helpful
- But what are these spatial models?
  - *Kernel models:* "basis functions" based on some easily observed auxiliary variable
    - E.g. GDP, population density, nightlights (Rayner et al, JGR, 2010; Oda & Maksyutov, ACP, 2011)
  - *Wavelet models:* A set of orthogonal polynomials, capable of capturing non-stationary behavior

Constructing a Gaussian kernel representation

- Was used for CO_2 inversion in the first talk
- Hypothesis:
  - FF emissions can be represented as a set of Gaussian kernels
    - Each kernel represents the spatial variation of emission in each "location"
  - Amplitude of each kernel to be inferred during the inversion
    - The shape of the Gaussian kernel (covariance of the bivariate Gaussian) is determined using DMSP-OLS nightlight image
- Questions
  - Given an upper limit of *M* kernels, how much of the nightlight can you capture?
    - Is this a function of the resolution at which the nightlights are measured?
  - What does the spatial coverage of kernels look like, *wrt* truth?

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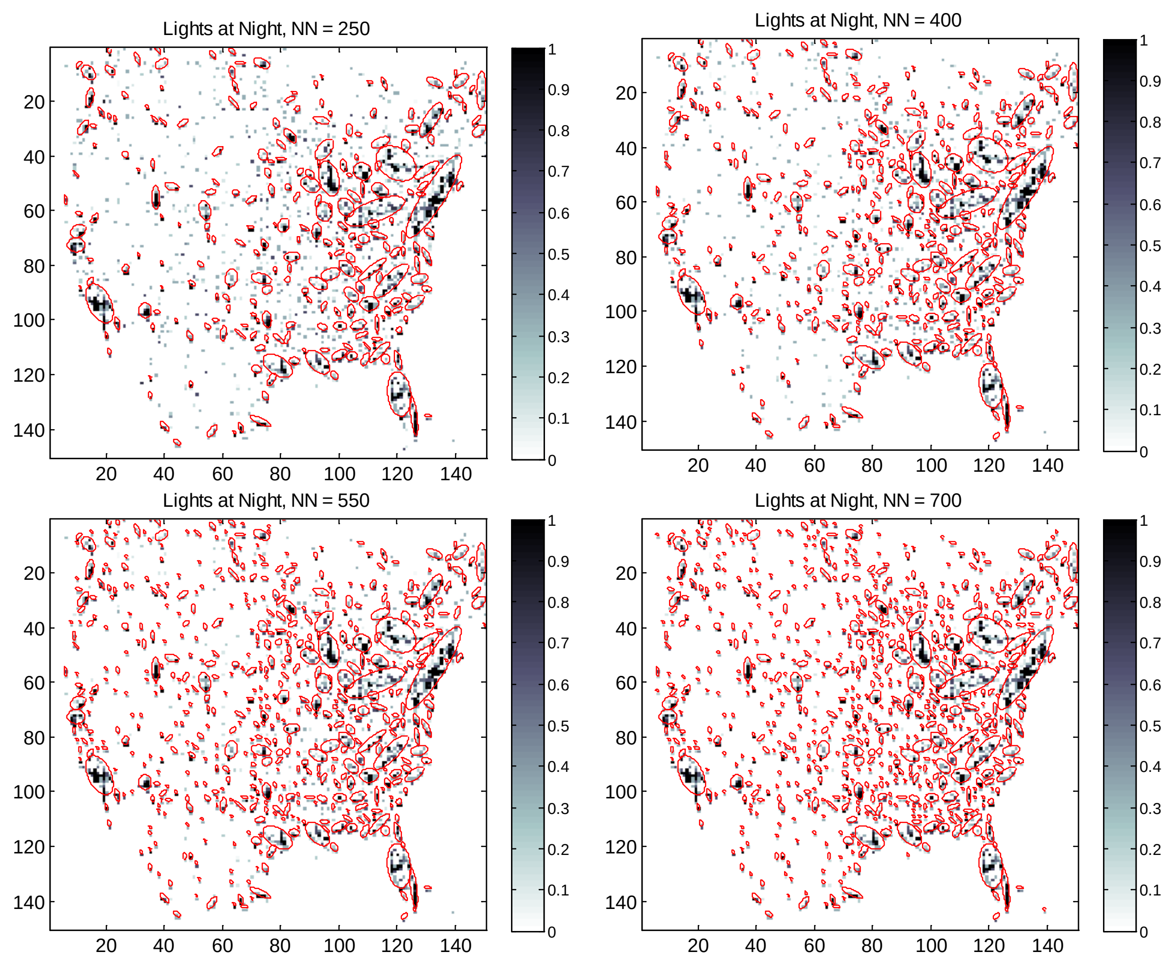
Lights at night: 150x150 Image

- Procedure
  - Threshold the nightlight intensity
  - Outline the thresholded clusters (red)
  - Fit an ellipse to the fluxes in cluster
    - Obtain the covariance needed for the kernel model
  - Place a Gaussian at the ellipse COM



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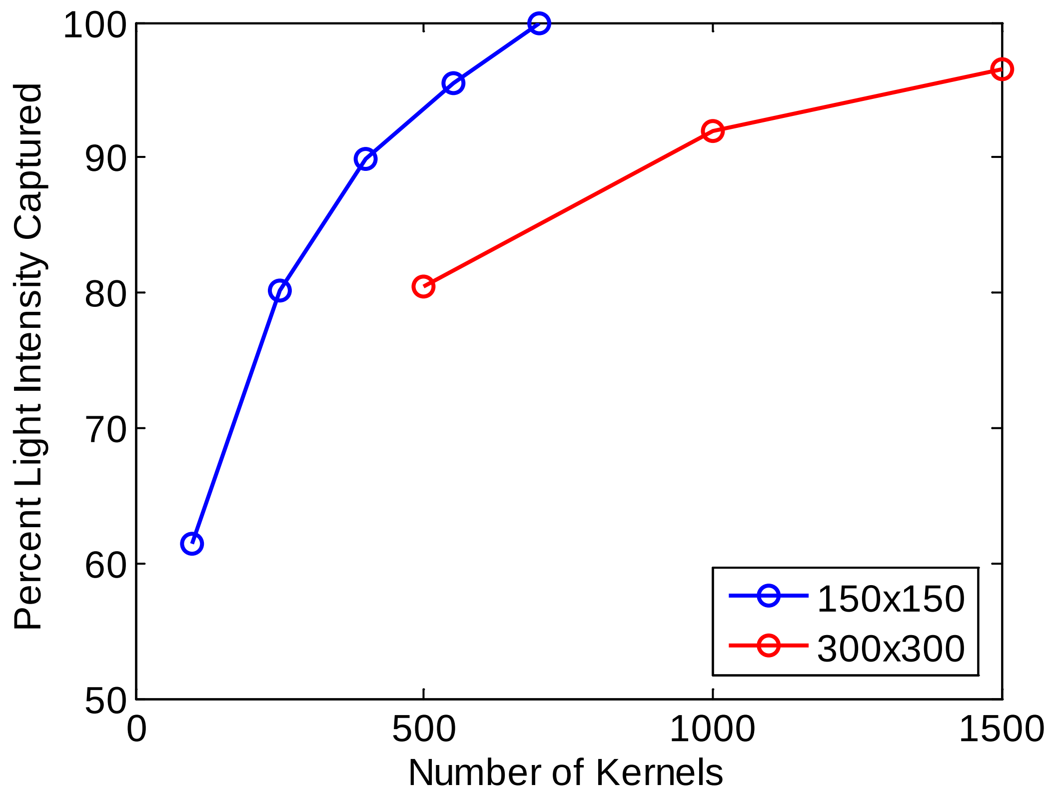
How much of the intensity did we capture?

Blue: 150x150 pixel nightlight image

Red : 300x300 pixel nightlight image



- The intensity capture as a function of *M* kernels depends on the resolution of the nightlight image
- About 1000 kernels does it
  - not very high dimensional for EnKF

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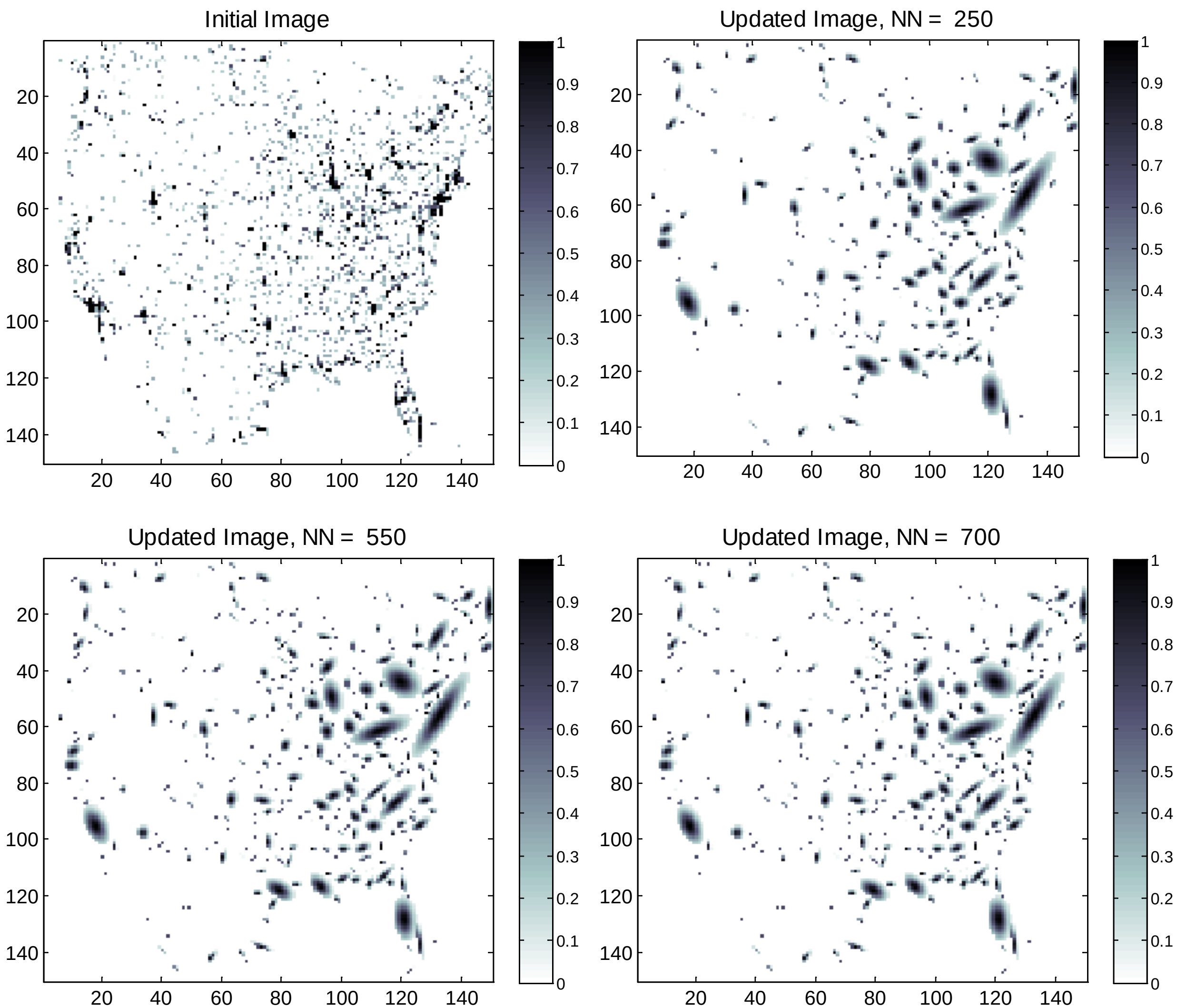
What is the spatial coverage of the kernels?



- Kernel amplitudes have been set to 1.0 arbitrarily
  - We're just checking spatial coverage
- 150x150 nightlight images were used to derive the kernels
- About 250-400 kernels may be OK

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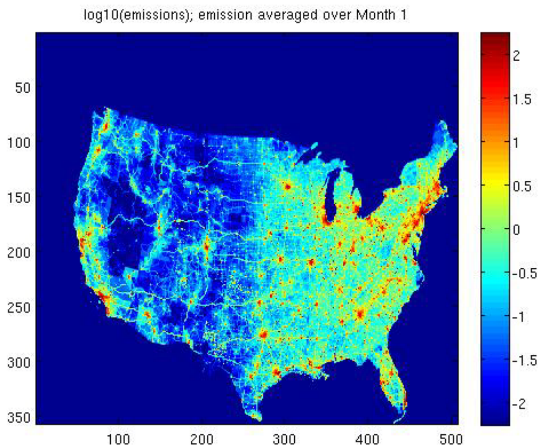
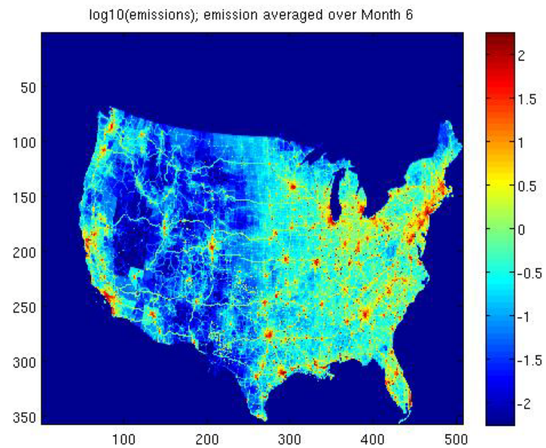
Constructing a wavelet spatial model

- Hypothesis: A wavelet basis will efficiently capture FF emissions
- Ramification:
  - Wavelet models are not low-dimensional, a priori
  - We hope that they may be sparse
- Procedure to construct such a model
  - Start with Vulcan (Gurney et al, *EST*, 2009)
  - Subject the FF emissions to wavelet decomposition using
    - Wavelets from a number of families and a number of orders
    - Choose the sparsest, simplest representation
  - Model the correlation between wavelet coefficients at different levels/scales/resolution
    - This will be used to constrain them when the spatial model is used in an inversion

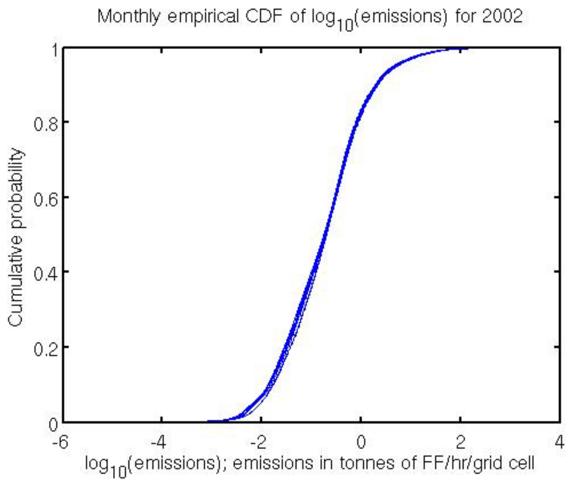
How do emissions vary month to month?



January, 2002



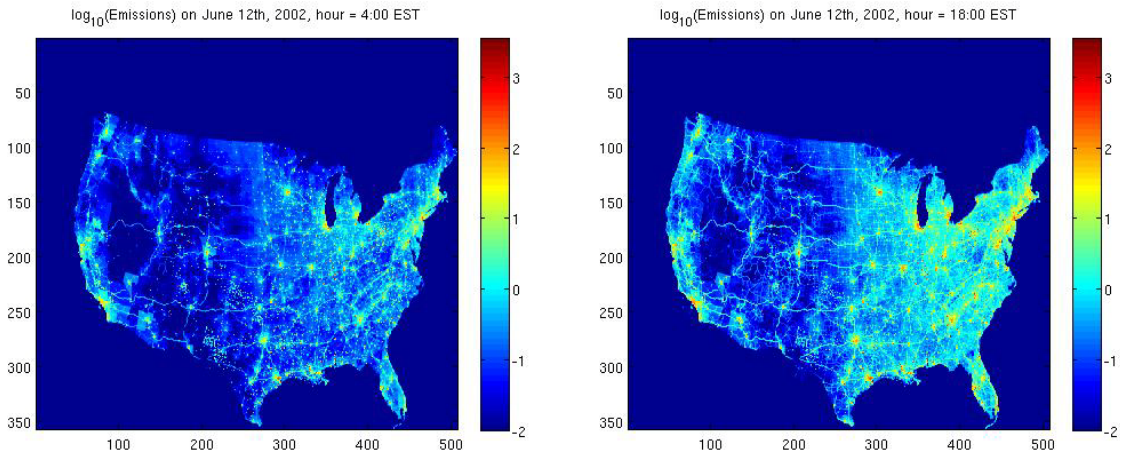
June, 2002

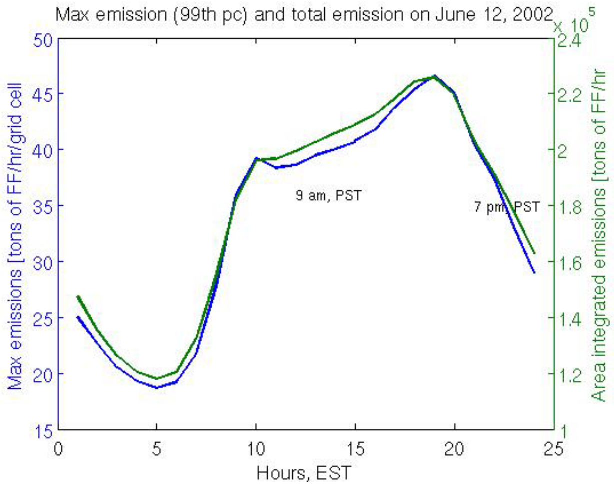


CDF of emissions for all 12 months

- Procedure
  - Read the hourly emissions from Vulcan; average them over a month
    - tons of FF emissions / hr / gridcell (10km x 10km)
- Result: same spatial models for January & June may work
  - Different model parameters, though

How do emissions change diurnally?





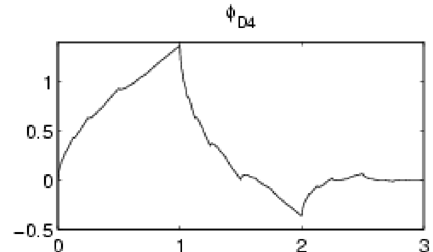
4 am, EST

6pm, EST

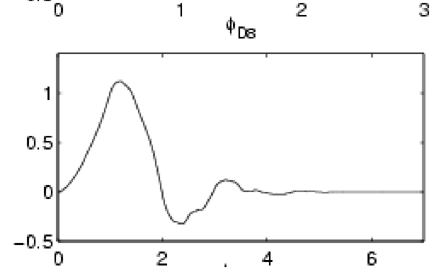
- Emissions for January 30, 2002
- Significant diurnal changes (in magnitude)
  - See eastern half for brightness
- Spatial distribution changes are on a small scale
  - No change in non-stationary nature – reweighting of a wavelet model may suffice

How does one represent emissions with wavelets?

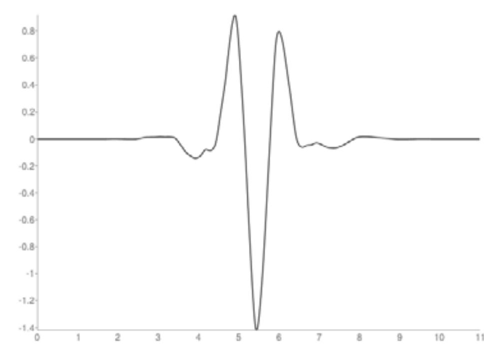
- Propose $$\boldsymbol{E(x)} = \sum_{s,l} w_{s,l}\phi_{s,l}(x)$$
  - $\phi_{s,l}(x)$ is a wavelet basis; *s, l* are its *scale* and *location* indices
  - $w_{s,l}$ are weights
- So what are wavelets?
  - Basis set with compact support
  - Belong to different families
  - Within a family, can have different orders (high order ~ smoother)
  - One chooses a family and an order, to expand E(x)
  - The expansion consists of varying
    - s, to get different frequency content
    - l, to shift in space (location)



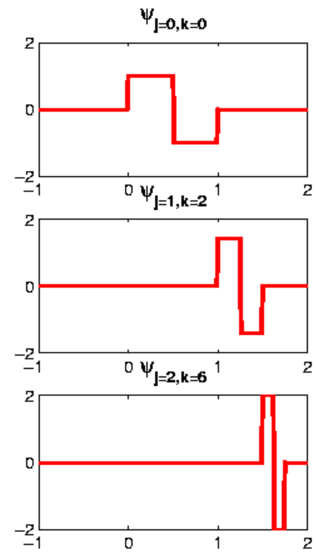
Daubechies, order 4



Daubechies, order 6



Symlet, order 6



Haars at different scales and locations

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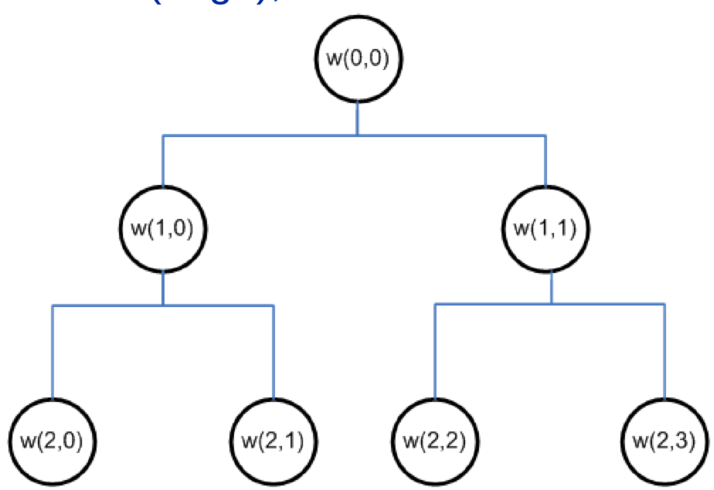
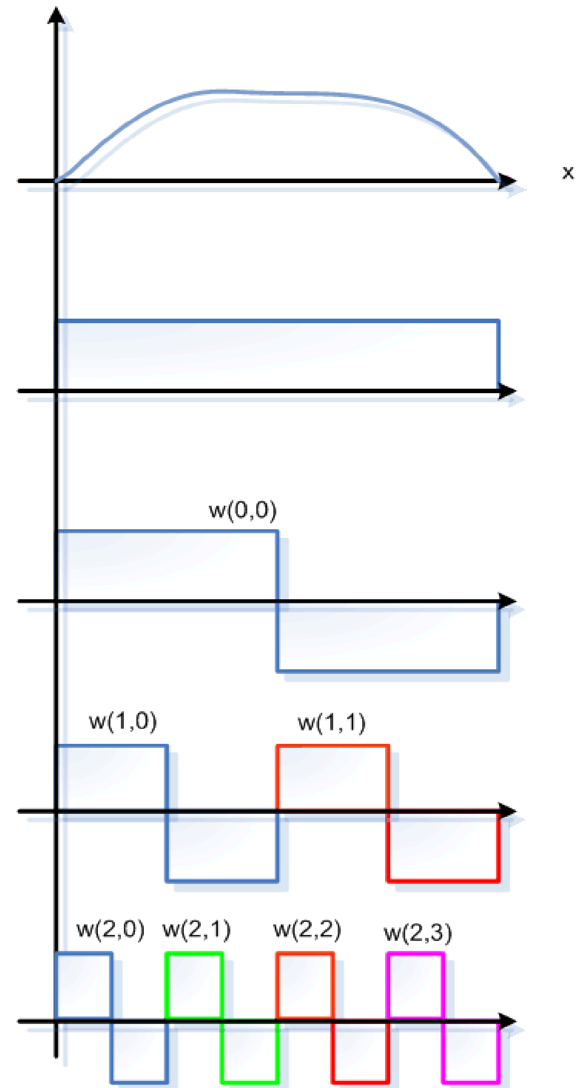
A wavelet decomposition of a field

- Decompose a field $E = \Phi w$ on wavelet bases
  - $w_{s,l}$ form a binary tree (quad-tree in 2D)
- If a field E is smooth, $w_{s,l}$, at large s are all small
  - So chop the binary tree; drastically reduce the elements in w
- If a field s is smooth, but with edges or splotchy structure
  - $w_{s,l}$ mostly zero, but some $w_{s,l}$ at large s may be non-zero
  - Sometimes if a $w_{s,l}$ is small (large), it's children are small (large) too





Posing the problem

- An emission field on 2^N x 2^N pixels
  - Can be decomposed on a wavelet basis, N deep
  - Each level s has 2^s x 2^s – (2^{s-1} x 2^{s-1}) weights
- Emissions

$$\mathbf{E}(\mathbf{x}) = \sum_{\mathbf{s}=1}^{\mathbf{N}} \sum_{\mathbf{i}=1}^{2^{\mathbf{s}}} \sum_{\mathbf{j}=1}^{2^{\mathbf{s}}} \mathbf{w}_{\mathbf{s},\mathbf{i},\mathbf{j}} \phi_{\mathbf{s},\mathbf{i},\mathbf{j}}(\mathbf{x})$$

- Conjecture
  - $w_{s,i,j}$ are mostly zero (i.e., is sparse)
  - $w_{s,i,j}$ and $w_{s+1,i,j}$ are correlated – parent-child relationship
- Procedure
  - Pick June 2002 from Vulcan
  - Subject them to a wavelet transform
    - Results in 9 levels
    - Try Daubechies, Symlet, Coiflet, ….. of many orders
  - Check the above conjectures

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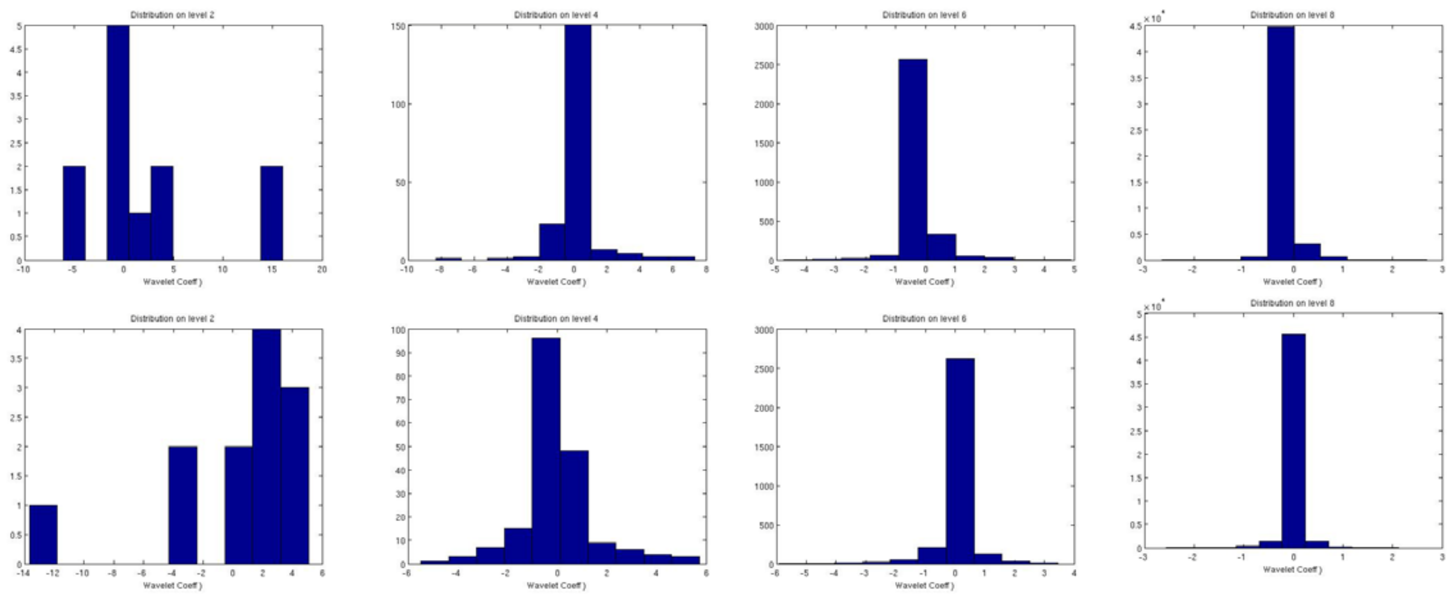
Is the wavelet distribution sparse?

- What is the distribution of *w* on each of the levels?

Haars (order = 2)

Symmlet (order = 6)



Level = 2 | Level = 4 | Level = 6 | Level = 8

- Sparse distribution, regardless of the wavelet chosen

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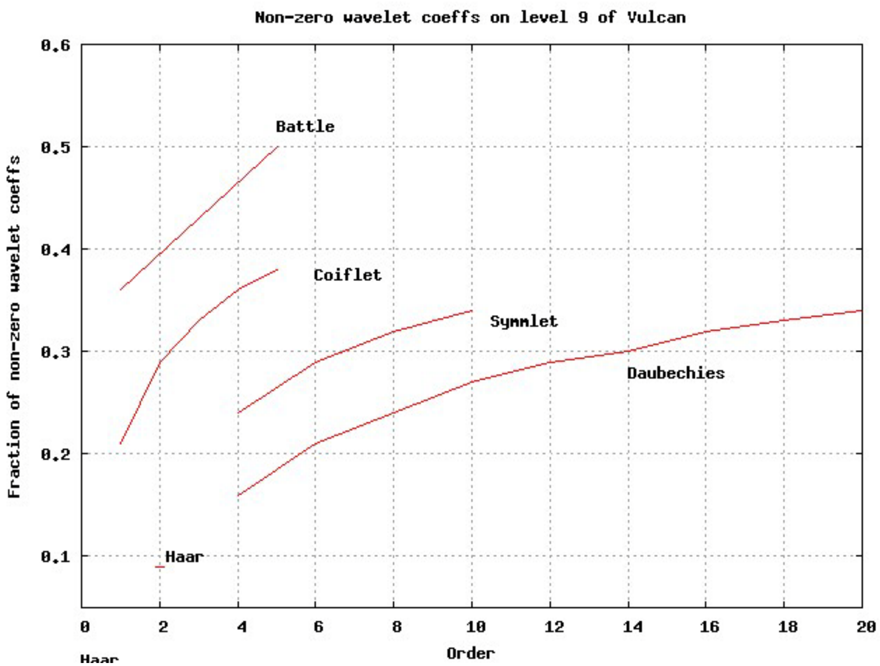
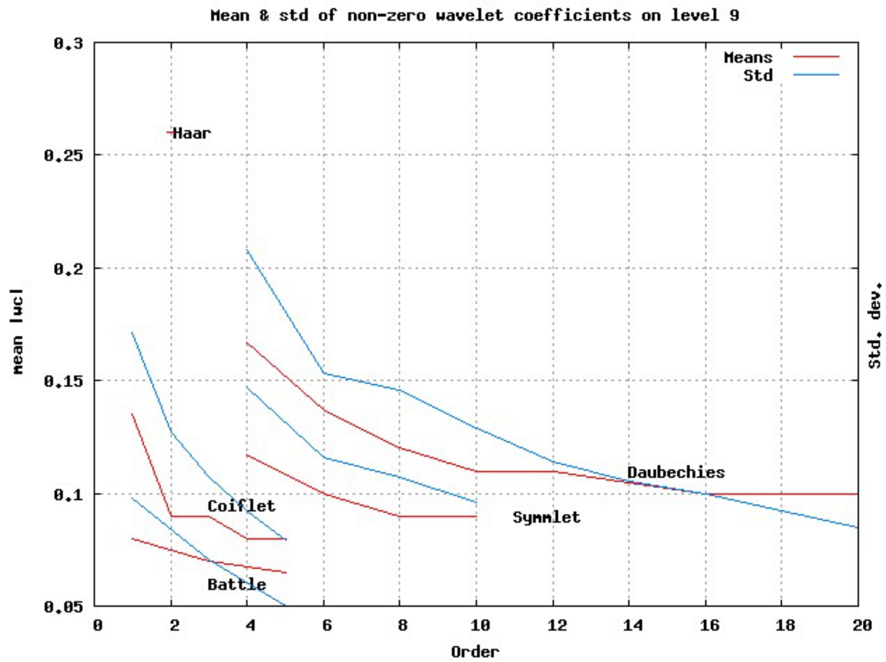
Wavelet coefficients and order at s = 9





- Sparsity of wavelet coefficients is good
  - Having a few large wavelet coefficients is bad – can't ignore them
- As order increases
  - Number of non-zero coefficients goes up i.e., sparsity decreases
  - There are no large coefficients

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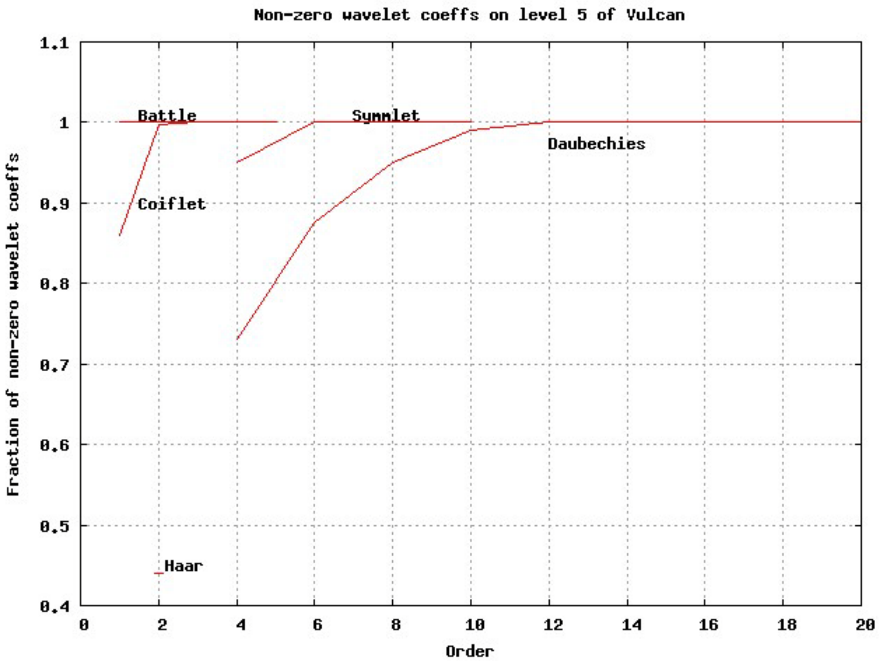
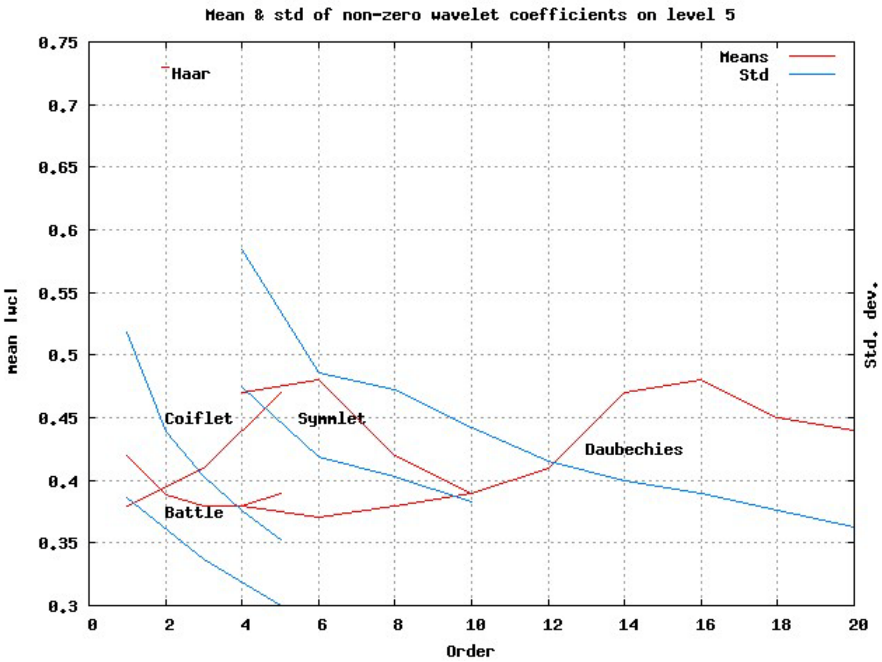
Wavelet coefficients and order at s = 5



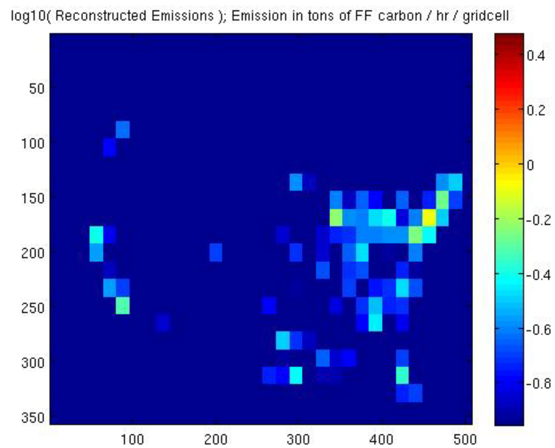


- As order increases
  - Sparsity plummets- but few non-zero coefficients are that large
- Live with low order wavelets like Haars?
  - Most coefficients on level 6 and above are small – set them to zero, or model them

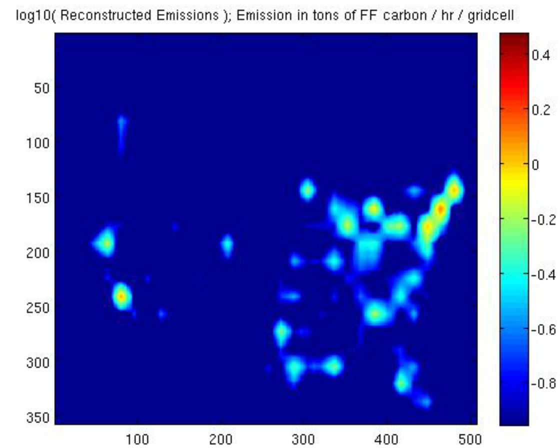
How important are the wavelets at high s?

- Choose 2 wavelet families
  - Haars: simplest possible, most wavelet coefficients beyond level 4 are zero, but the non-zero ones are big
    - Ignoring the non-zero ones will make a big mistake (may be)
  - Symlet: a sophisticated, high-order wavelet. All wavelet coefficients beyond level 4 are small, but aren't non-zero
- Procedure
  - Take June 2002 emissions from Vulcan database
  - Do a wavelet decomposition
  - Set all wavelet coefficients on s = {6, 7, 8, 9} to zero. Reconstruct
  - See difference
- There are $2^5 \times 2^5$ non-zero wavelet coefficients

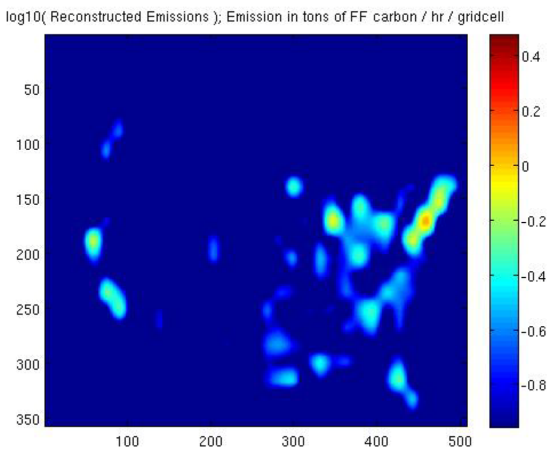
Reconstructed fields, ignoring high s



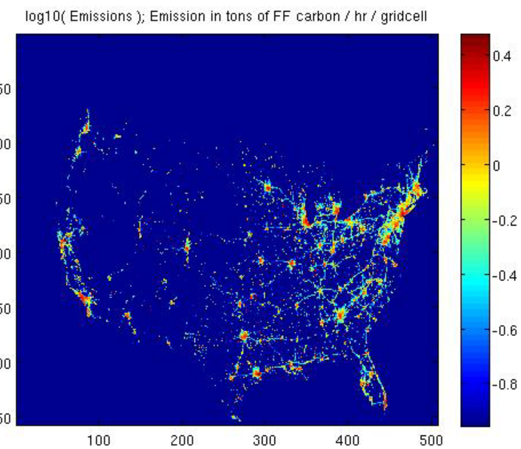
Haars



Daubechies 6



Symlet 6



Original

- Lessons learnt
- Haars and the more complicated wavelets don’t have very different distributions – may be smoother, and little more structure
- If you consider that we will be working with sparse data, the choice of wavelets may not matter at all
- Sophisticated wavelets (symlet) may not give better answers

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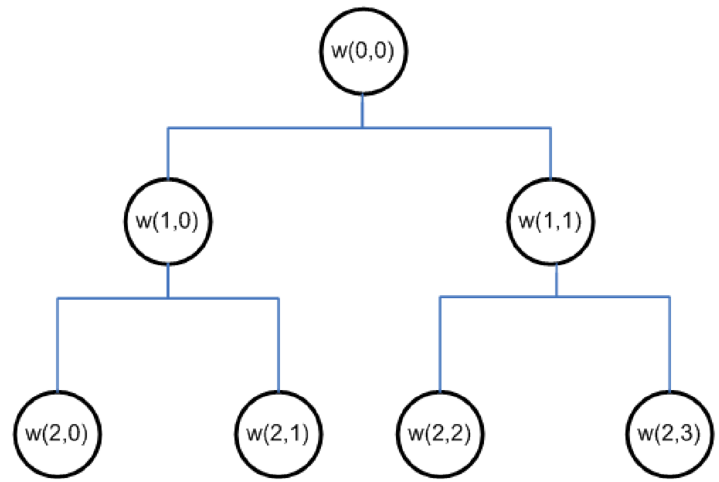
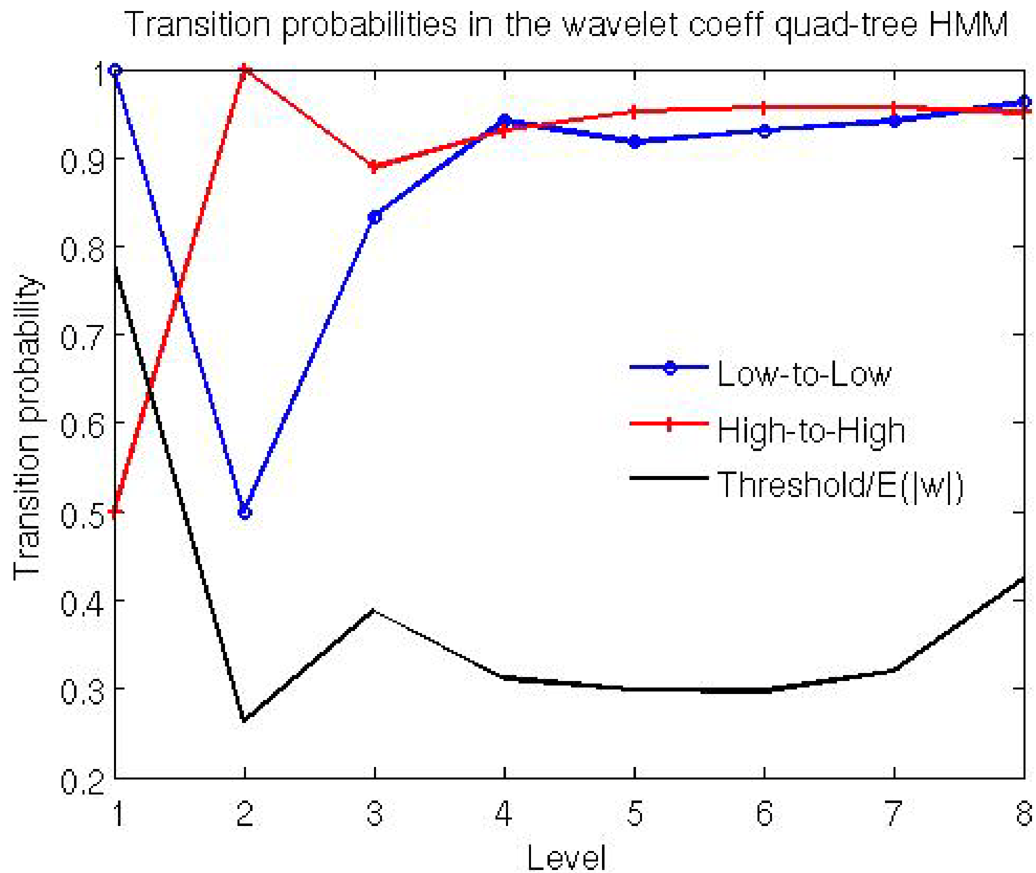
Walking down the quad-tree

- Are there inter-level correlations?
  - Can result in parent-child correlations
  - Or at least a HMM on low/high weight classification
- Consider the average (μ_s) and standard deviation σ_s of the non-zero wavelet coefficients on level 's'
  - If $|w_{s,l}| < (\mu_s - 0.5\ \sigma_s)$ consider it a "low-valued" coefficient
- If I am a high-valued coefficient, is my child high-valued too?
  - Does my child modify the signal structure significantly?
  - Compute P(high -> high) transition between me and my children
- If I am a low-valued coefficient, is my child low-valued too?
  - If I get ignored, can you ignore the sub-tree rooted at me?
  - Compute P(low -> low) transition between me and my children



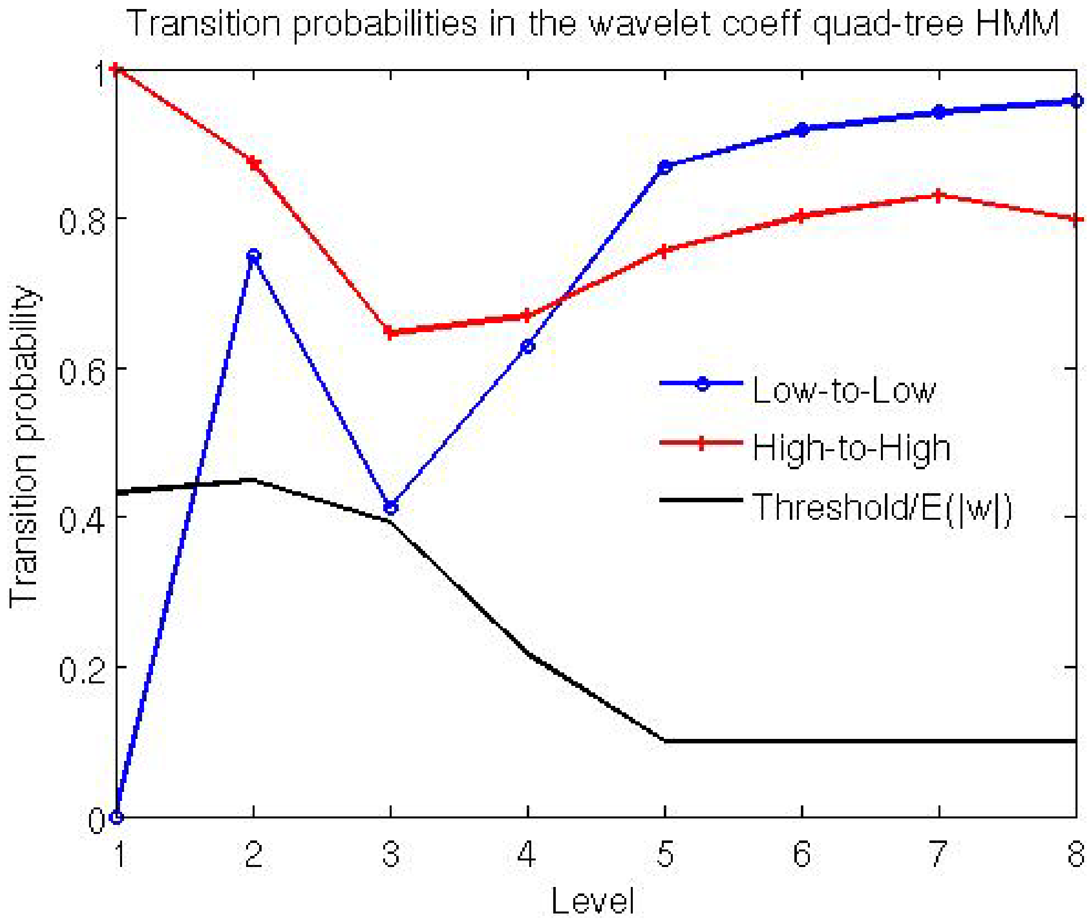
P(L->L) and P(H->H) for Haars

- Persistence behavior beyond level 3
  - High wavelet coefficients have high children, low wavelet coefficients have low children (90% transition probability)
  - High/low threshold as a ratio to mean (non-zero) wavelet coefficient is about 0.3
  - Could lead to a simple HMM between levels



P(L->L) and P(H->H) for symlet 6

- With symlet 6 (a high-order sophisticated wavelet) transition probabilities are a lot more variable
- A hard one to model with an HMM



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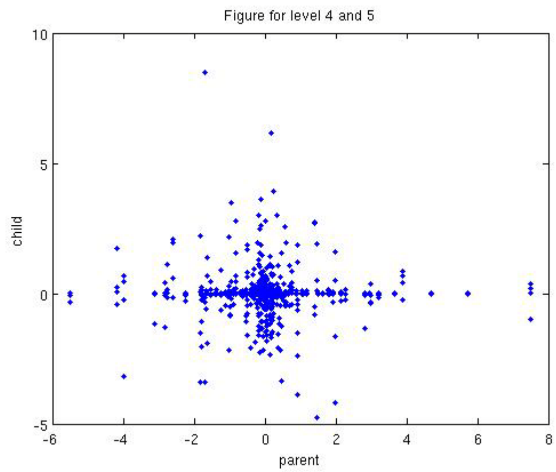
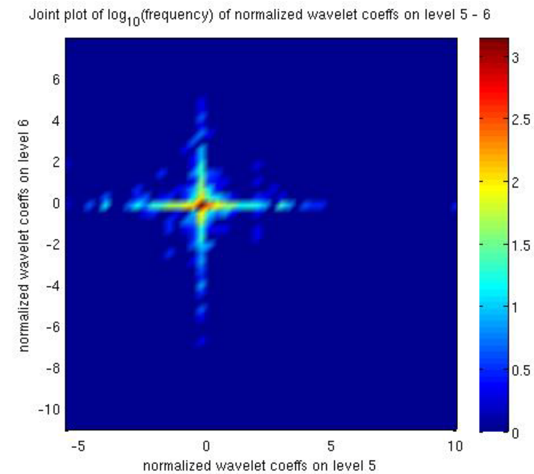
Are there correlations between levels?

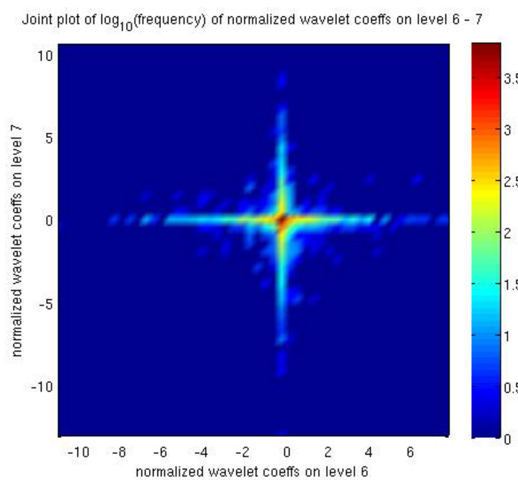
- Is there a correlation between wavelet coefficients on level '*s*' and '*s+1*'?
  - Not a simple one
  - Star-shaped distribution is often modeled as a "shrinkage prior" called the "horseshoe prior"



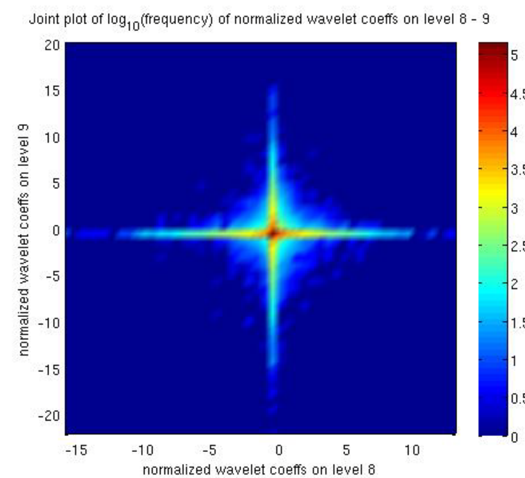
Level 4 and 5



Level 5 and 6



Level 6 and 7



Level 8 and 9

Summary

- Both Gaussian kernels and Haars may be used to capture FF emissions
  - Inversion using Gaussian kernels has been attempted (van BloemenWaanders, yesterday)
  - Wavelet modeling & inversion a lot harder
- Future work
  - Do an EnKF inversion with wavelets
    - Priors on weights given from nightlights, GDP, population density
    - Basically looking to see which wavelets to permanently set to zero
  - Compare and seek to answer these questions:
    - Do we need to bother with fine wavelets at all?
    - How does this perform vis-à-vis Gaussian kernels?

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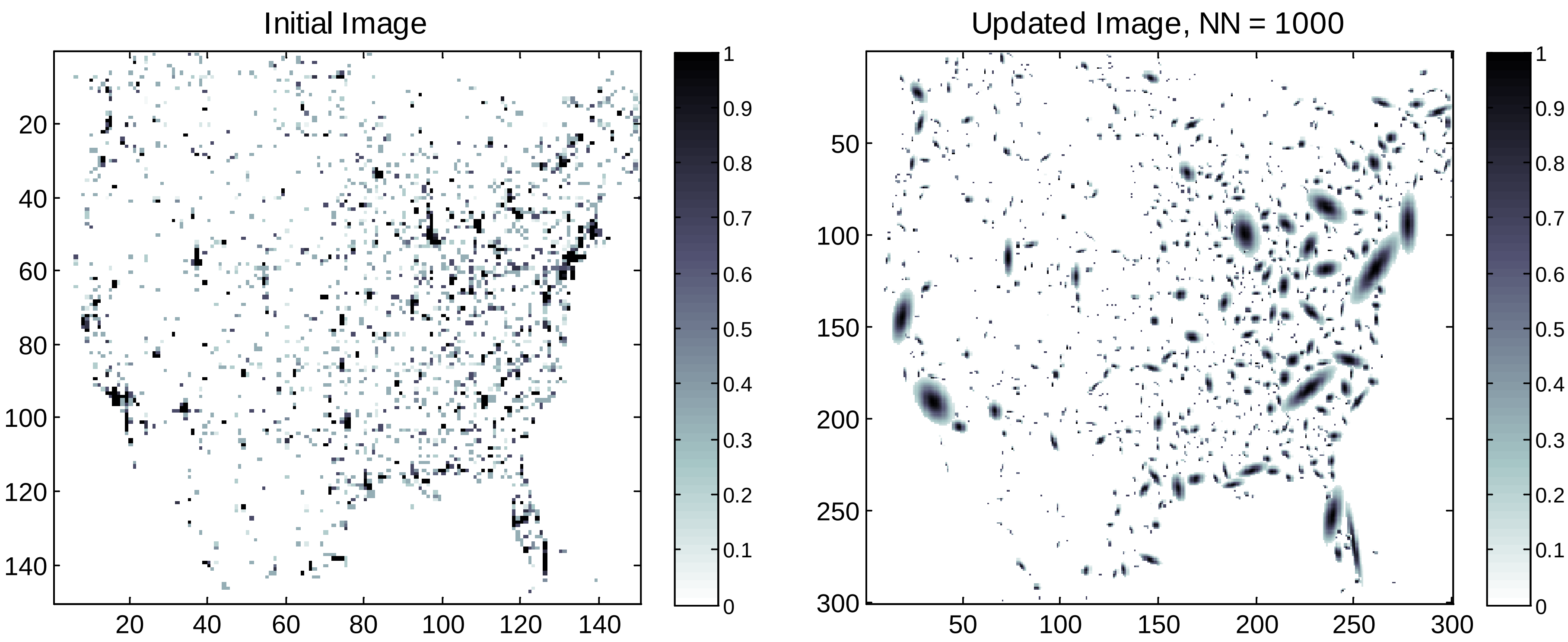
Gaussian Kernel representation



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