Modeling Dynamic Fracture with Peridynamics, Finite Element Modeling, and Contact

11th US National Congress on Computational Mechanics

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What is Peridynamics?

- Peridynamics is a *nonlocal* extension of continuum mechanics
- Balance of linear momentum is based on an *integral equation*
- Capable of modeling bodies in which discontinuities occur spontaneously
- Well suited for *fracture modeling*

S.A. Silling. Reformulation of elasticity theory for discontinuities and long-range forces. *Journal of the Mechanics and Physics of Solids*, 48(1):175-209, 2000.

S. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari. Peridynamic states and constitutive modeling. *Journal of Elasticity*, 88(2):151-184, 2007.

S.A. Silling and R.B. Lehoucq. Peridynamic theory of solid mechanics. SAND Report 2010-1233J, Sandia National Laboratories, Albuquerque, NM, 2010.

David J. Littlewood. Simulation of dynamic fracture using peridynamics, finite element modeling, and contact. Proceedings of the ASME 2010 International Mechanical Engineering Congress and Exposition. Vancouver, Canada, 2010.

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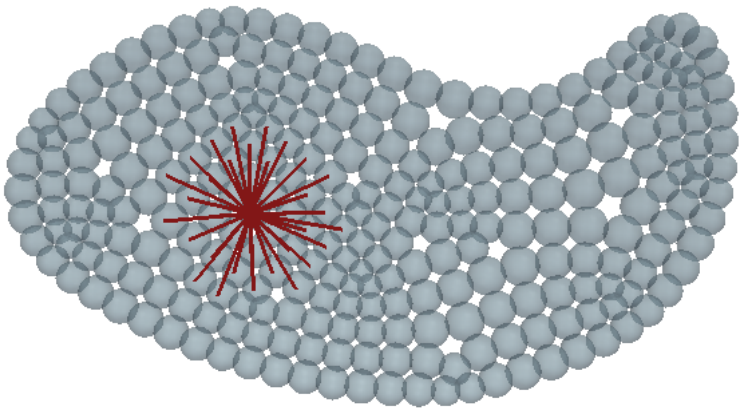
Peridynamics is a Nonlocal Extension of Continuum Mechanics

Balance of linear momentum formulated as an *integral* equation

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{\mathscr{B}} \left\{ \underline{\mathbf{T}}[\mathbf{x},t]\,\langle \mathbf{x}' - \mathbf{x} \rangle - \underline{\mathbf{T}}'[\mathbf{x}',t]\,\langle \mathbf{x} - \mathbf{x}' \rangle \right\} \, \mathrm{d}V_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t)$$

A *bond* connects any two elements that interact directly

Pairwise forces are determined by a force state acting on a bond



$$\underbrace{\underline{\mathbf{T}}[\mathbf{x},t]}_{\text{Force State}} \quad \underbrace{\langle \mathbf{x}_i' - \mathbf{x} \rangle}_{\text{Bond}}$$

Adaptation of Classical Material Models for Peridynamics

- Approximate deformation gradient based on positions and deformations of all elements in the neighborhood

Approximate Deformation Gradient

$$\bar{\mathbf{F}} = \left(\sum_{i=0}^{N} \underline{\omega}_i \, \underline{\mathbf{Y}}_i \otimes \underline{\mathbf{X}}_i \, \Delta V_{\mathbf{x}_i} \right) \mathbf{K}^{-1}$$

Shape Tensor

$$\mathbf{K} = \sum_{i=0}^{N} \underline{\omega}_i \, \underline{\mathbf{X}}_i \otimes \underline{\mathbf{X}}_i \, \Delta V_{\mathbf{x}_i}$$

- Deformation gradient (or strain, or strain rate) passed to classical material model, which computes stress

- Stress converted to pairwise forces

$$\underline{\mathbf{T}} \left\langle \mathbf{x}' - \mathbf{x} \right\rangle = \underline{\omega} \, \sigma \mathbf{K}^{-1} \left\langle \mathbf{x}' - \mathbf{x} \right\rangle$$

S. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari. Peridynamic states and constitutive modeling. *Journal of Elasticity*, 88(2):151-184, 2007.

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Suppression of Zero-Energy Modes

Predicted Location of Neighbor

$$\mathbf{x}_n'^{\star} = \mathbf{x}_n + \bar{\mathbf{F}}_n \, (\mathbf{x}_o' - \mathbf{x}_o)$$

Hourglass Vector

$$\mathbf{\Gamma}_{\mathrm{hg}} = \mathbf{x}_n'^{\star} - \mathbf{x}_n'$$

Hourglass Vector Projected onto Bond

$$\gamma_{\mathrm{hg}} = \mathbf{\Gamma}_{\mathrm{hg}} . (\mathbf{x}_n' - \mathbf{x}_n)$$

Hourglass Force

$$\mathbf{f}_{\mathrm{hg}} = -\mathrm{C}_{\mathrm{hg}} \underbrace{\left(\frac{18k}{\pi\delta^4}\right)}_{\text{micro-modulus}} \underbrace{\frac{\gamma_{\mathrm{hg}}}{\|\mathbf{x}_o' - \mathbf{x}_o\|}}_{\text{hourglass stretch}} \underbrace{\frac{\mathbf{x}_n' - \mathbf{x}_n}{\|\mathbf{x}_n' - \mathbf{x}_n\|}}_{\text{bond unit vector}} \Delta V_{\mathrm{x}} \, \Delta V_{\mathrm{x}'}$$

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Iterative Penalty Contact Algorithm for Explicit Transient Dynamics

1. Element faces divided into triangular facets
2. Proximity search identifies potential contact
3. Apply projection operator and compute signed distance between facets
4. Apply penalty kernel if interpenetration is detected
5. Iterative enforcement scheme:
   - ① Assemble contact forces and update configuration
   - ② Re-evaluate gaps between contact facets
   - ③ Penalize non-zero gaps
   - ④ Iterate to drive gap to zero

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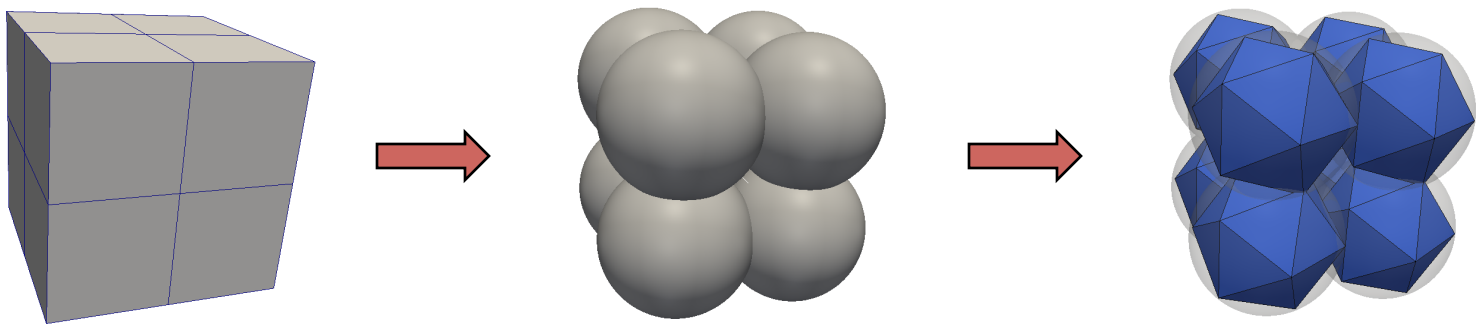
Integrating Peridynamics with Contact Algorithm

- Contact algorithm operates on planar facets
- Peridynamics algorithm operates on sphere elements
- Lofted geometry allows for coupling of peridynamics and contact algorithm



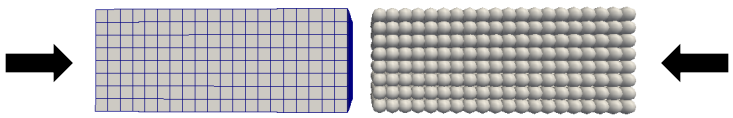
Initial hex mesh

Conversion to sphere mesh

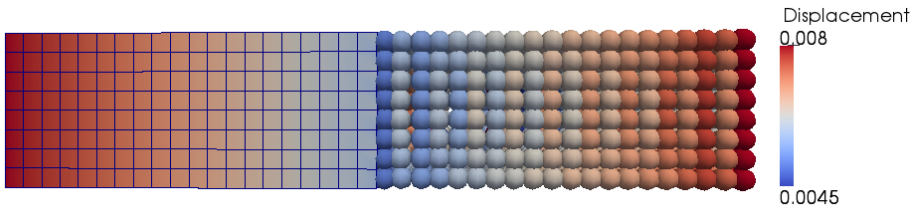
Create planar facets for contact algorithm

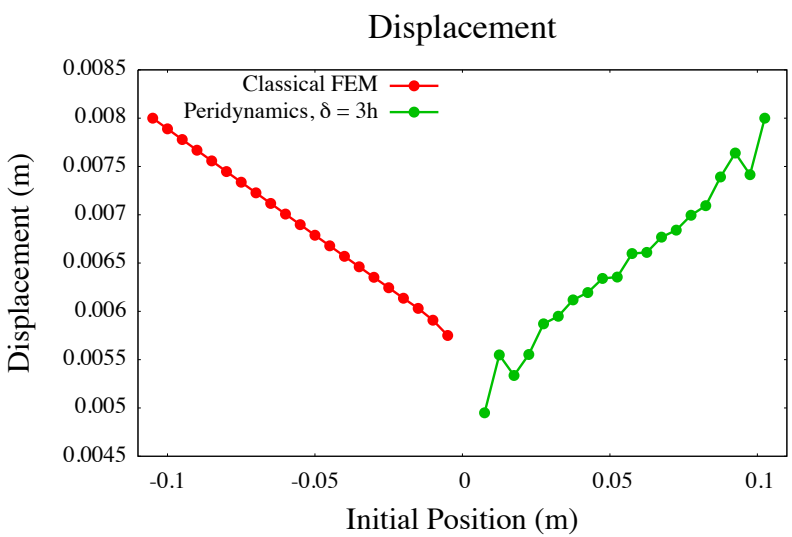
Local / Nonlocal Interface Effects

Simple Example:
Bars Contacting in Compression

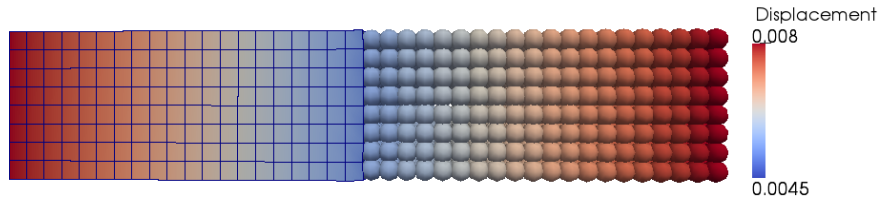


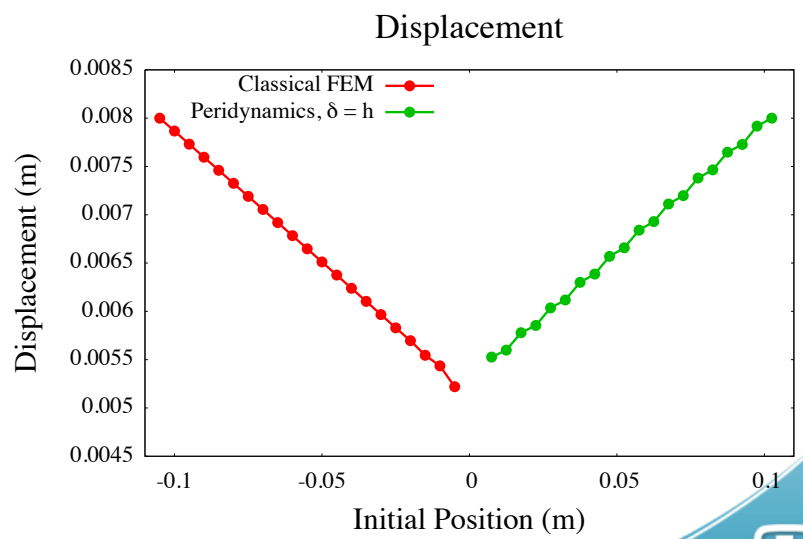
Standard Horizon
Horizon = 3 * Mesh Spacing





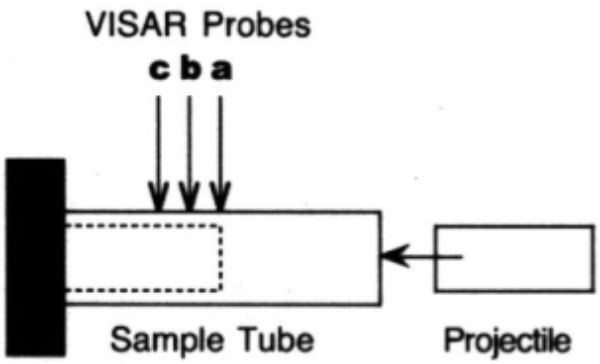
Reduce Horizon
Horizon = Mesh Spacing





Expanding Tube Experiment

- Experimental setup:
  - Tube expansion via collision of Lexan projectile and plug within AerMet tube
  - Accurate recording of velocity and displacement on tube surface
- Modeling approach:
  - AerMet tube modeled with peridynamics, elastic-plastic material model with linear hardening
  - Lexan plugs modeled with traditional FEM, EOS-enabled Johnson-Cook material model

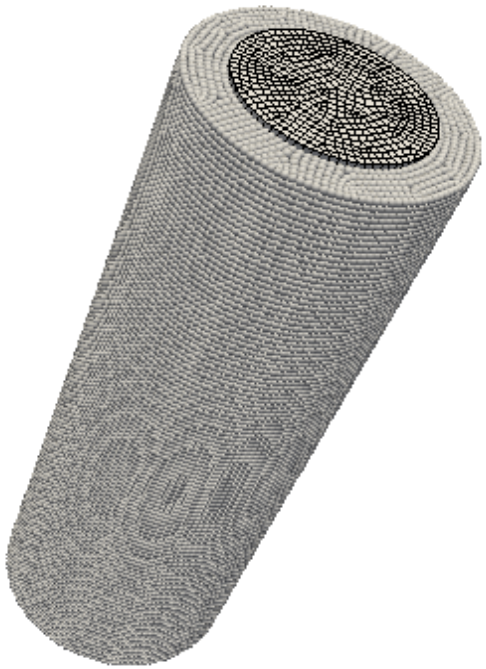


[Vogler et. al]

Vogler, T.J., Thornhill, T.F., Reinhart, W.D., Chhabidas, L.C., Grady, D.E., Wilson, L.T., Hurricane, O.A., and Sunwoo, A. Fragmentation of materials in expanding tube experiments. *International Journal of Impact Engineering*, 29:735-746, 2003.

Model Discretization

- AerMet tube
  - Peridynamics
  - Elastic-plastic material model
  - 73,676 sphere elements
  - Horizon set to five times element radius
- Lexan projectile / plug
  - Classical FEM
  - Johnson-Cook material model
  - 53,214 hexahedron elements



Model Discretization

Material Parameters

AerMet Tube

Elastic-Plastic Constitutive Model

Parameter	Value
Density	7.87 g/cm^3
Young's Modulus	194.4 GPa
Poisson's Ratio	0.3
Yield Stress	1.72 GPa
Hardening Modulus	1.94 GPa
Critical Stretch	0.02

Lexan Projectile/Plug

Johnson-Cook Constitutive Model

Parameter	Value
Density	1.19 g/cm^3
Young's Modulus	2.54 GPa
Poisson's Ratio	0.344
Yield Stress	75.8 MPa
Hardening Constant *B*	68.9 MPa
Rate Constant C	0.0
Hardening Exponent *N*	1.0
Thermal Exponent *M*	1.85
Reference Temperature	70.0 °F
Melting Temperature	500.0 °F

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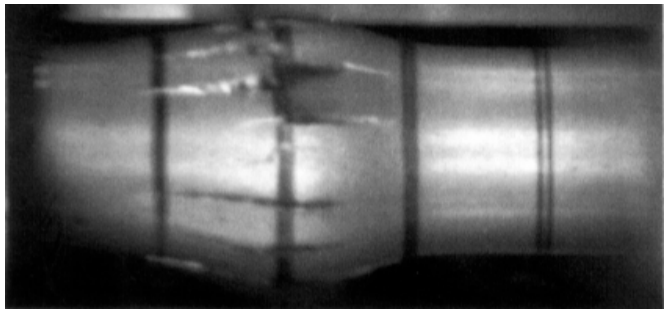
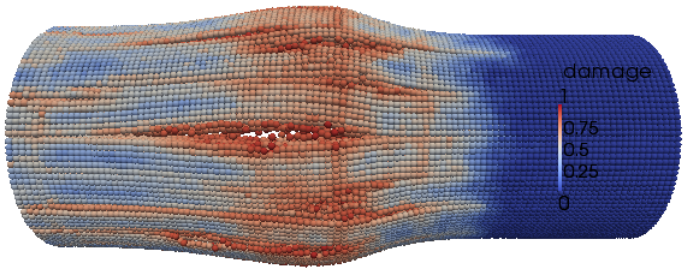
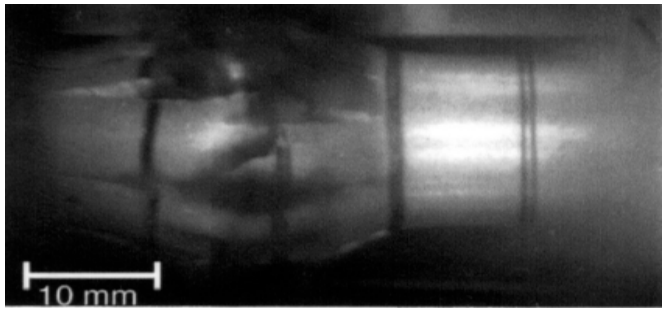
Predicted Damage Profiles



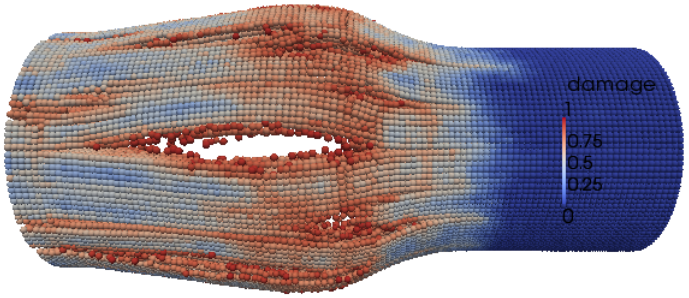
Experimental image at 15.4 microseconds [Vogler et. al]



Simulation at 15.4 microseconds



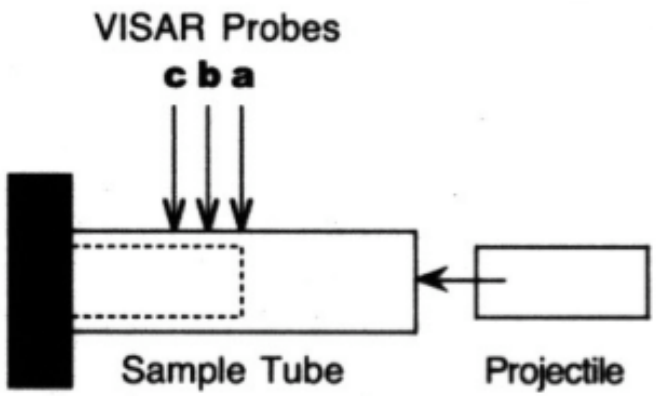
Experimental image at 23.4 microseconds [Vogler et. al]

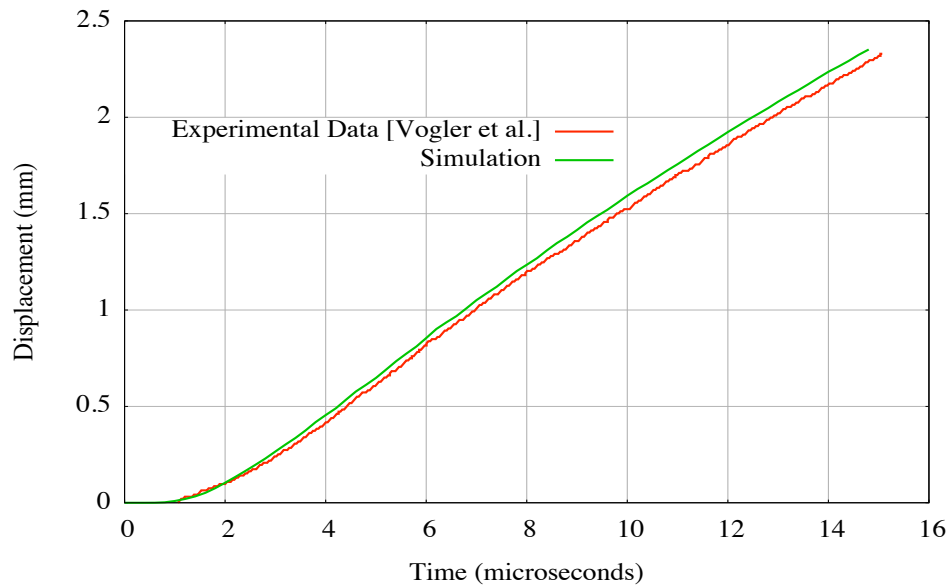


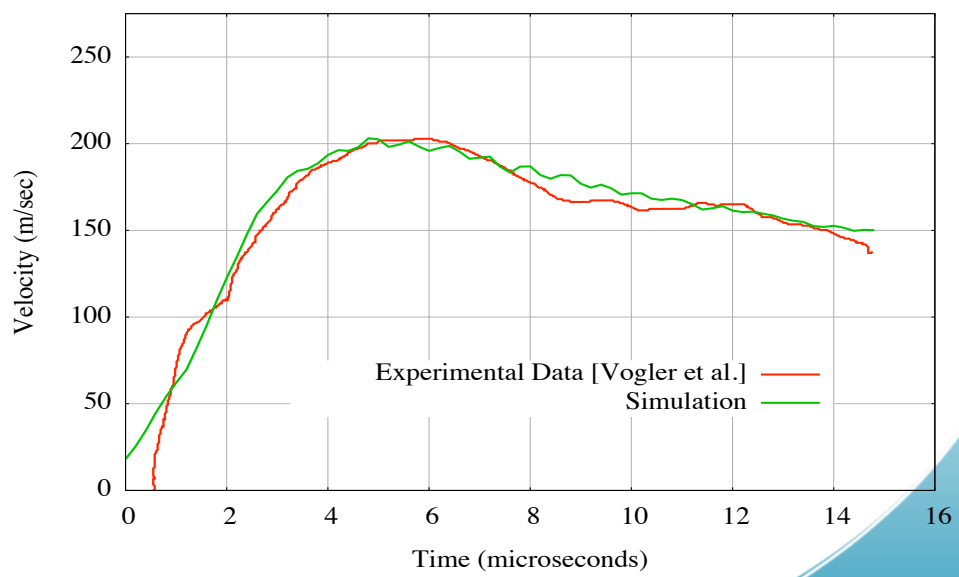
Simulation at 23.4 microseconds

Predicted Displacement and Velocity on Tube Surface

Displacement and velocity
on tube surface
at probe position A



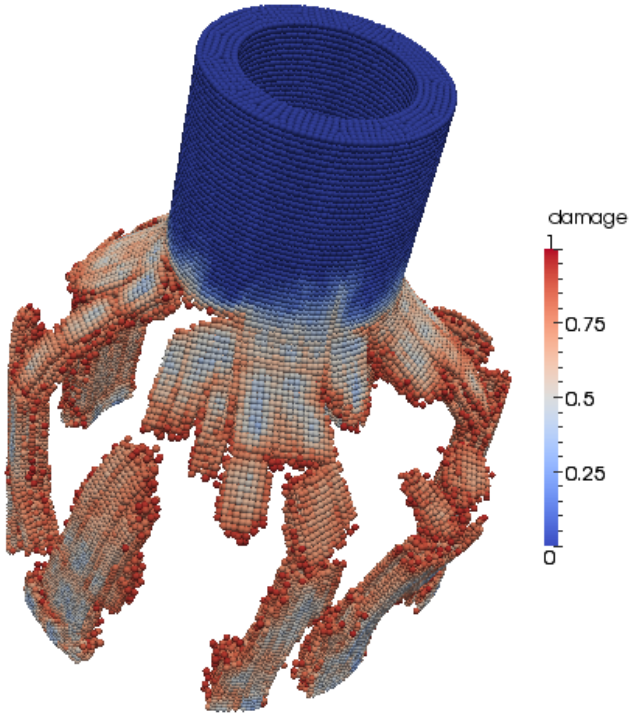




Fragmentation Pattern

<u>Qualitative Comparison of Fragmentation Results</u>

- Vogler et. al reported significant uncertainty in results at late time
- Approximately half the tube remained intact
- Vogler et. al recovered 14 fragments with mass greater than one gram



Simulation at 84.8 microseconds

Questions?

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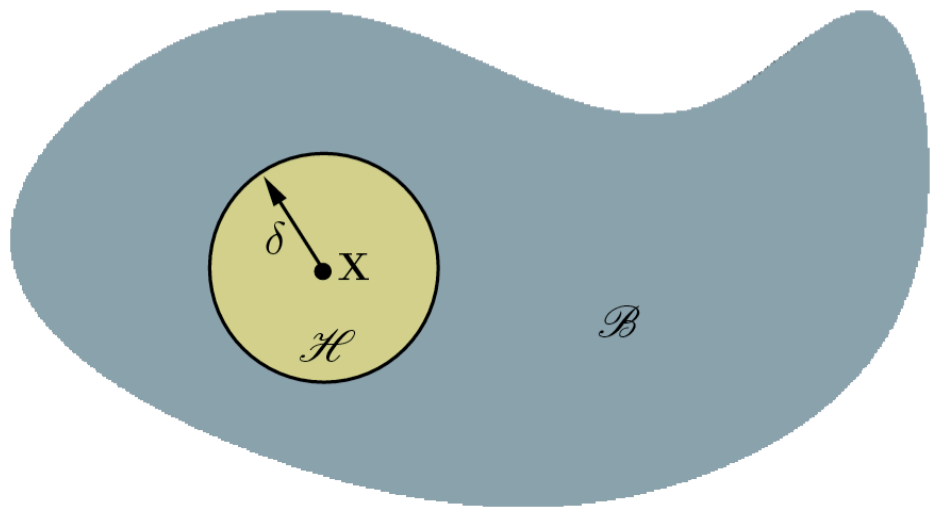
Back-Up Slides

Peridynamics is a Nonlocal Formulation of Continuum Mechanics

- Balance of linear momentum formulated as an integral equation

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{\mathscr{B}} \left\{ \underline{\mathbf{T}}[\mathbf{x},t]\,\langle \mathbf{x}'-\mathbf{x}\rangle - \underline{\mathbf{T}}'[\mathbf{x}',t]\,\langle \mathbf{x}-\mathbf{x}'\rangle \right\} \mathrm{d}V_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t)$$

$\mathbf{X}$	Point in body $\mathscr{B}$
δ	Horizon of $\mathbf{X}$
$\mathscr{H}$	Family of $\mathbf{X}$

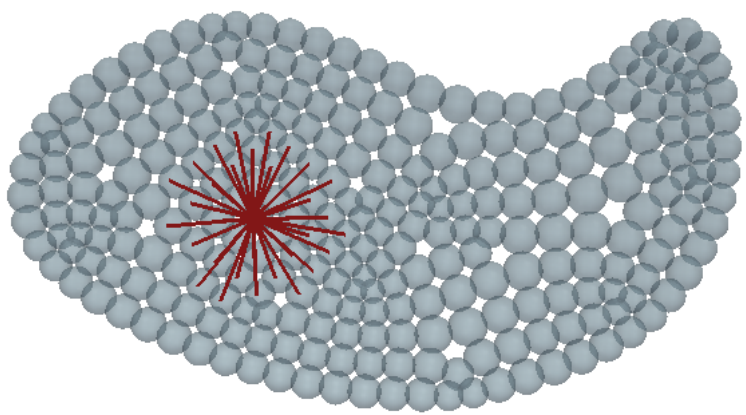


Discretization of a Peridynamic Body

Body may be represented with a finite number of sphere elements

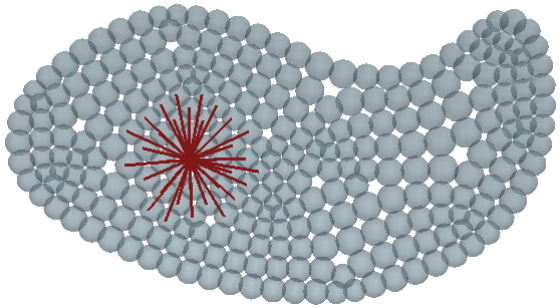
$$\rho(\mathbf{x})\ddot{\mathbf{u}}_h(\mathbf{x},t) = \sum_{i=0}^{N} \left\{ \underline{\mathbf{T}}[\mathbf{x},t]\left\langle \mathbf{x}_i' - \mathbf{x} \right\rangle - \underline{\mathbf{T}}'[\mathbf{x}_i',t]\left\langle \mathbf{x} - \mathbf{x}_i' \right\rangle \right\} \Delta V_{\mathbf{x}_i'} + \mathbf{b}(\mathbf{x},t)$$

- Each element interacts with all elements within its neighborhood
- A *bond* connects any two elements that interact directly
- Pairwise forces determined by a forc state, $\underline{\mathbf{T}}[\mathbf{x},t]$, acting on a bond, $\langle \mathbf{x}_i' - \mathbf{x} \rangle$



Constitutive Laws in Peridynamics

$$\underbrace{\underline{\mathbf{T}}[\mathbf{x}, t]}_{\text{Force State}} \quad \underbrace{\langle \mathbf{x}_i' - \mathbf{x} \rangle}_{\text{Bond}}$$

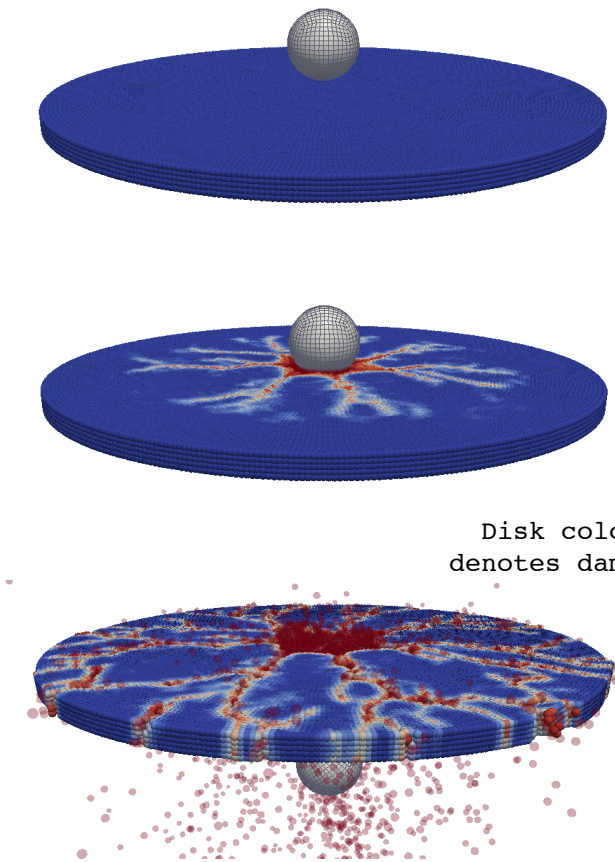


- Force states are determined by constitutive laws
- Force states are functions of the deformations of all points within a neighborhood
- Force states act on bonds, resulting in a pairwise force between elements
- Pairwise forces combine to determine the net force on a given element

Combined Peridynamics / Classical FEM Modeling Approach

- Utilize peridynamics to capture crack formation and propagation
- Interaction of perdynamics and classical FEM via contact
- Classical constitutive models
  - Elastic-plastic with linear hardening
  - Johnson-Cook

Brittle fracture:
FEM projectile impacting PD disk

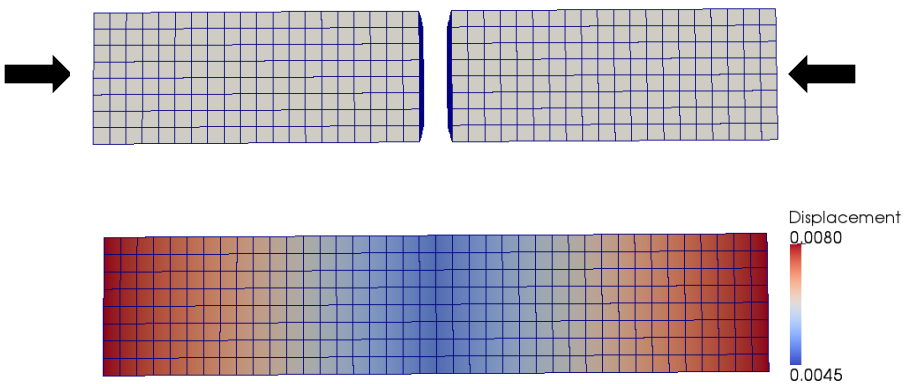


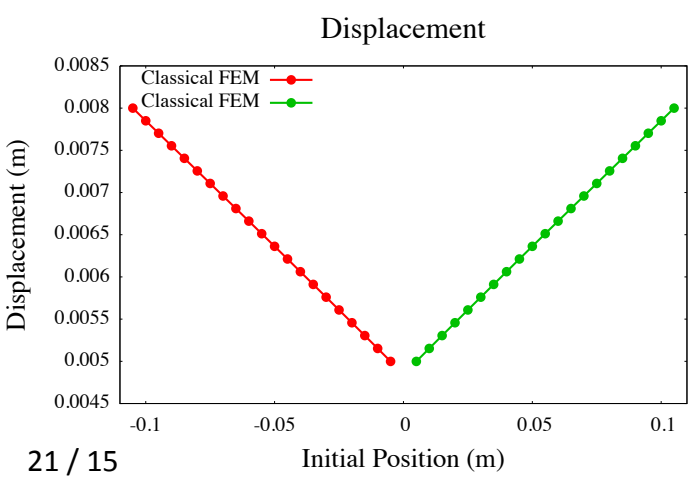
Disk color
denotes damage

Local / Nonlocal Interface Effects

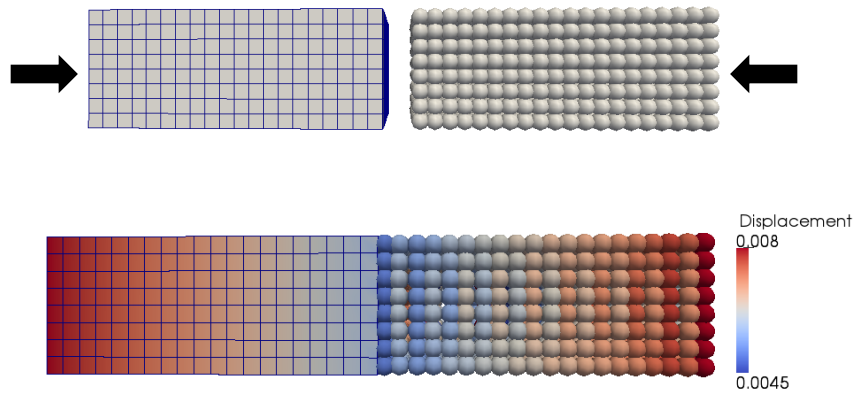
Simple Example: Rectangular Bars Pressed Together In Contact

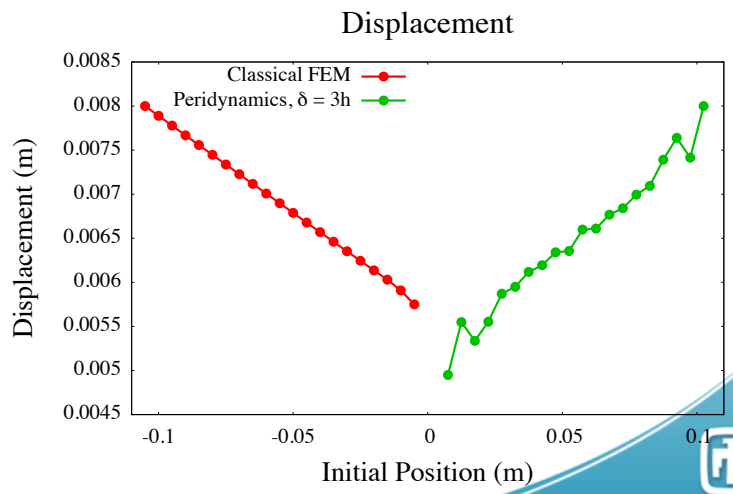
Classical Finite Elements





Classical FEM / Peridynamics
Horizon = 3 * Mesh Spacing





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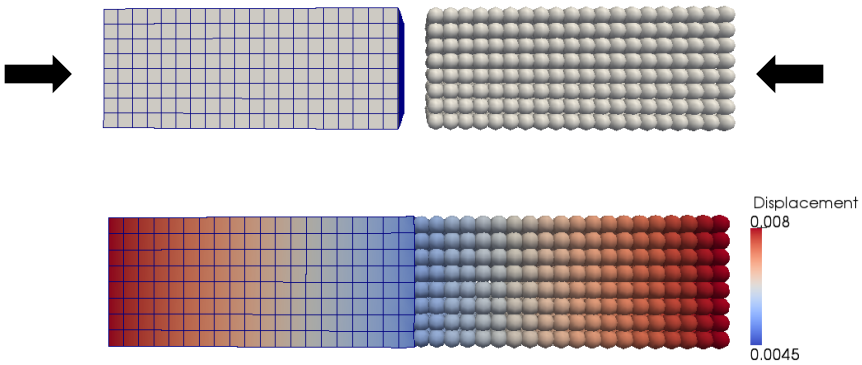
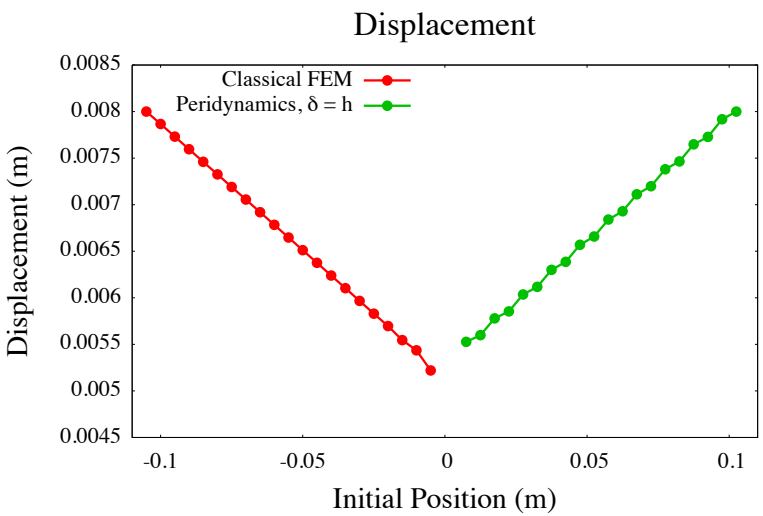
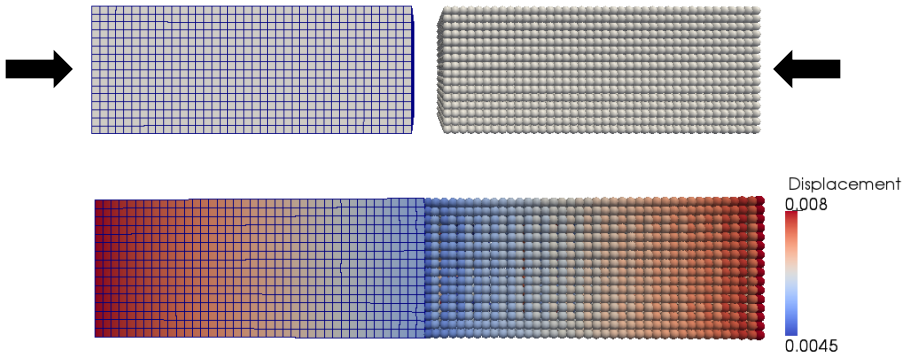
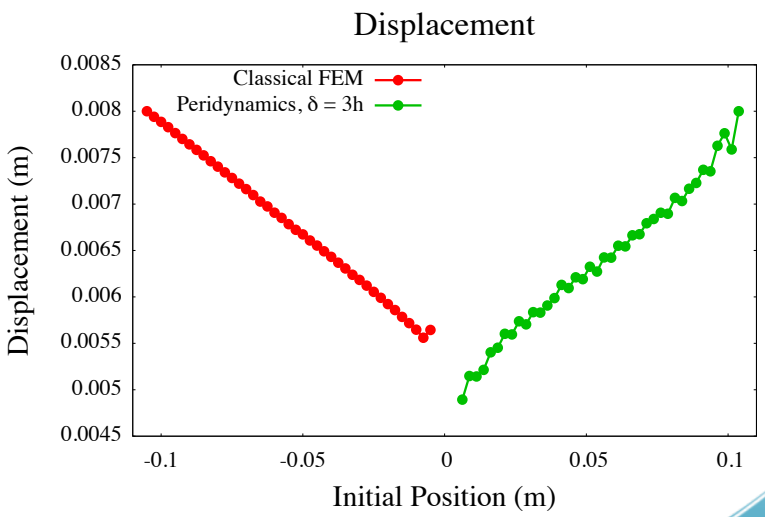
Mitigation Strategies

Reduce Horizon
Horizon = Mesh Spacing





Refine Mesh