Recent Extensions of the Discontinuous Enrichment Method for Variable-Coefficient Advection-Diffusion Problems in the High Péclet Regime

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The discontinuous enrichment method (DEM) is among a number of different finite element approaches that have been proposed for addressing the challenge of solving the advection-diffusion equation, $-\kappa\Delta c(\mathbf{x}) + \mathbf{a}(\mathbf{x}) \cdot \nabla c(\mathbf{x}) = f(\mathbf{x})$, accurately and efficiently in the advection dominated (high Péclet) regime. The basic idea of DEM is to construct a finite element basis that is related to the operator governing the problem being solved, and therefore has a natural potential for resolving difficult features in the problem's solution, such as sharp gradients. To this effect, the approximation space in DEM is defined as the set of free-space solutions of the homogeneous form of the governing PDE, obtained in analytical form. Since the enrichment in DEM is *not* constrained to vanish at the element boundaries, continuity of the solution across element interfaces in DEM is not automatic; rather, it is enforced weakly using Lagrange multipliers.

Originally [1], DEM was developed and evaluated for the constant-coefficient advection-diffusion equation. Attention is now turned to the variable-coefficient advection-diffusion equation. It is shown that the original constant-coefficient methodology developed in [1] can be naturally extended to variable-coefficient transport problems. The enrichment functions within each element are exponentials that exhibit a steep gradient in the advection direction. These functions can be parametrized nicely with respect to a flow direction parameter so as to make possible the systematic design and implementation of DEM elements of arbitrary orders. It is shown that the approximation can be improved by augmenting the enrichment space with polynomial free-space solutions to the constant-coefficient advection-diffusion equation, as well as a "higher-order" enrichment function that solves the advection-diffusion equation with $\mathbf{a}(\mathbf{x})$ linearized to first order. Numerical results for several benchmark problems with a spatially-varying $\mathbf{a}(\mathbf{x})$ reveal that these new DEM elements outperform their Galerkin counterparts of comparable computational complexity by a large margin. This suggests a serious potential of DEM for the solution of realistic advection-dominated transport problems in various fluid mechanics applications.

References

[1] I. Kalashnikova, C. Farhat, R. Tezaur. "A Discontinuous Enrichment Method for the Solution of Advection-Diffusion Problems in High Péclet Number Regimes". *Fin. El. Anal. Des.* **45** (2009) 238–250.