Magneto- Rayleigh-Taylor Instability: Theory and Experiments

Y.Y. Lau, R.M. Gilgenbach, D.A. Chalenski, J.C. Zier,[1] P. Zhang, I. M. Rittersdorf, M. Weis, S.G. Patel, D.M. French,[2] M.R. Gomez,[3] A.M. Steiner, M.A. Franzi

Plasma, Pulsed Power and Microwave Laboratory,
Nuclear Engineering and Radiological Sciences Dept.,
University of Michigan
Ann Arbor, MI 48109-2104 USA

S. A. Slutz, M.G. Mazarakis, M.E. Cuneo, M.R. Lopez

Sandia National Laboratories
Albuquerque, NM 87185 USA

[1]J.C. Zier is now with the US Naval Research Laboratory
[2]D.M. French is now with the US Air Force Research Laboratory
[3]M.R. Gomez is now with Sandia National Laboratories

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Outline

- **RT vs MRT, anisotropy and feedthrough[1]**
- **Effects of magnetic shear on MRT[2]**
- **MRT experiments on foil driven by MA-LTD[3]**
- **Ongoing works**

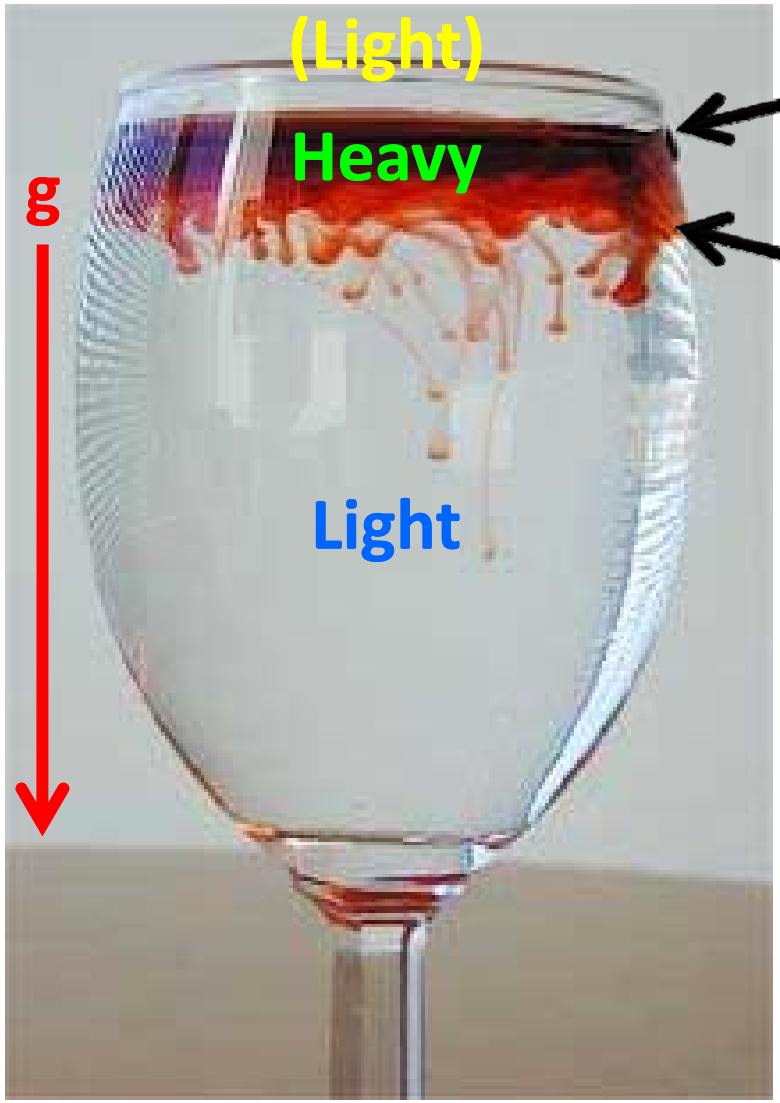
[1]Y. Y. Lau, *et al*., Phys. Rev. E 83, 066405 (2011).

[2]P. Zhang, *et al*., Phys. Plasmas (Feb 2012 issue).

[3]J. Zier, *et al*., Phys. Plasmas (March 2012 issue).

(magneto)-Rayleigh-Taylor Instability

- Rayleigh-Taylor (RT)
  - Driven by fluid or kinetic pressures
  - Isotropic on interface
- Magneto-Rayleigh-Taylor (MRT)
  - Driven by magnetic pressure and kinetic pressure
  - B-field creates anisotropy

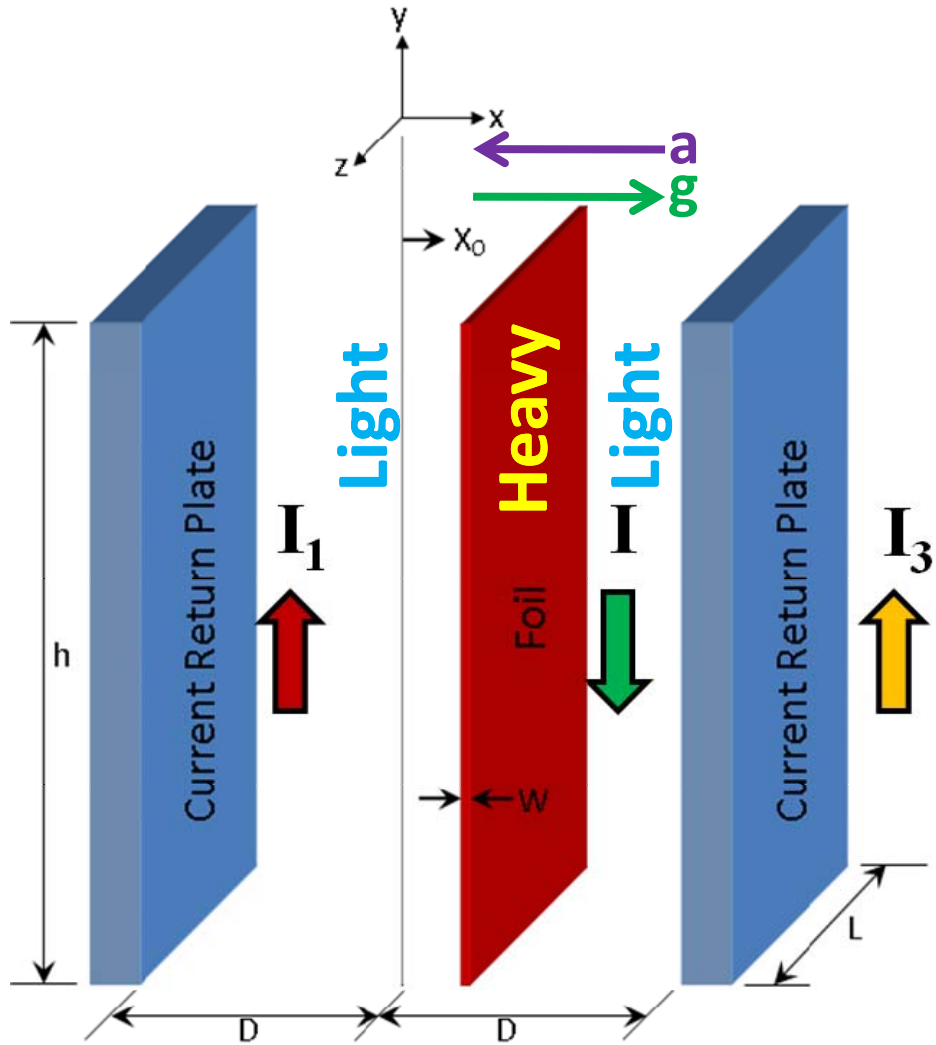


Stable Interface

Unstable Interface

Load Geometry for MAIZE Experiments

- Thin foil between current return plates
  - 400 nm thick Al
  - 1 cm wide
- Double return plates reduce load inductance
  - 1 cm tall interaction region
  - 1 cm between return plates
- MRT acceleration is controlled with offset distance from the mid-plane
  - Small Offset → Less acceleration
  - Large Offset → More acceleration



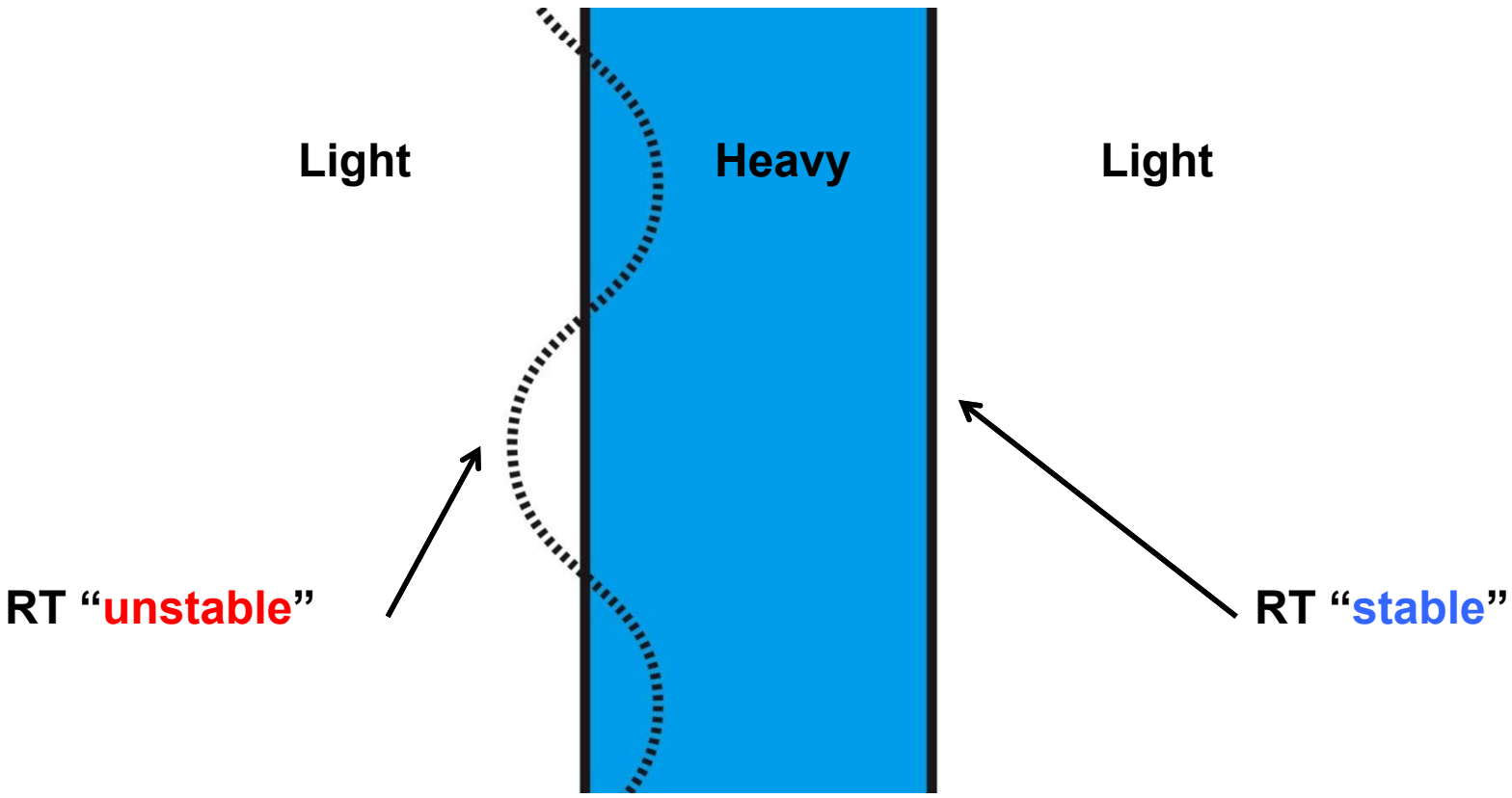
Anisotropy

Degree of MRT anisotropy depends on the fraction of magnetic pressure in providing the acceleration.

What else?

Feedthrough Factor:

ξ(stable interface)/ξ(unstable interface)



$\vec{g}$ (gravity in rest frame)

a (= $-\vec{g}$ = acceleration in lab frame)

Our Model : Ideal MHD

(in rest frame of interfaces)

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\mathbf{J}\times\mathbf{B}-\rho g\mathbf{x}$$

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0$$

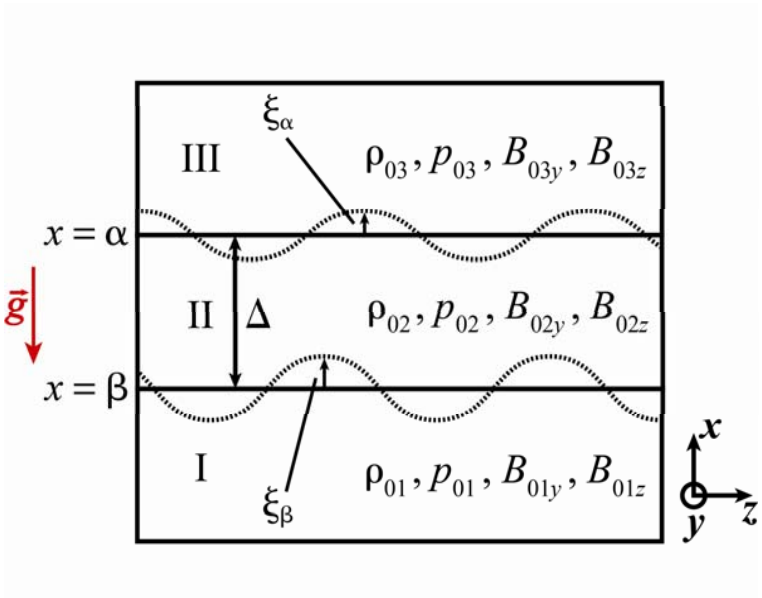
$$\nabla\cdot\mathbf{v}=0$$ **Fluid incompressible**

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{B})$$ **Frozen-in Law**

$$\nabla\times\mathbf{B}=\mu_0\mathbf{J}$$

Equilibrium Conditions:

in *Rest Frame* of Interfaces



$\rho_{01}, \rho_{02}, \rho_{03}, B_{01}, B_{02}, B_{03}, \Delta$, g, are arbitrary constants

$$\mathbf{B}_0 = \hat{\mathbf{y}} B_{0y} + \hat{\mathbf{z}} B_{0z}$$

p_0 is a linear function of x within each region I, II, and III.

Weight of Region II supported by pressure difference between I and III:

$$g\rho_{02}\Delta = \left[p_{01}\left(\beta^-\right) + \frac{\left|\mathbf{B}_{01}\right|^2}{2\mu_0} \right] - \left[p_{03}\left(\alpha^+\right) + \frac{\left|\mathbf{B}_{03}\right|^2}{2\mu_0} \right]$$

Pressure balance across the upper and lower interface:

$$\left[p_{03}\left(\alpha^+\right) + \frac{\left|\mathbf{B}_{03}\right|^2}{2\mu_0} \right] = \left[p_{03}\left(\alpha^-\right) + \frac{\left|\mathbf{B}_{02}\right|^2}{2\mu_0} \right] \quad ; \quad \left[p_{02}\left(\beta^+\right) + \frac{\left|\mathbf{B}_{02}\right|^2}{2\mu_0} \right] = \left[p_{01}\left(\beta^-\right) + \frac{\left|\mathbf{B}_{01}\right|^2}{2\mu_0} \right]$$

Perturbation Analysis

Within each region (I, II, and III):

$$p = p_0(x) + p_1(x)e^{i\omega t - ik_y y - ik_z z},$$

$$\mathbf{v} = \mathbf{v_0} + \mathbf{v}_1 = \mathbf{v}_1 = [\mathrm{v}_{1x}(x), \mathrm{v}_{1y}(x), \mathrm{v}_{1z}(x)]e^{i\omega t - ik_y y - ik_z z},$$

$$\mathbf{B} = \mathbf{B_0} + \mathbf{B}_1 = \mathbf{B_0} + [\mathrm{B}_{1x}(x), \mathrm{B}_{1y}(x), \mathrm{B}_{1z}(x)]e^{i\omega t - ik_y y - ik_z z}.$$

Subscript 0: Unperturbed quantities
Subscript 1: Perturbation quantities

Total perturbation pressure is continuous at the upper and lower interfaces => MRT dispersion relation

MRT Dispersion Relation

$$\omega^4 - R\omega^2 + S = 0$$

$$R = R(\rho_{01}, \rho_{02}, \rho_{03}, \mathbf{B}_{01}, \mathbf{B}_{02}, \mathbf{B}_{03}, g, \Delta, k_y, k_z),$$
$$S = S(\rho_{01}, \rho_{02}, \rho_{03}, \mathbf{B}_{01}, \mathbf{B}_{02}, \mathbf{B}_{03}, g, \Delta, k_y, k_z).$$

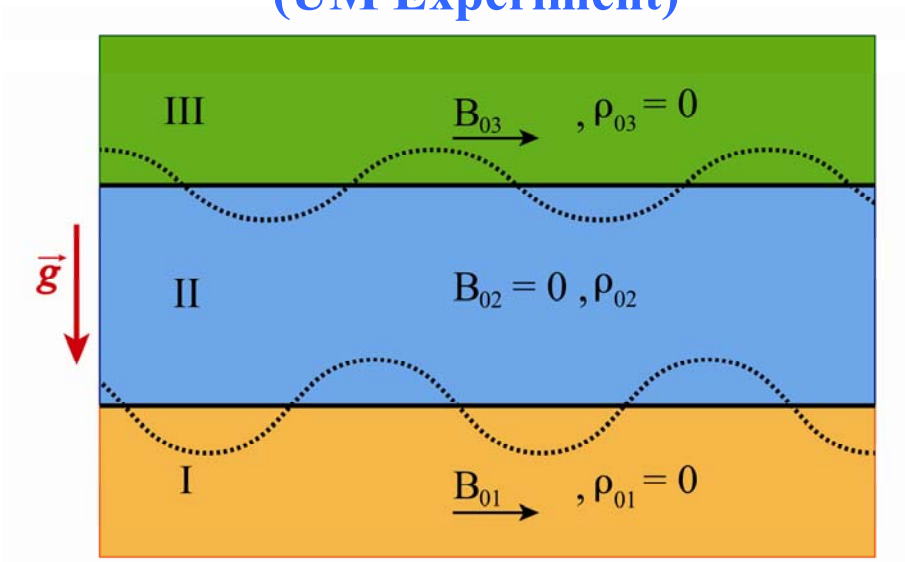
Four modes: $\omega = \pm\omega_1, \qquad \omega = \pm\omega_3$

$\omega_1 \equiv$ most unstable mode's eigenfrequency

$$\omega_1^2 = \frac{R}{2} - \sqrt{\frac{R^2}{4} - S}$$

Extension of Harris' Model: Nonzero B_{03}

(UM Experiment)

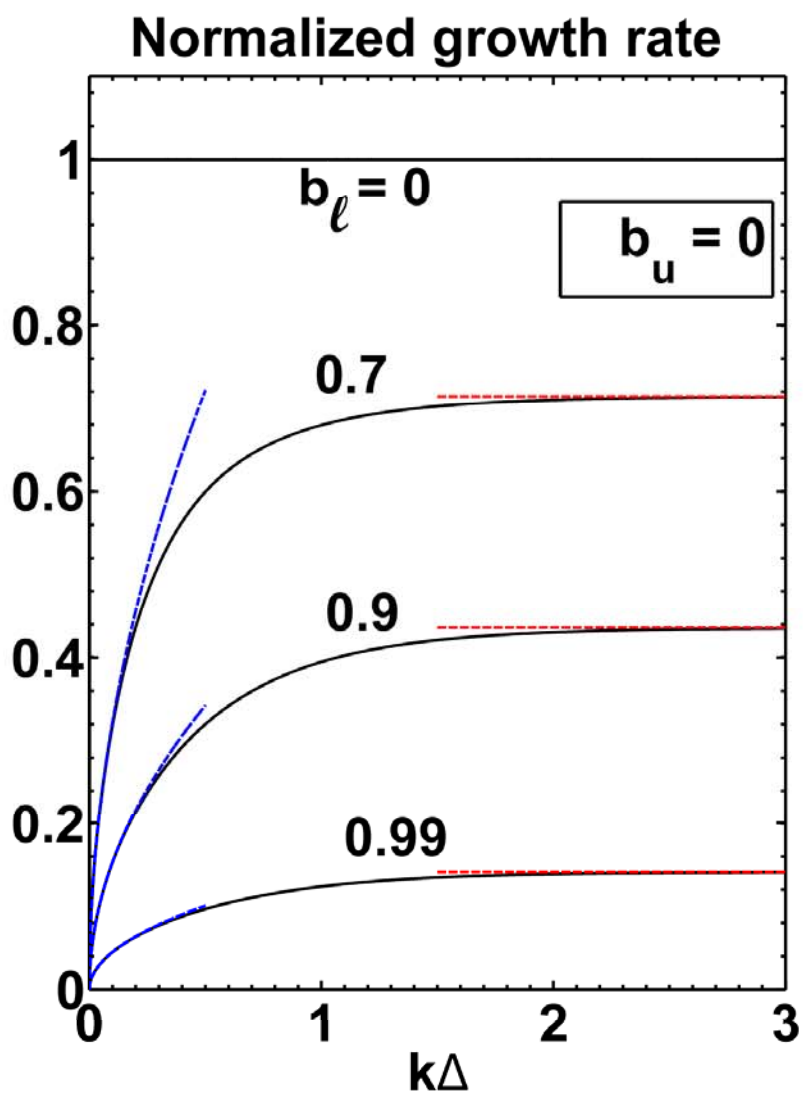


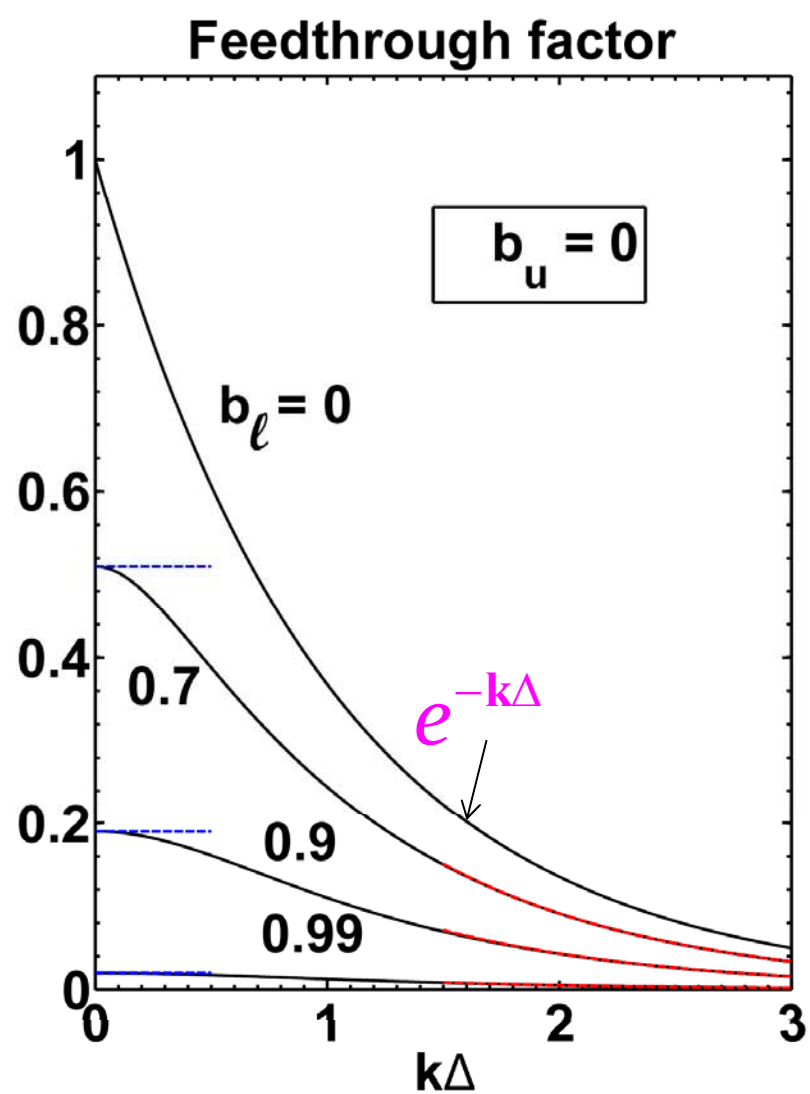
$$V_u \equiv \sqrt{B_{03}^2 / \mu_0 \rho_{02}}$$

$$V_\ell \equiv \sqrt{B_{01}^2 / \mu_0 \rho_{02}}$$

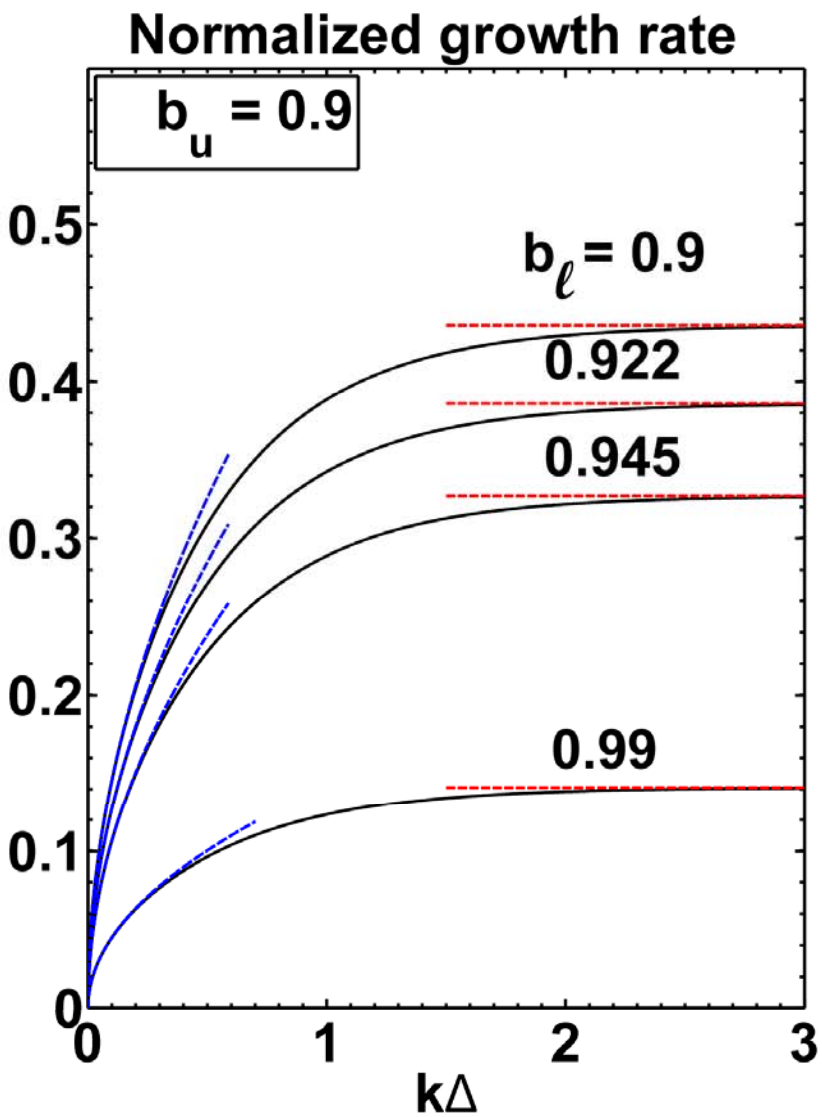
$$\mathrm{Im}(\omega_1) = \begin{cases} \sqrt{k\Delta}\sqrt{\dfrac{\left(gk - k_z^2 V_l^2\right)\left(gk + k_z^2 V_u^2\right)}{k_z^2 V_l^2 + k_z^2 V_u^2}} & , k\Delta \to 0 \\ \sqrt{kg - k_z^2 V_l^2} & , k\Delta >> 1 \end{cases}$$

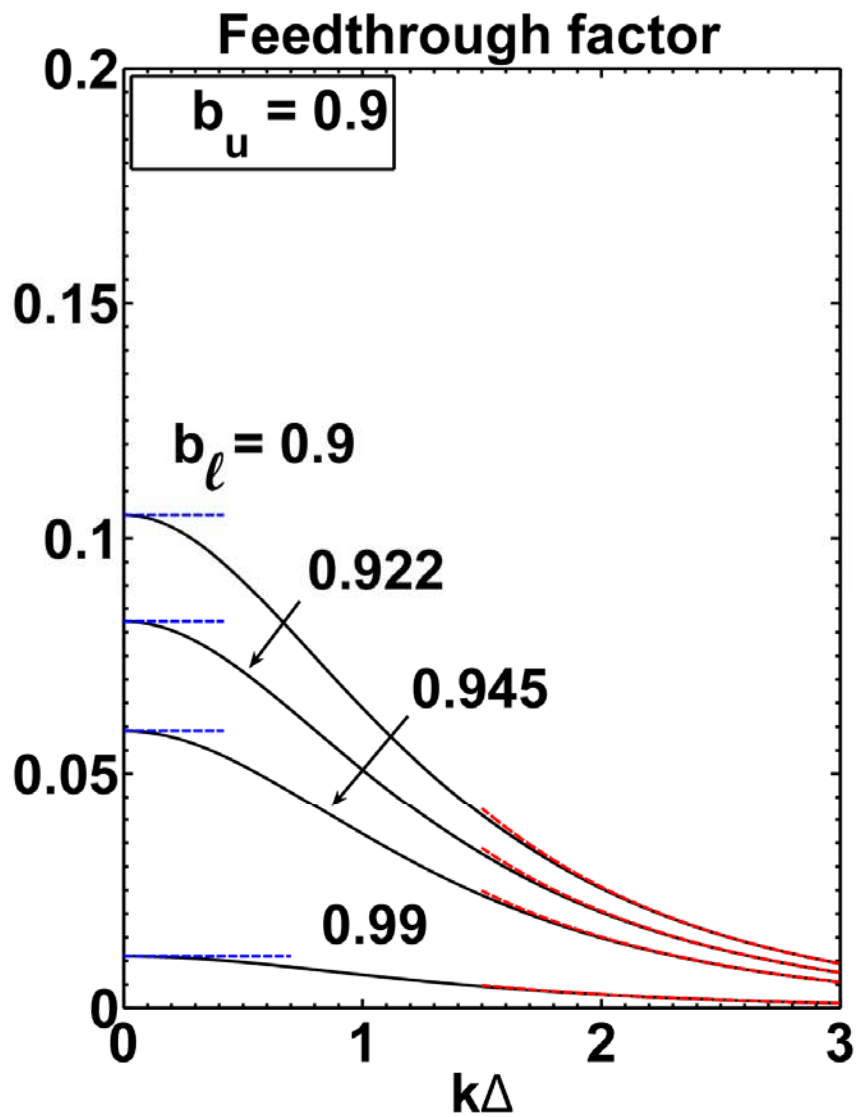
$$F_1 \approx \begin{cases} \dfrac{gk - k_z^2 V_l^2}{gk + k_z^2 V_u^2} & , k\Delta \to 0 \\ e^{-k\Delta} \dfrac{2\left(gk - k_z^2 V_l^2\right)}{2kg - k_z^2 V_l^2 + k_z^2 V_u^2} & , k\Delta >> 1 \end{cases}$$





$$\bar{\gamma} = \gamma / \sqrt{kg}, \quad b_u^2 = k_z^2 V_u^2 / kg, \quad b_\ell^2 = k_z^2 V_\ell^2 / kg \ (<1 \text{ for MRT})$$


Normalized growth rate
$b_u = 0.9$
$b_\ell = 0.9$
0.922
0.945
0.99
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
kΔ


Feedthrough factor
$b_u = 0.9$
$b_\ell = 0.9$
0.922
0.945
0.99
0.2
0.15
0.1
0.05
0
0
1
2
3
kΔ

The two interfaces are decoupled when

λ = 1mm, Δ = 1mm

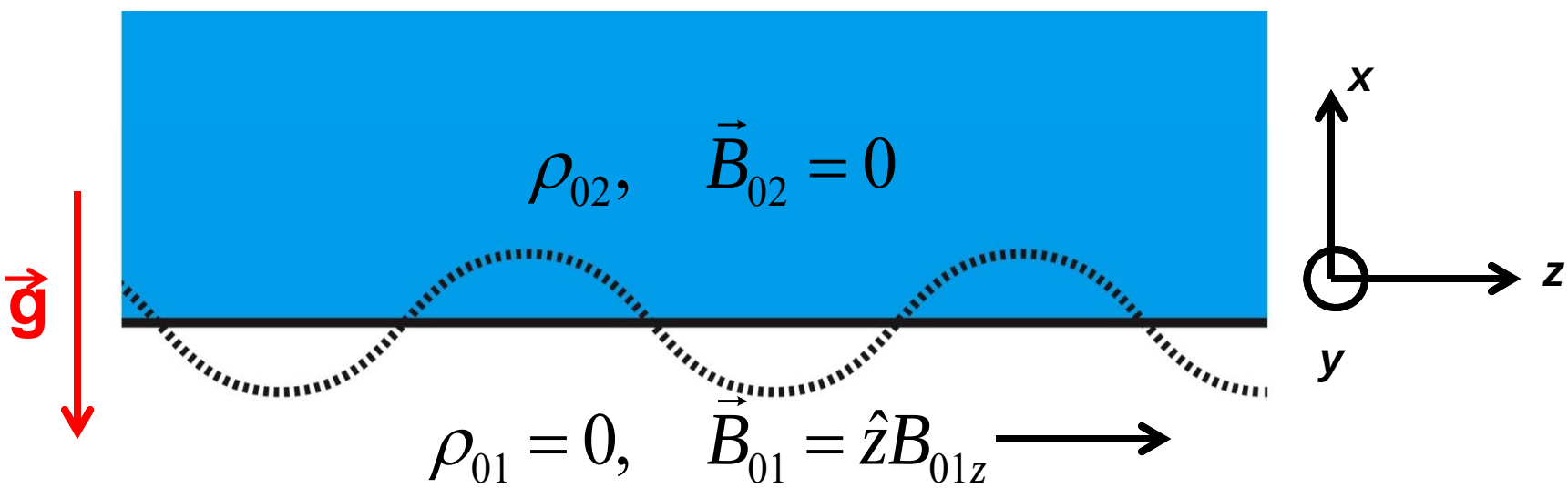
$$k\Delta = \frac{2\pi}{1mm} \times 1mm = 6.28$$

$$\text{Feedthrough Factor } < e^{-k\Delta} = 0.0019, \text{as if } \Delta \to \infty.$$

=> MRT on one interface cannot affect the opposite interface.

One Interface (kΔ >2):

Kruskal-Schwarzchild (1954)
Chandrasekhar (1961)



$$\omega^2 = -kg + k_z^2 v_\ell^2 \quad \text{, where} \quad v_\ell^2 = \frac{B_{01z}^2}{\mu_0 \rho_{02}} \quad , \quad k = \sqrt{k_y^2 + k_z^2}$$

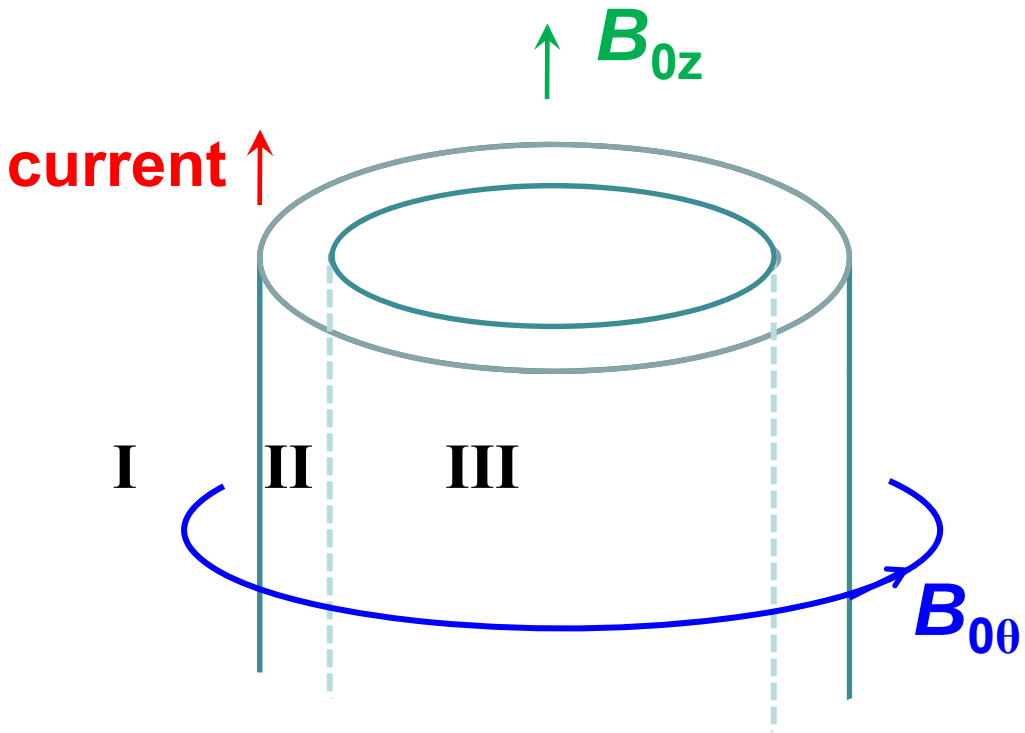
MRT is stable if $k_z^2 v_\ell^2 > kg$

Conclusions

- **Anistotropy and feedthrough in MRT was solved for 3 regions with 2 interfaces, with an arbitrary mix of kinetic and magnetic pressure in each region**
- **Classical theories of Taylor, Chandrasekhar, Harris, are recovered as limiting cases**
- **MRT retains significant growth if it is not stabilized by magnetic field line bending**
- **MRT feedthrough is significantly reduced in modes that bend the magnetic field line**

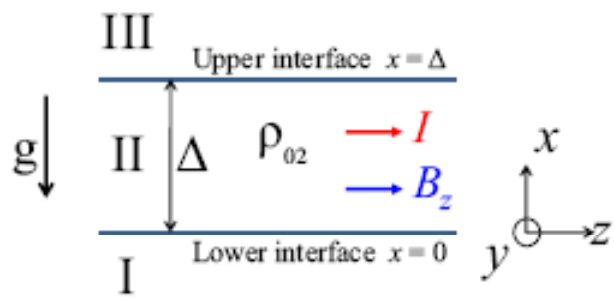
Ref.: Y. Y. Lau et al., Phys. Rev. E 83, 066405 (2011)

Effect of Magnetic Shear on MRT*



Cause: diffusion of $B_{0\theta}$ within liner
+ pre-imposed B_{0z}

*S. Slutz et al., Phys. Plasmas 17, 056303 (2010).



Equilibrium B_0 :

III. $B_{03} = (0, \quad 0, \quad B_{03z})$

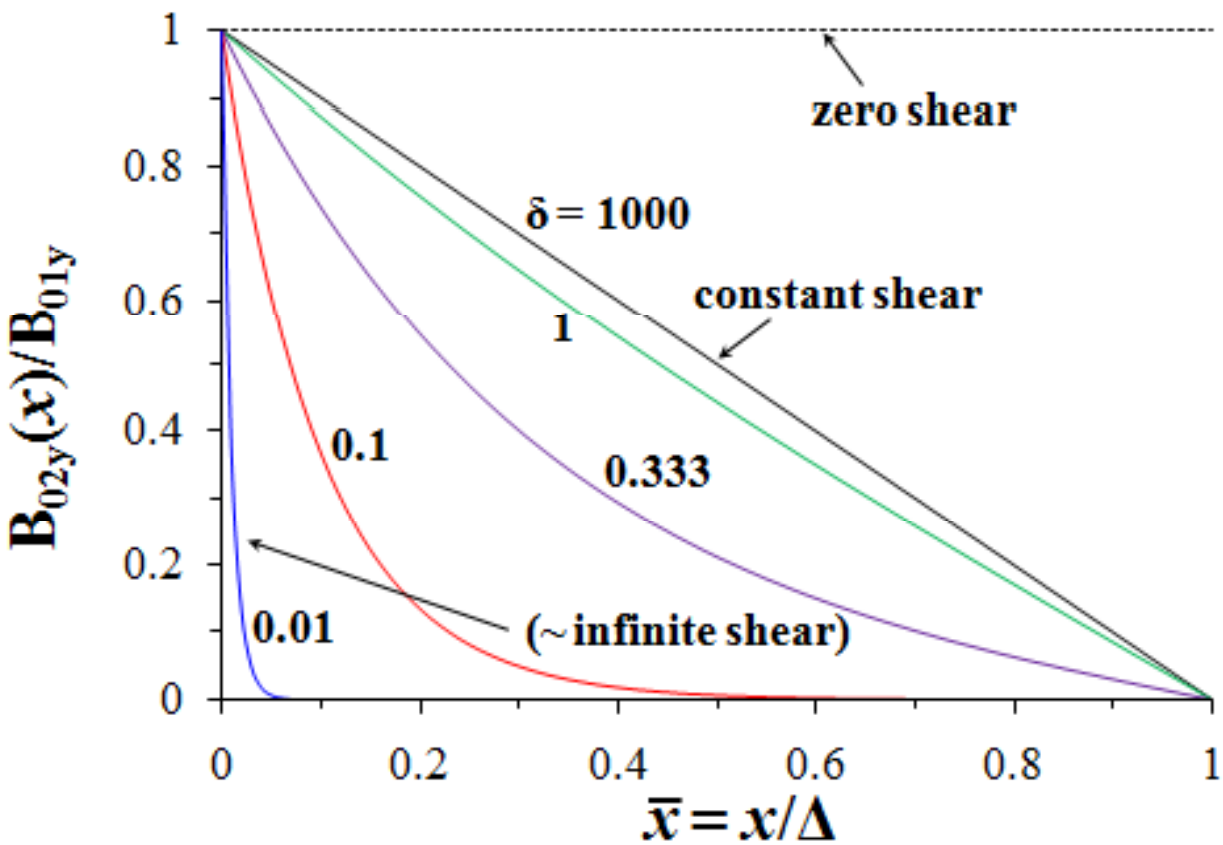
II. $B_{02} = (0, \quad B_{02y}(x), \quad B_{03z})$

I. $B_{01} = (0, \quad B_{01y}, \quad B_{03z})$

B_{01y} = const, B_{03z} = const.

$B_{02y}(x)$ profile

$$B_{02y}(x) = B_{01y} \times \frac{\sinh\left[\frac{1}{\delta}\left(1-\frac{x}{\Delta}\right)\right]}{\sinh\frac{1}{\delta}}; \qquad 0 \le x \le \Delta$$



δ= "skin depth", in units of Δ

Analytic Results on Two Limits

- $B_{02y}(x) = B_{01y}$ (Zero magnetic shear)
  - $b_\ell = 0$ mode always unstable, regardless of b_u

- $\delta=0$ (Infinite magnetic shear)
  - MRT is stabilized if $b_u > 1$

$$b_l^2 = \frac{(\vec{k}\cdot\vec{B}_{01})^2}{\mu_0\rho_{02}(kg)};\quad b_u^2 = \frac{(\vec{k}\cdot\vec{B}_{03})^2}{\mu_0\rho_{02}(kg)}$$

Solve for eigenvalue ω and eigenfunction ξ_x :

$$\frac{d}{dx}\left\{\left[\rho_0\left(-\omega^2\right)+\frac{1}{\mu_0}\left(\vec{k}\cdot\vec{B}_0\right)^2\right]\frac{d\xi_x}{dx}\right\}-k^2\left[\rho_0\left(-\omega^2\right)-g\frac{d\rho_0}{dx}+\frac{1}{\mu_0}\left(\vec{k}\cdot\vec{B}_0\right)^2\right]\xi_{1x}=0,$$

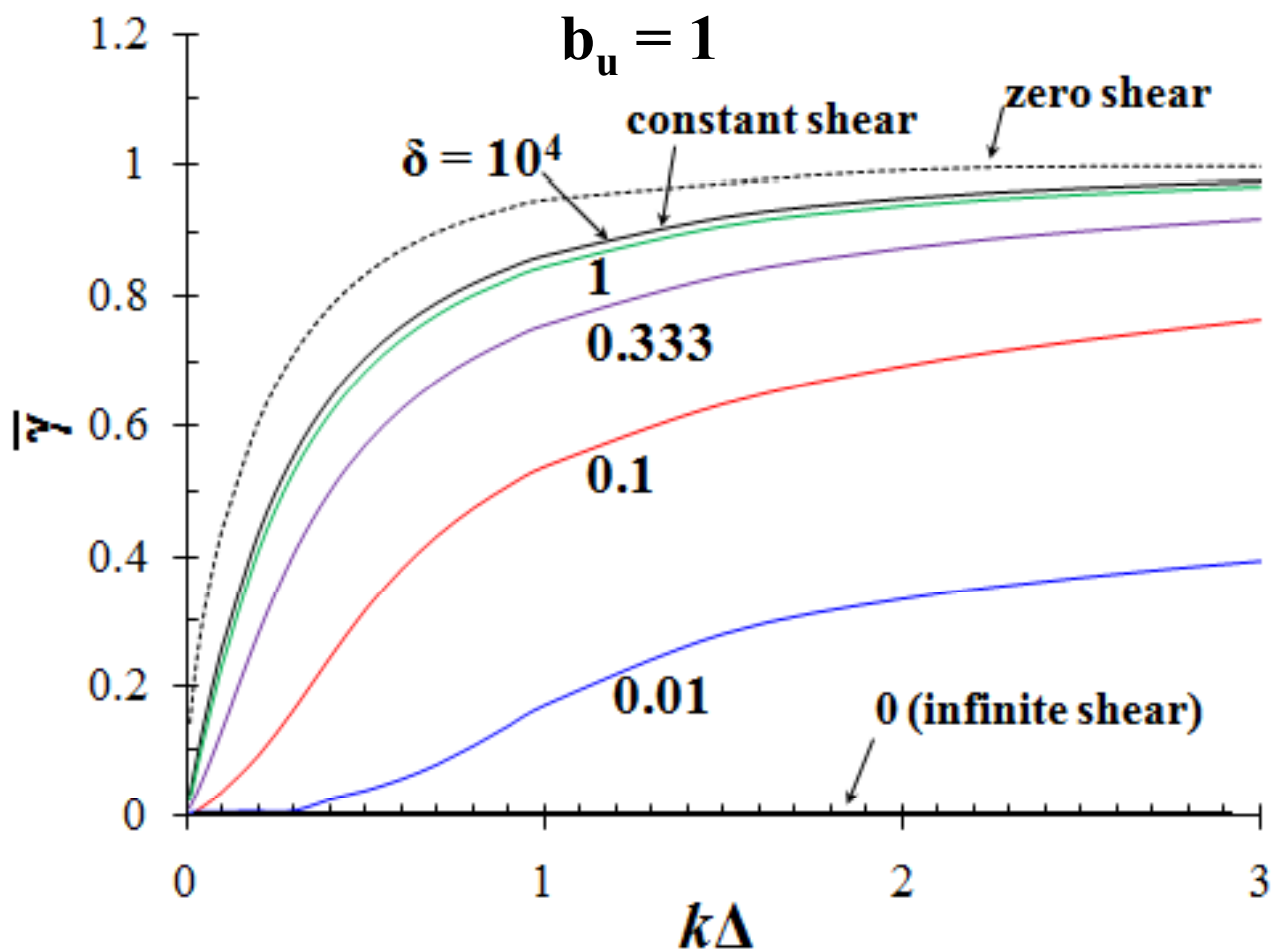
$$-\infty < x < \infty$$

$$\vec{k}=(0,k_y,k_z),\quad k=\sqrt{k_y^2+k_z^2}$$

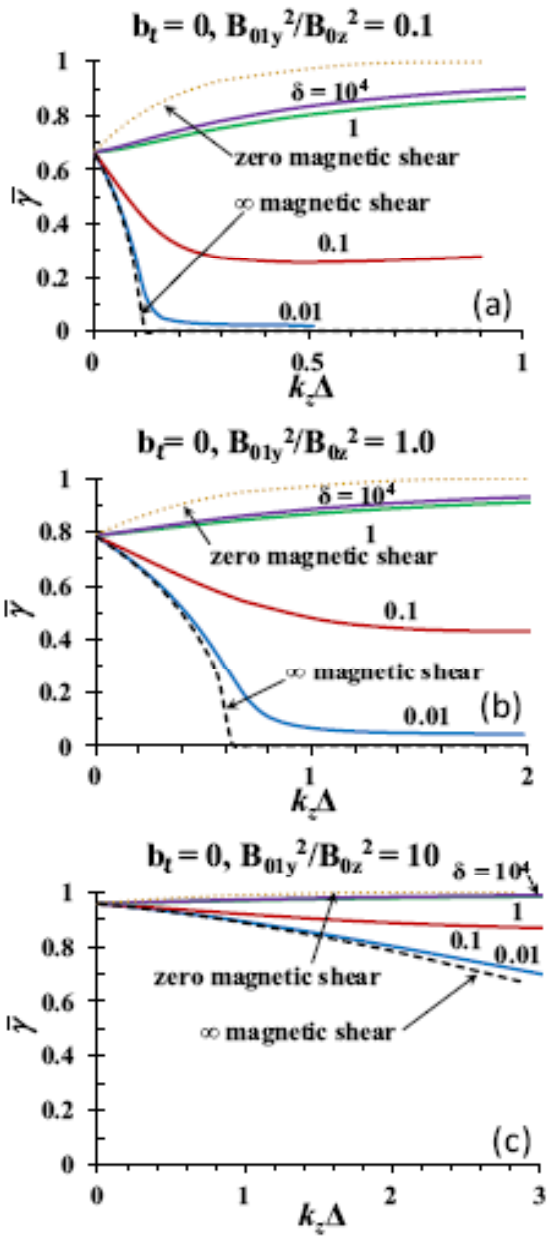
$$b_l^2=\frac{(\vec{k}\cdot\vec{B}_{01})^2}{\mu_0\rho_{02}(kg)};\quad b_u^2=\frac{(\vec{k}\cdot\vec{B}_{03})^2}{\mu_0\rho_{02}(kg)}$$

Can show: $b_\ell = 0$ **modes are most unstable**

Growth Rate ($b_\ell = 0$)



Increasing magnetic shear reduces MRT growth rate, but cannot completely eliminate it

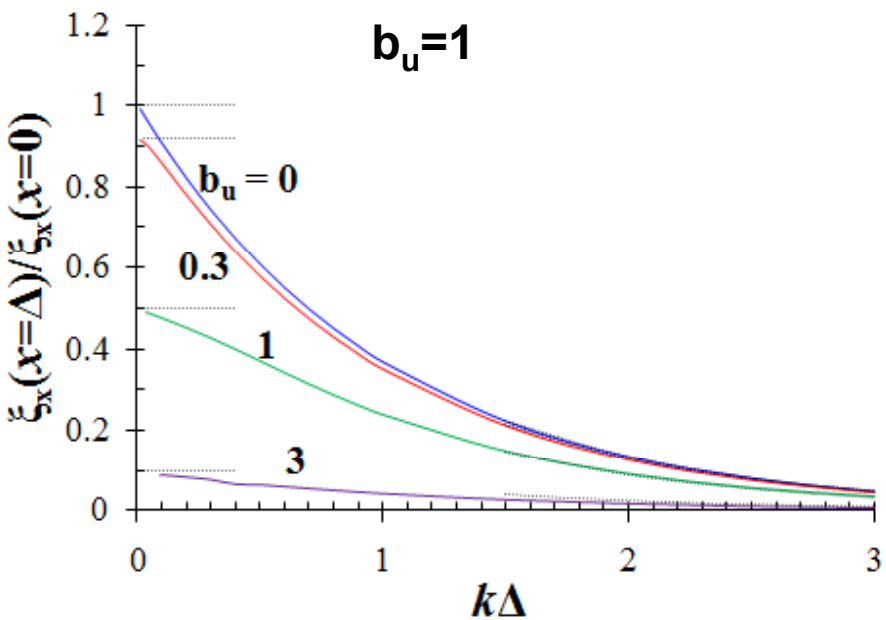


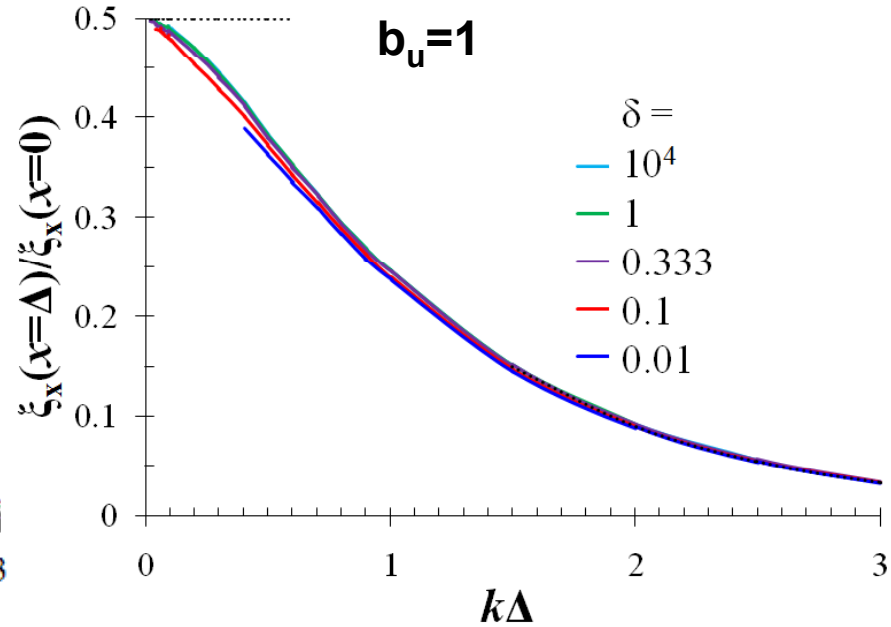
$B_{0z} \sim 3\ B_{0\theta}$

$B_{0z} \sim B_{0\theta}$

$B_{0z} \sim 0.3\ B_{0\theta}$

Feedthrough Factor ($b_\ell = 0$)



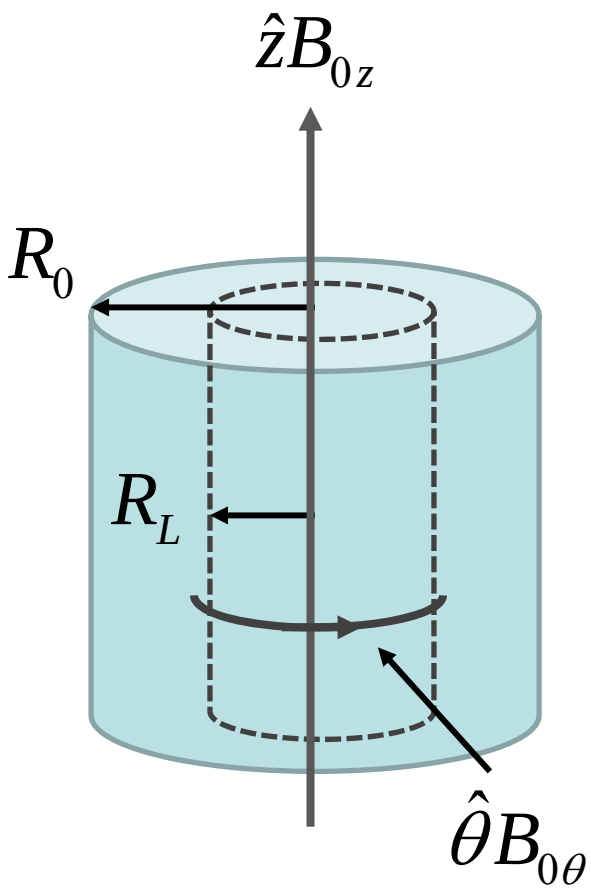


The feedthrough factor is virtually independent of magnetic shear

Conclusions on Effects of Magnetic Shear on MRT

- The $b_\ell = 0$ modes are the most unstable
- Increasing magnetic shear reduces MRT growth rate, but cannot completely eliminate it. A large B_{0z} is required to affect MRT growth.
- The feedthrough factor is vitually independent of magnetic shear

Use of low density foam to create a final axial magnetic field of order $B_{0\theta}$ upon compression



Conservation of Mass

$$\pi(R_0^2 - R_L^2)\rho_{F0} = 2\pi R_L \Delta R \rho_L$$

Conservation of Magnetic Flux

$$\pi(R_0^2 - R_L^2)B_{0z} = 2\pi R_L \Delta R B_{0\theta}$$

$$\Rightarrow \frac{\rho_{F0}}{\rho_L} = \frac{B_{0z}}{B_{0\theta}}$$

$$\Rightarrow R_0 = R_L\sqrt{1 + \frac{2}{A_R}\frac{B_{0\theta}}{B_{0z}}} \qquad A_R = \frac{R_L}{\Delta R}$$

Assume:

$$A_R = 20$$

$$B_{0z} = 30 \text{ T}$$

$$B_{0\theta} = 1500 \text{ T}$$

$$R_L = 3.3 \text{ mm}$$

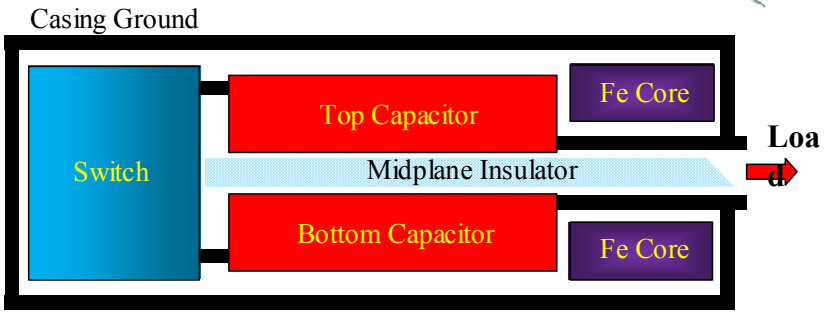
$$\Rightarrow R_0 = 8.08 \text{ mm}$$

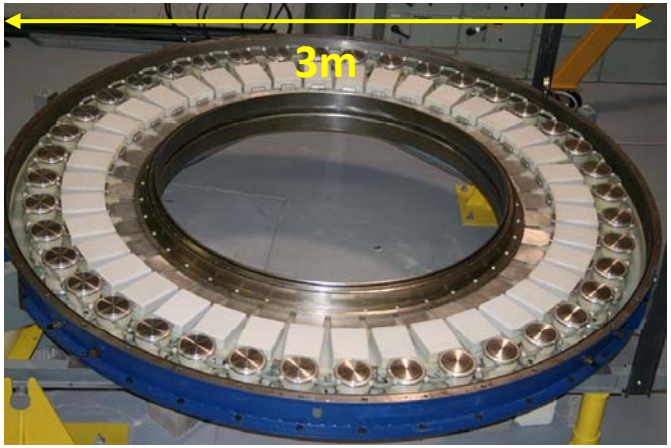
$$\Rightarrow \rho_{F0} = \frac{B_{0z}}{B_{0\theta}} \rho_L = \frac{\rho_L}{50} = 0.037 \text{ g/cc for Be}$$

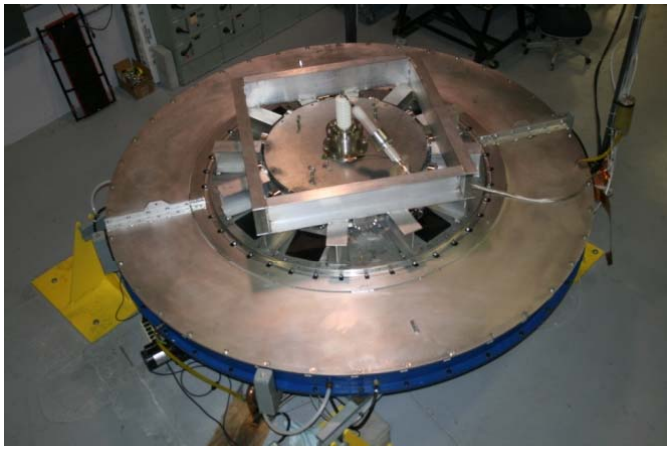
Such a low density foam or gas, upon compression, may create an applied axial magnetic field that exerts some control of MRT by magnetic shear.

MAIZE Linear Transformer Driver (LTD)

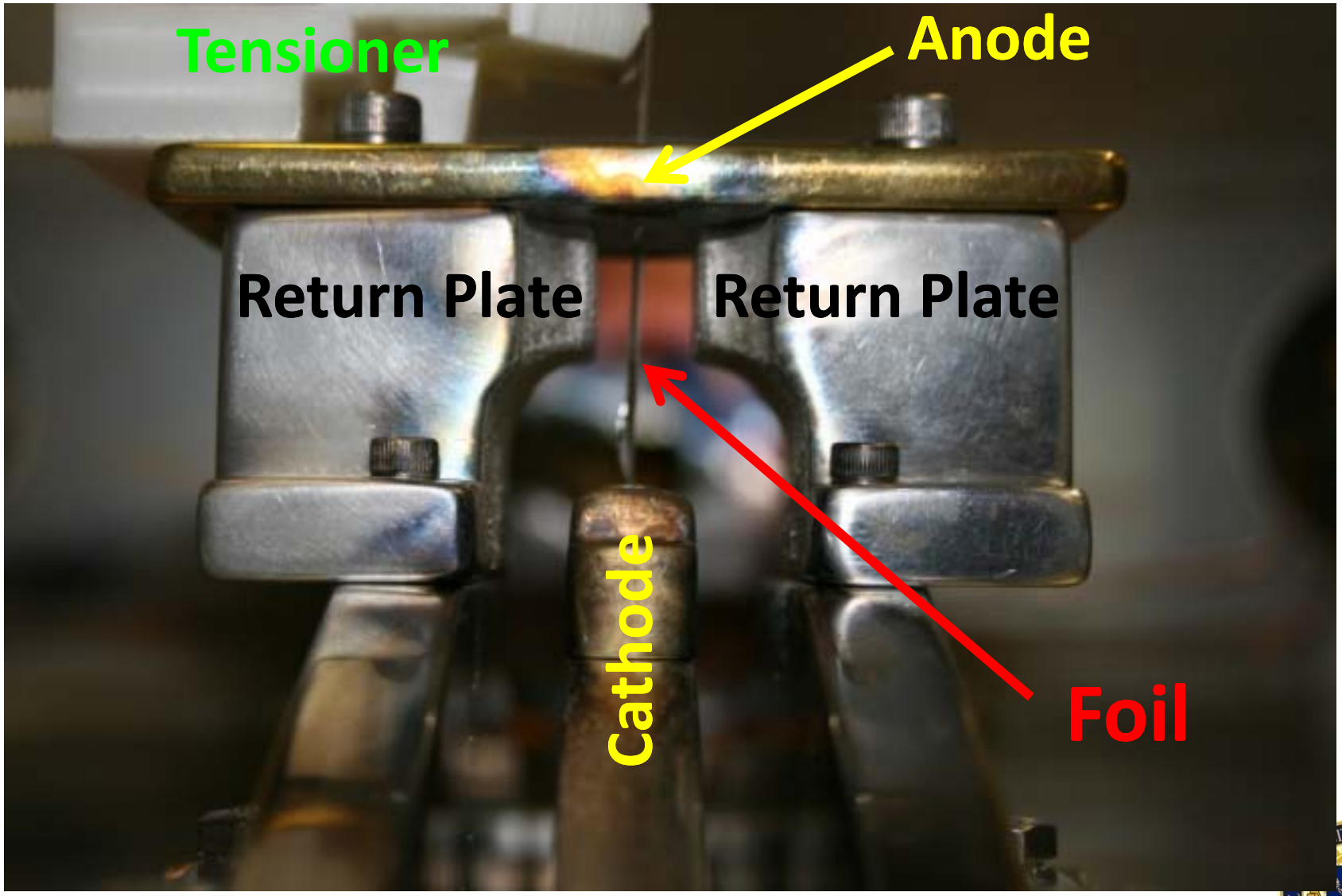
- Parameters
  - 40 Bricks
  - 2 Ferromagnetic cores
  - 1.6 m ID
  - 3.06 m OD
  - 0.22 m Thick
- Matched load (0.1 ohm)
  - **± 100 kV Charge**
  - 1000 kA peak current
  - 100 kV load
  - ~100 ns rise-time
- Foil load
  - **± 70 kV Charge**
  - ~600 kA peak current
  - ~40 kV load
  - 170 ns rise-time





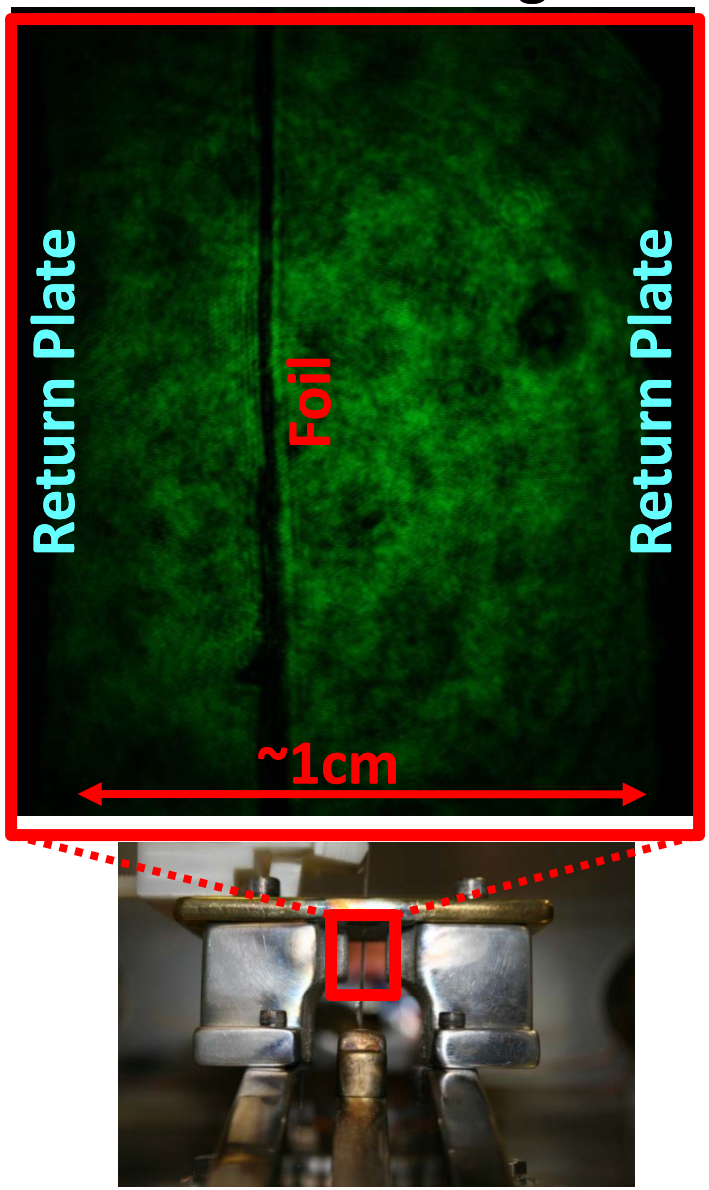


Load Hardware

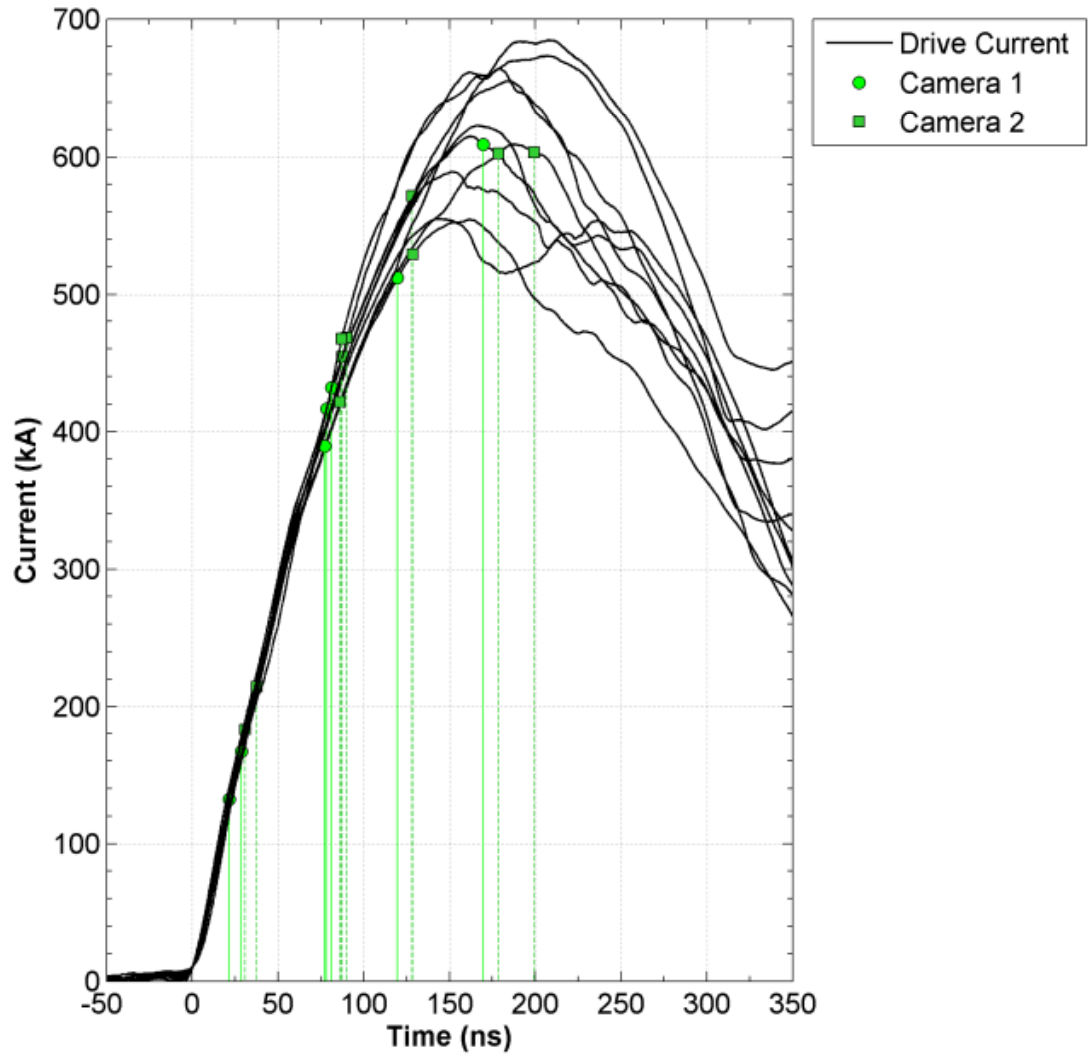


Large Offset Case

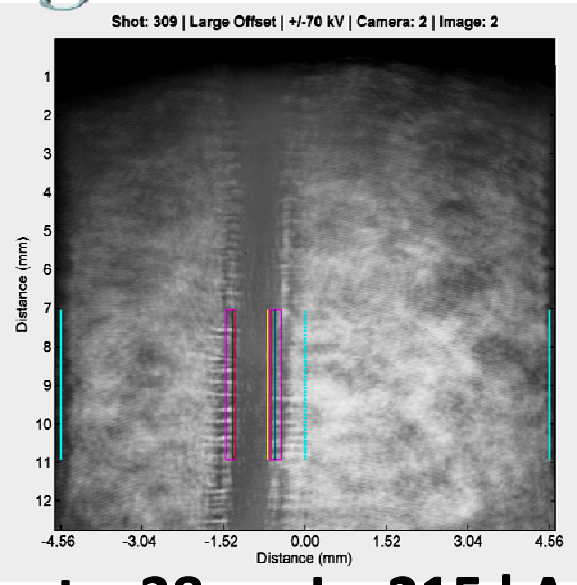
Preshot Image



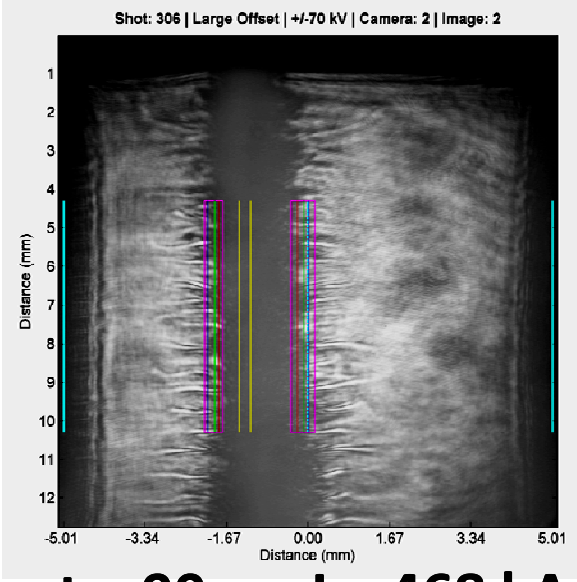
Laser Image Times



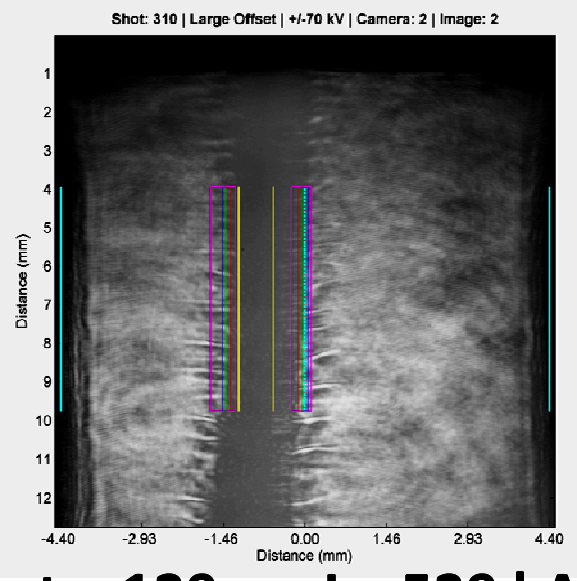
Large Offset: Foil Plasma Evolution



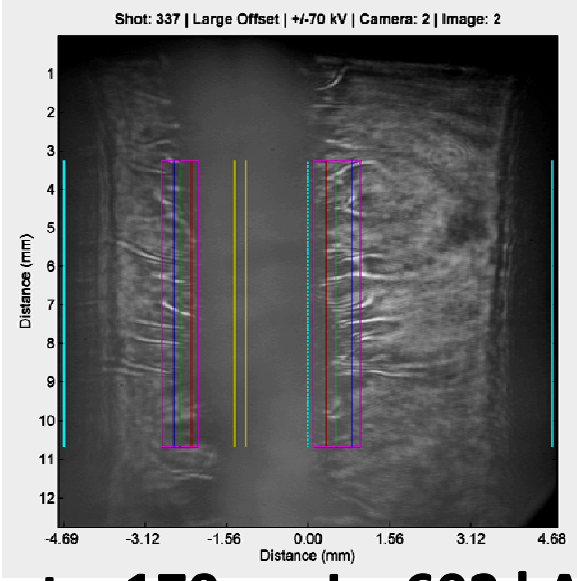
t = 38 ns, I = 215 kA



t = 90 ns, I = 468 kA

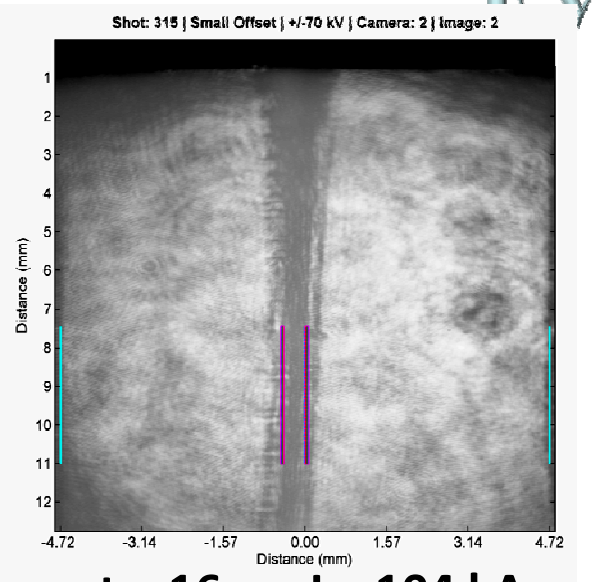


t = 129 ns, I = 529 kA

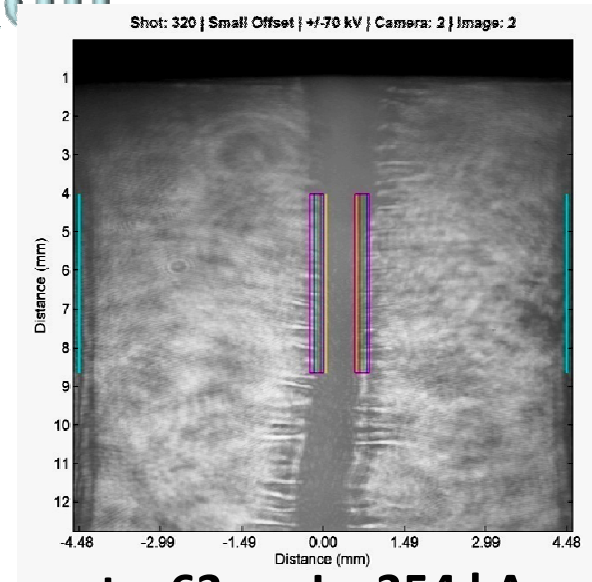


t = 179 ns, I = 602 kA

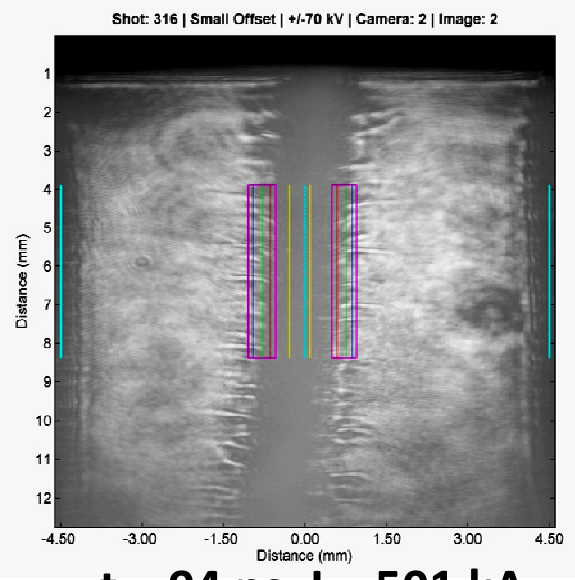
Centered Case: Foil Plasma Evolution



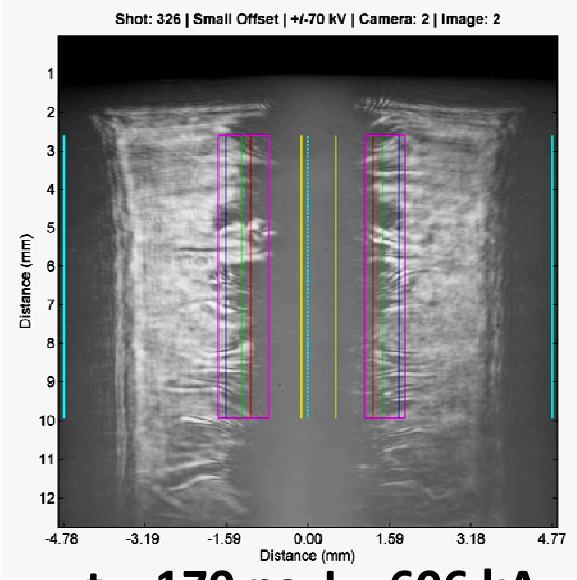
t = 16 ns, I = 104 kA



t = 63 ns, I = 354 kA

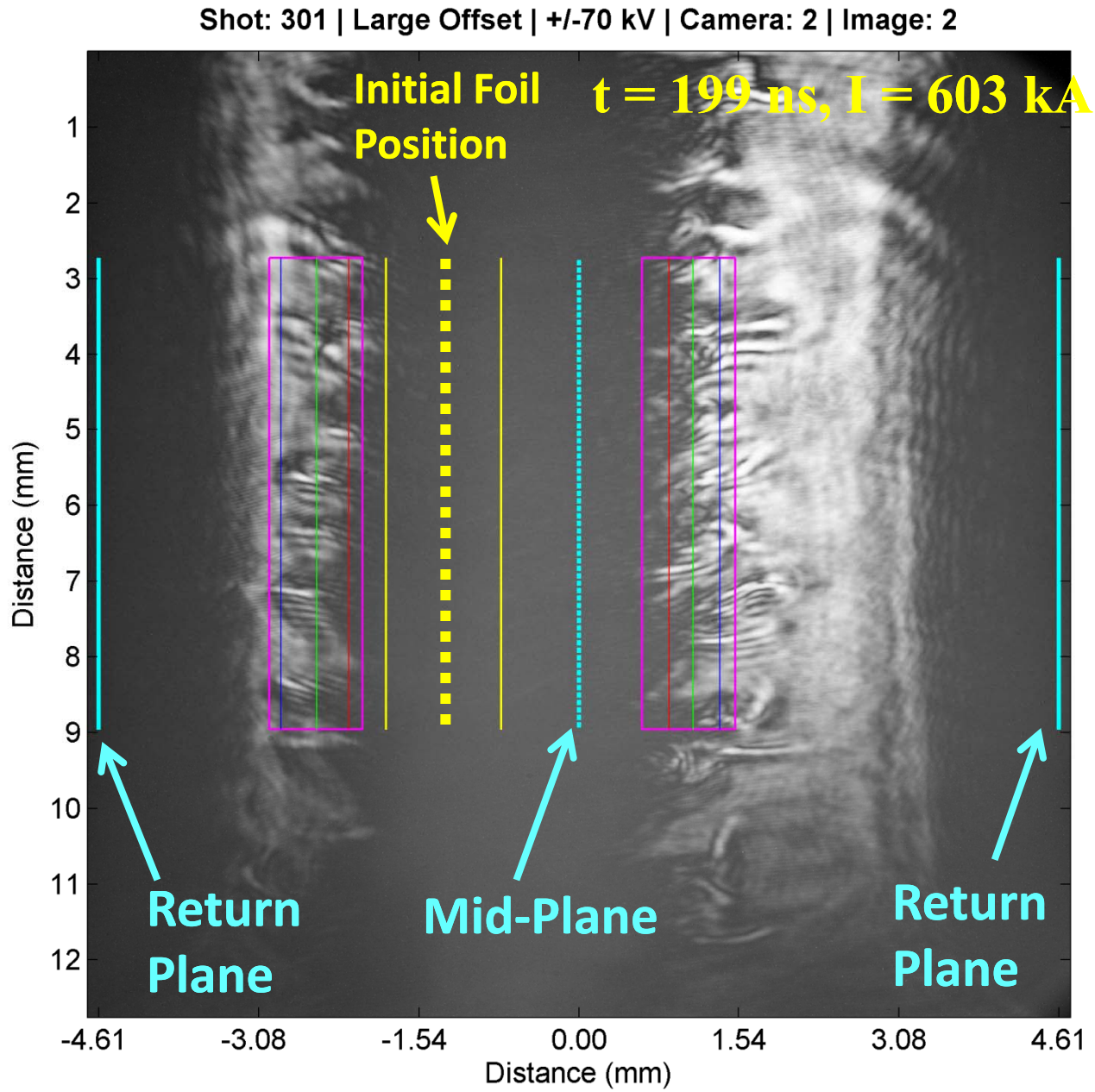


t = 94 ns, I = 501 kA

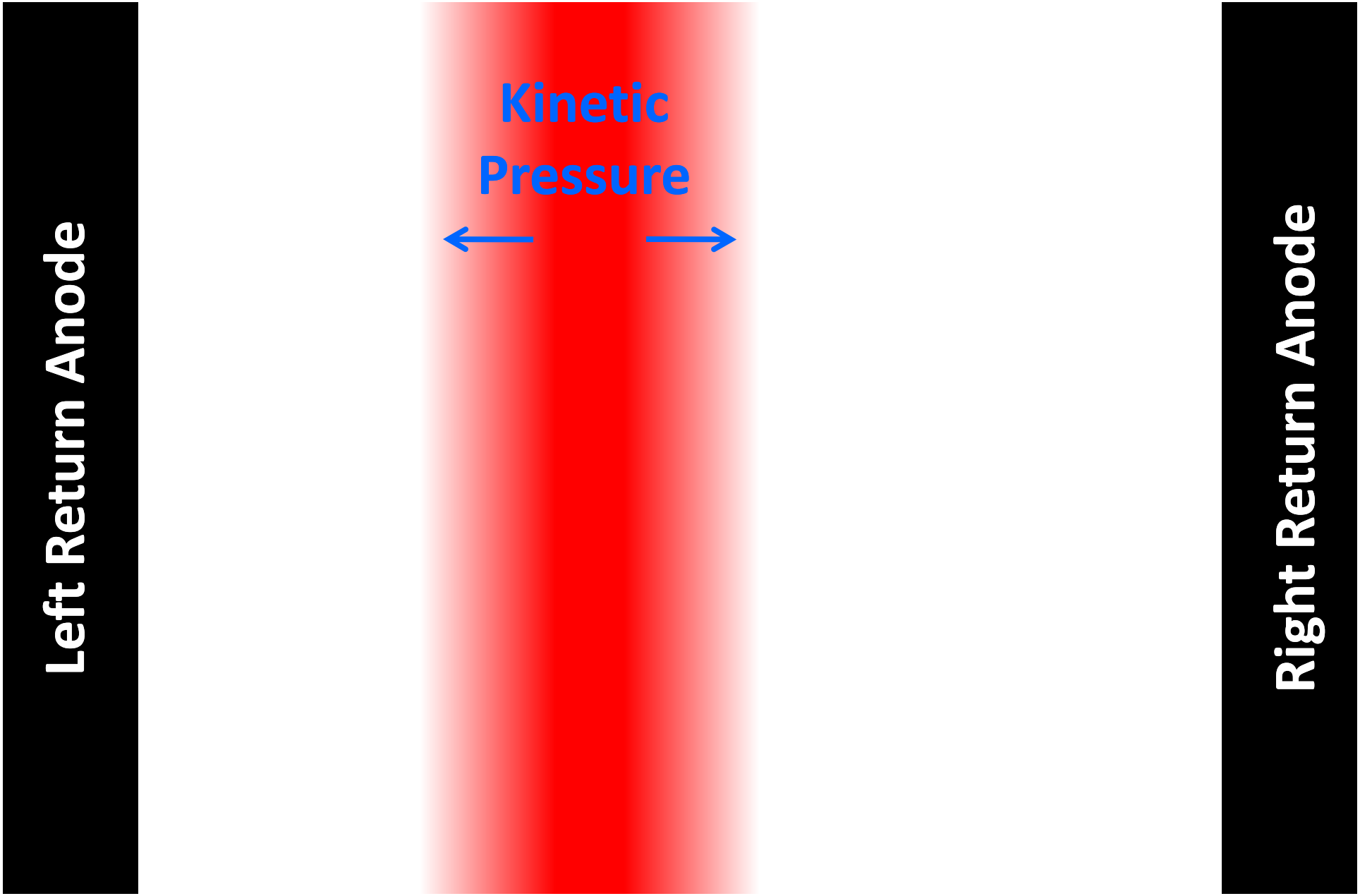


t = 179 ns, I = 606 kA

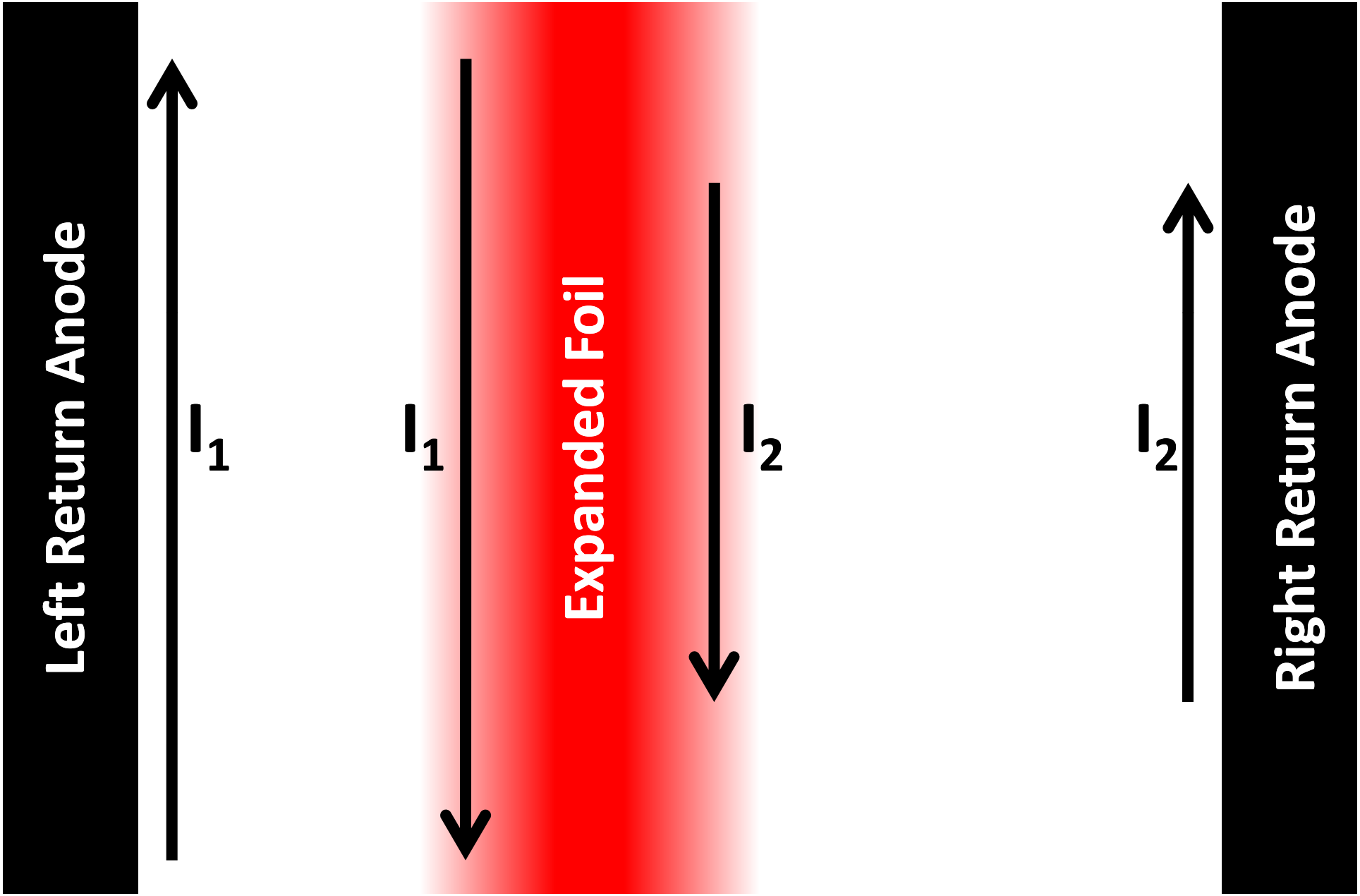
Peak Current



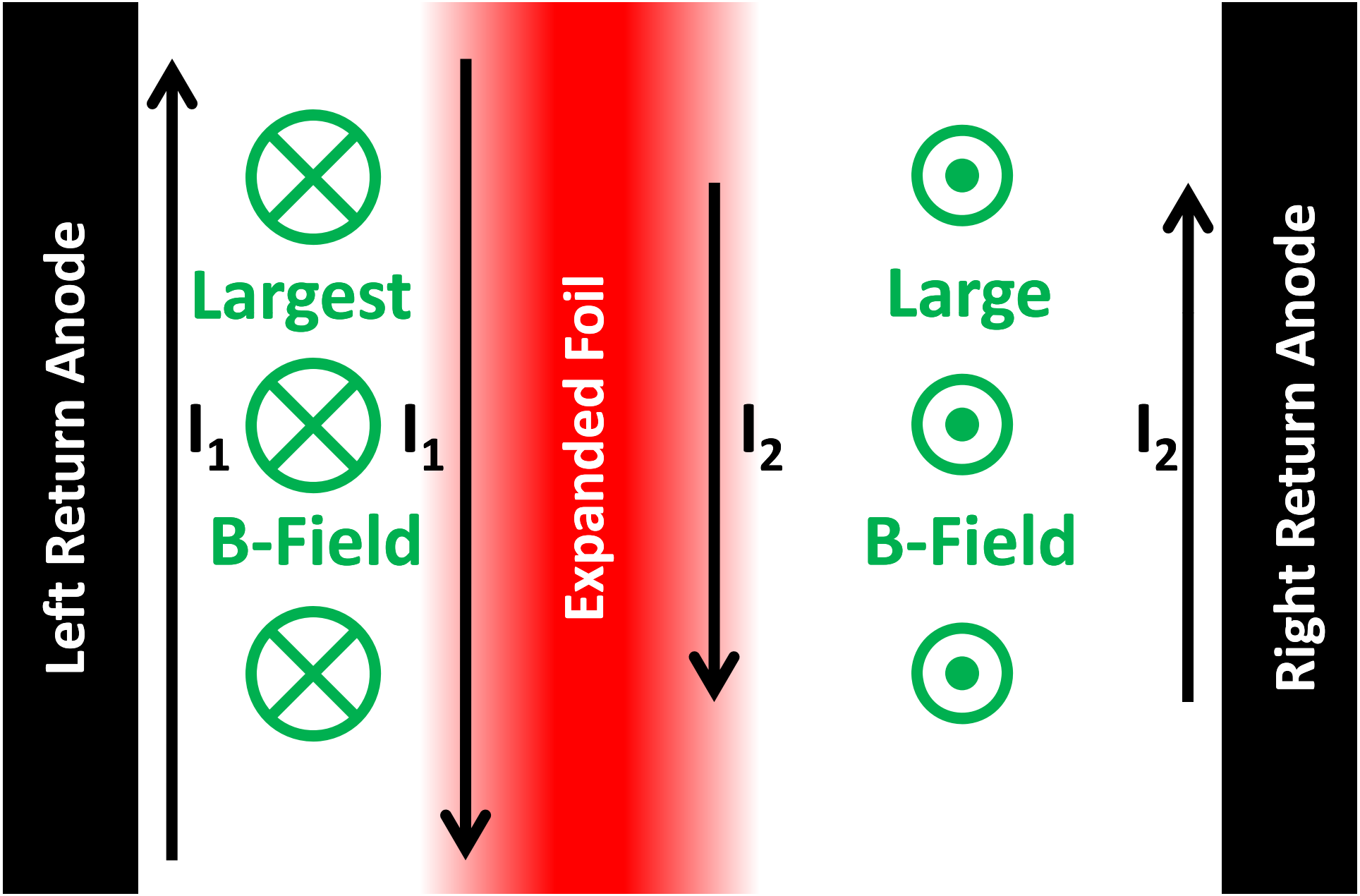
Interface Instability to MRT



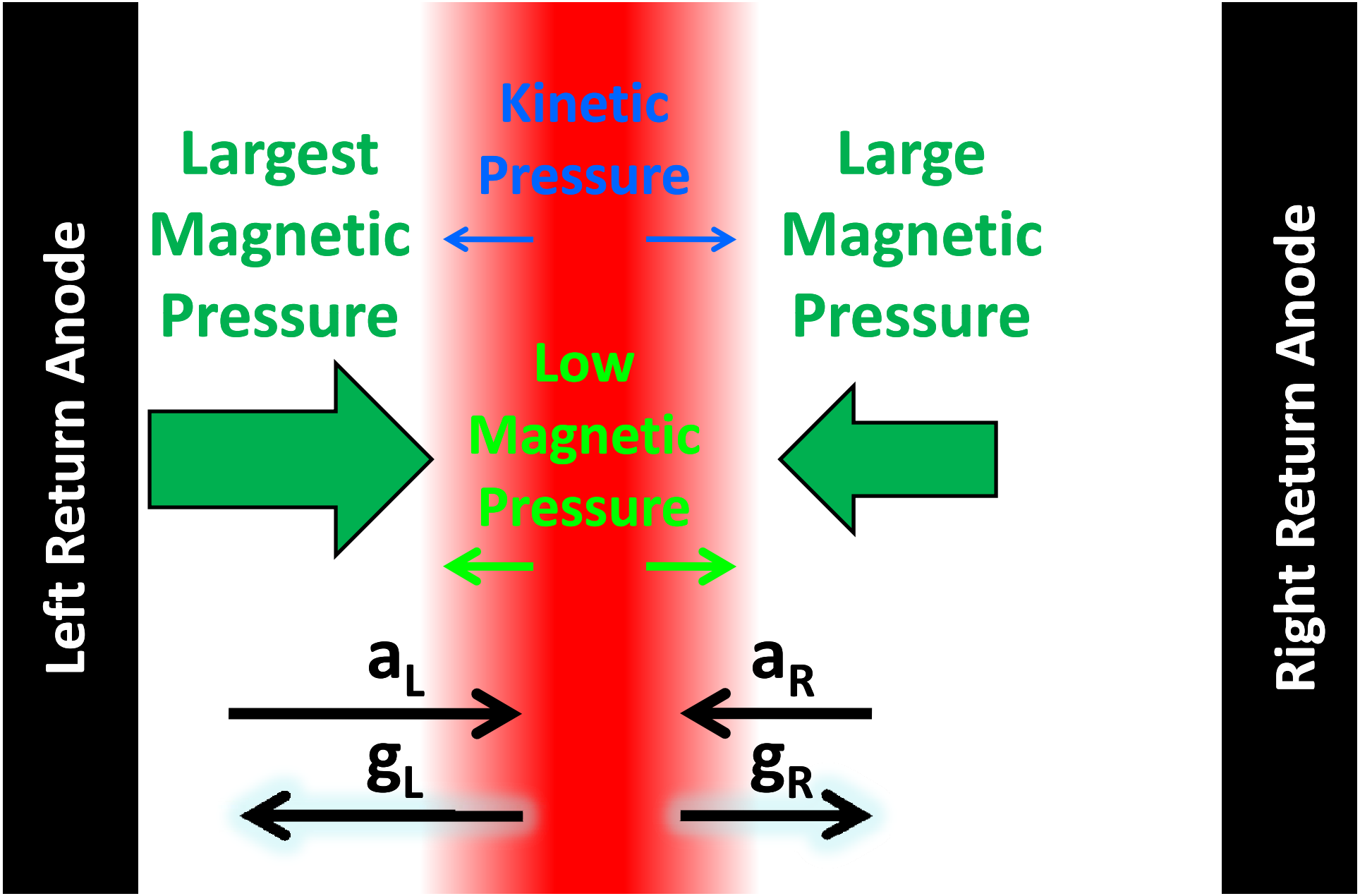
Interface Instability to MRT

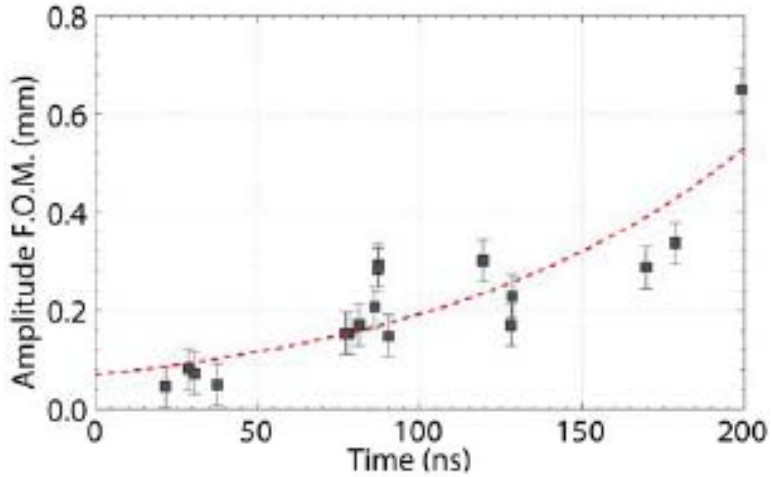


Interface Instability to MRT

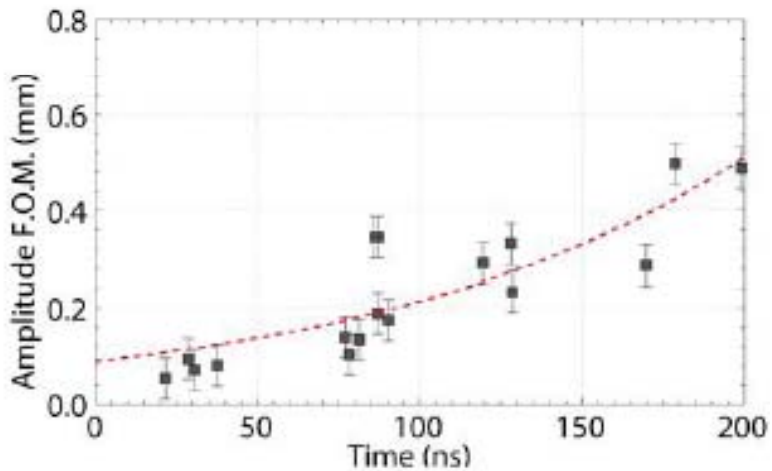


Interface Instability to MRT

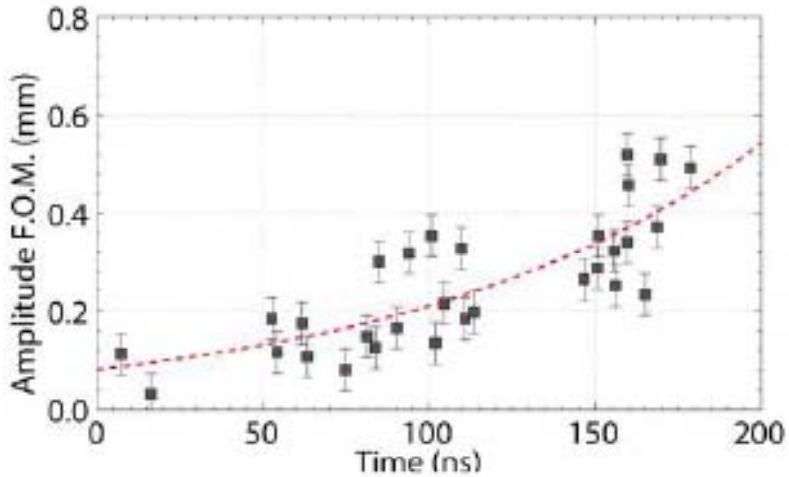




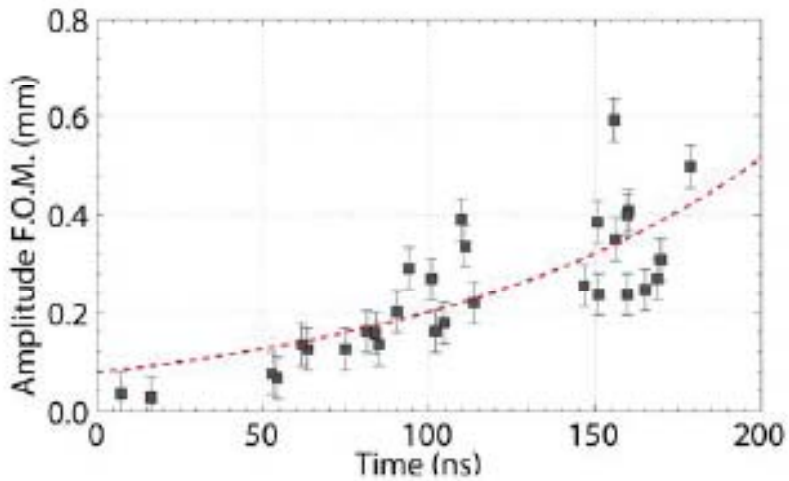
a) Large Offset, Left Side



b) Large Offset, Right Side



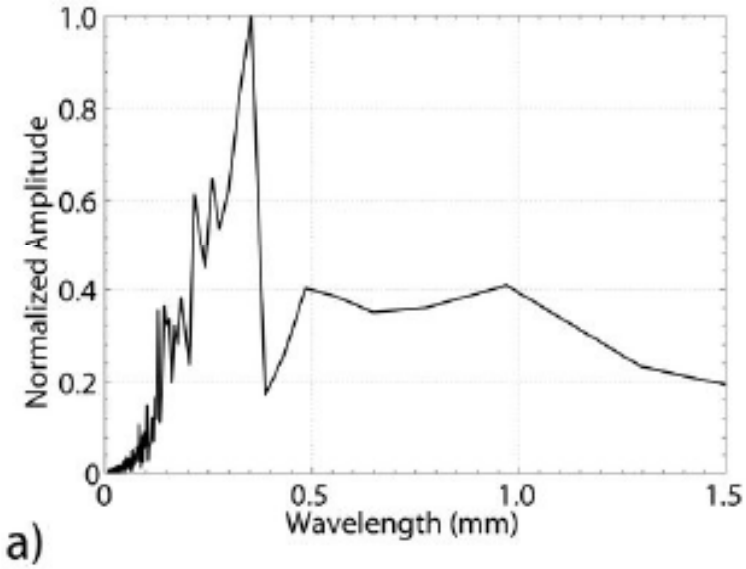
a) Small Offset, Left Side



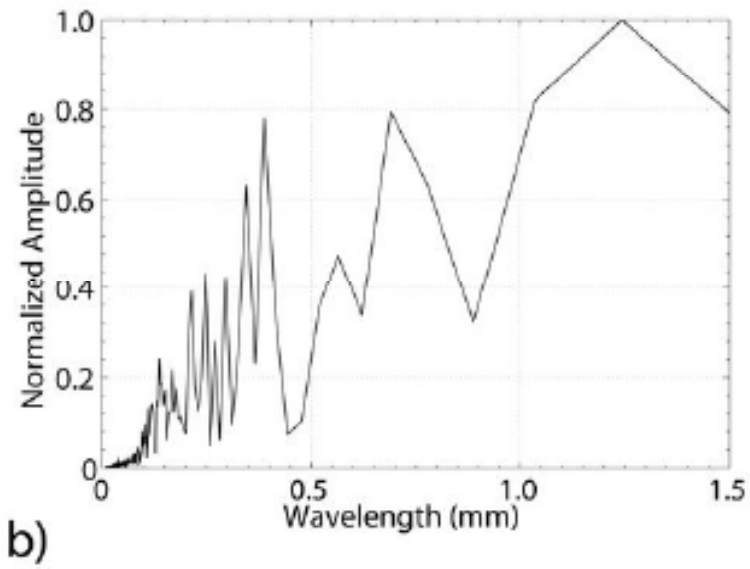
b) Small Offset, Right Side

Summary of MRT Observations and e-folding Times

Case	Expected Fluid Configuration	Observed Fluid Configuration	e-folding time (ns)
Large Offset Left Side	'Light' B-field accelerates 'Heavy' Plasma	Highest magnetic pressure drives fastest growth on MRT unstable interface	98
Large Offset Right Side	'Light' B-field decelerates 'Heavy' Plasma	Early expansion allows right side magnetic pressure to produce MRT unstable interfaces after interfaces decouple; unstable after crosses axis	114
Nearly Centered	No driving acceleration	Both sides show similar growth rate due to early expansion followed by later time magnetic compression	~ 106



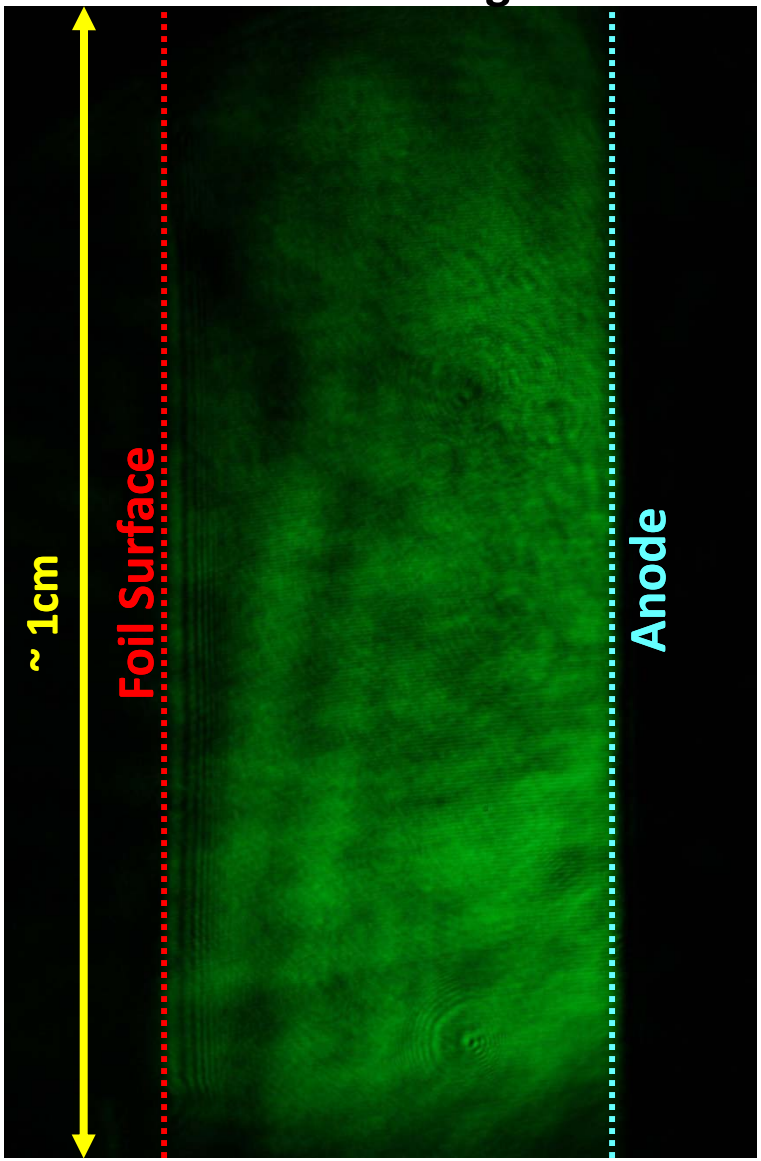
Shot 309
T = 38 ns, I = 215 kA



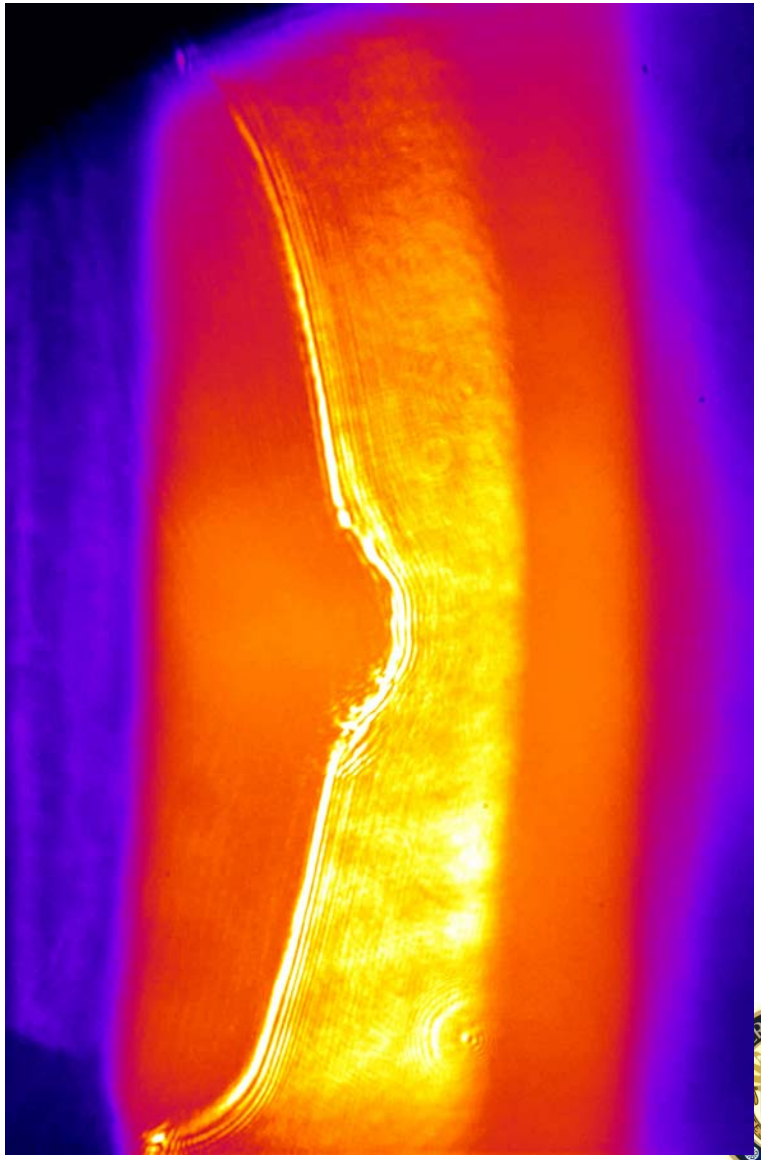
Shot 301
T = 199 ns, I = 603 kA

Vertical Laser Images

Preshot Image



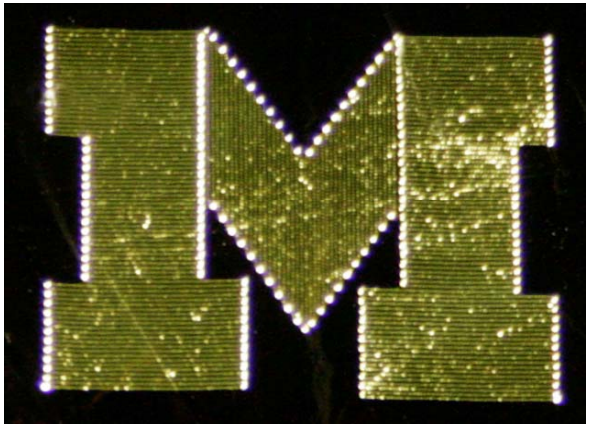
t = ~190 ns



µMachined foils – a brief intro

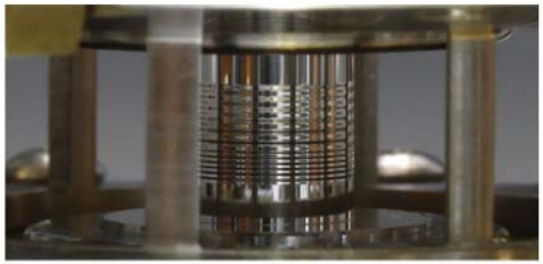
- ~3 mW power incident on foils
- Holes ~40µm diameter
- Spacing ~150µm
- Line spacing ~150µm
- Holes fully penetrate material 400 nm Al foil
  - Mass removal determined by hole size

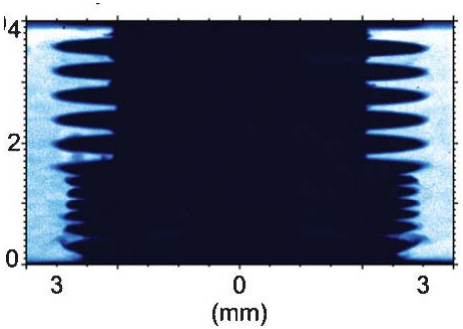
Example of patterned foil



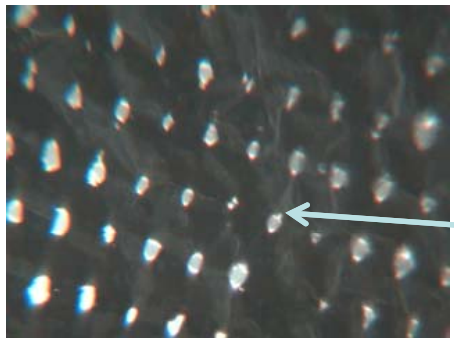
1cm

Image from Sinars et al.





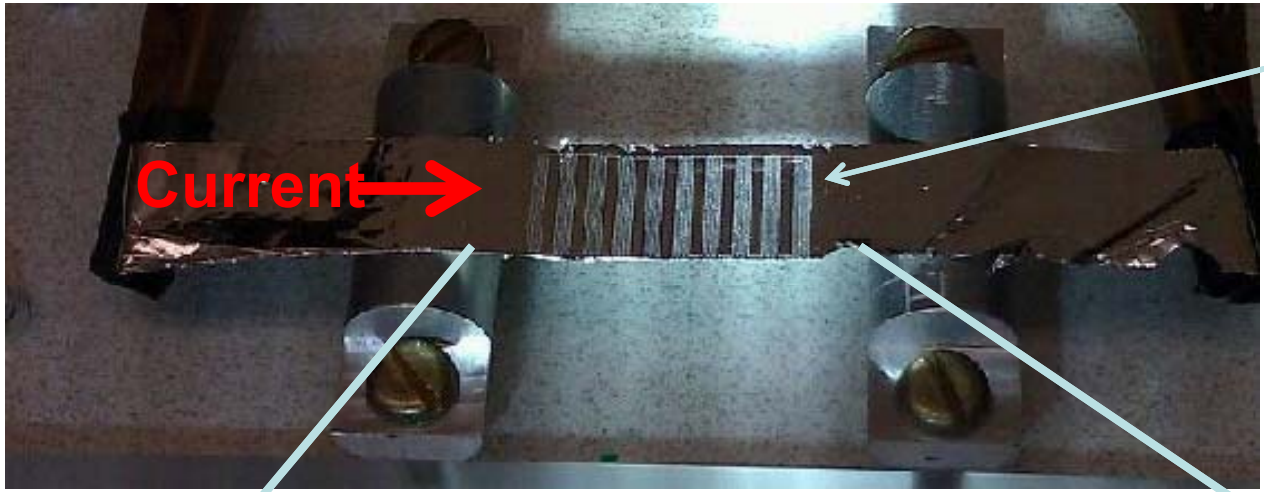
Patterned aluminum 400nm foil



~40µm

Our UMμM facility, 2mm wavelength mass perturbation on 400 nm Al foil

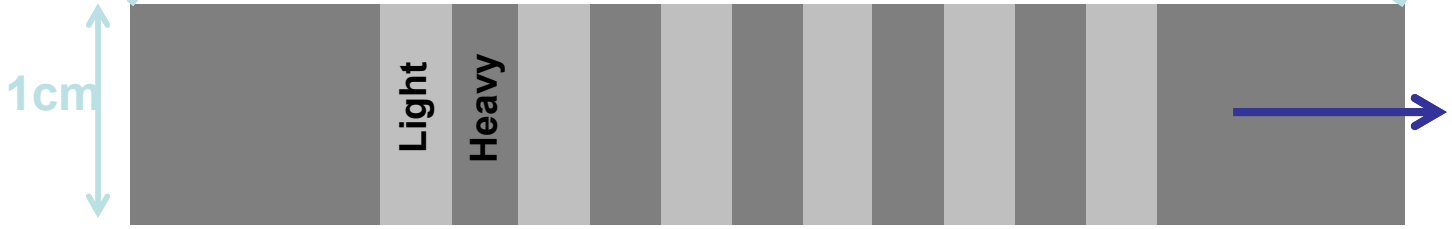
Micro-pattern of removed holes used to create macro pattern of mass removal corresponding to the wavelength of MRT which we are attempting to excite



Light shines through μMachined holes

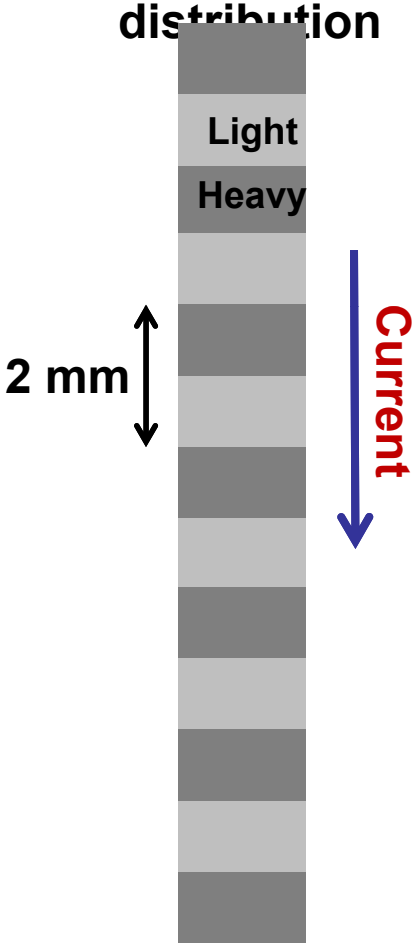
1cm

Resulting pattern of mass perturbation

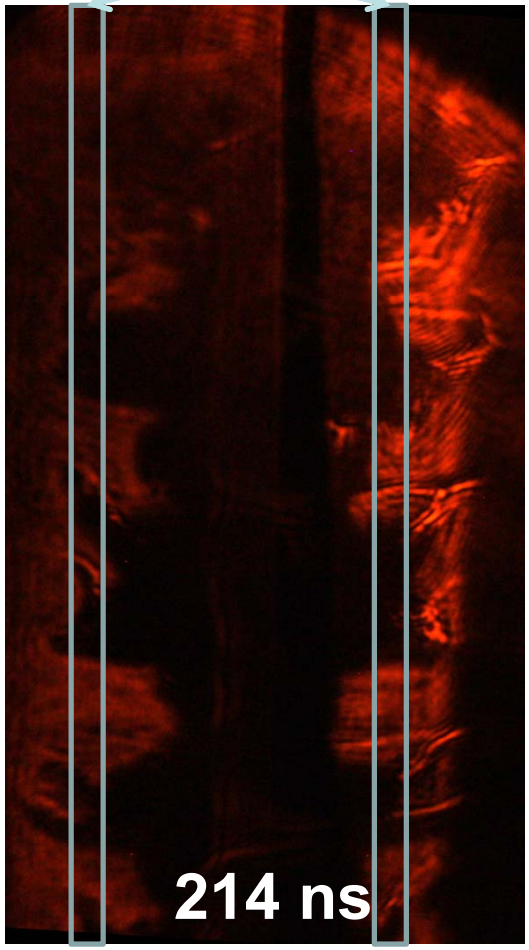


Shot 441 – ~2 mm seeding wavelength

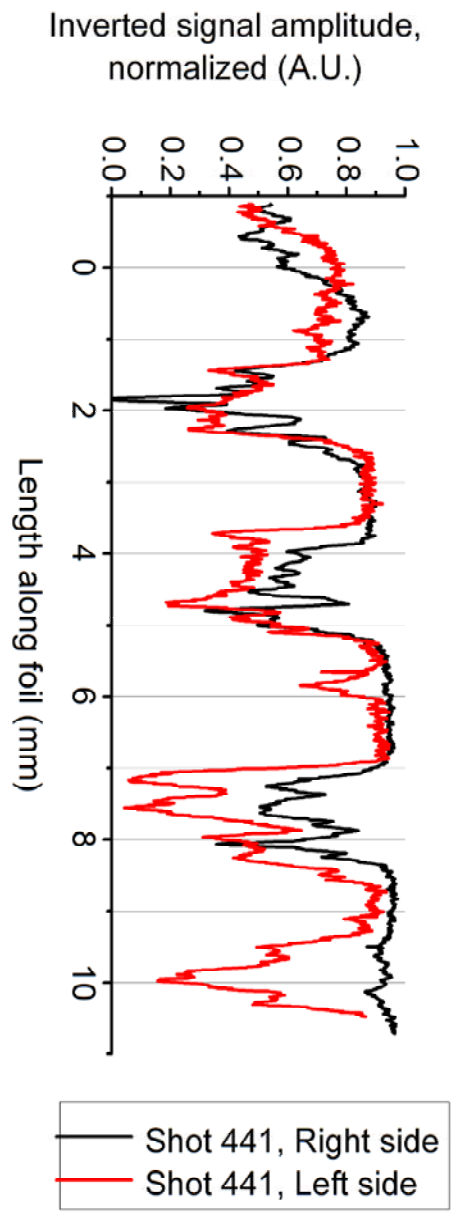
Original foil mass distribution



Boxes used – location/size arbitray. Simply demonstrating effect

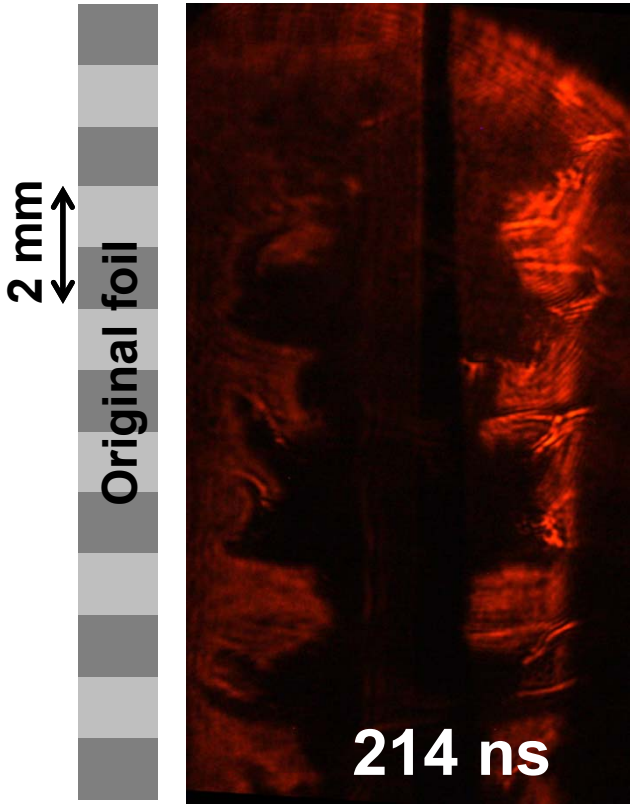


Max I: 467 kA @ 154 ns
Laser at 214 ns



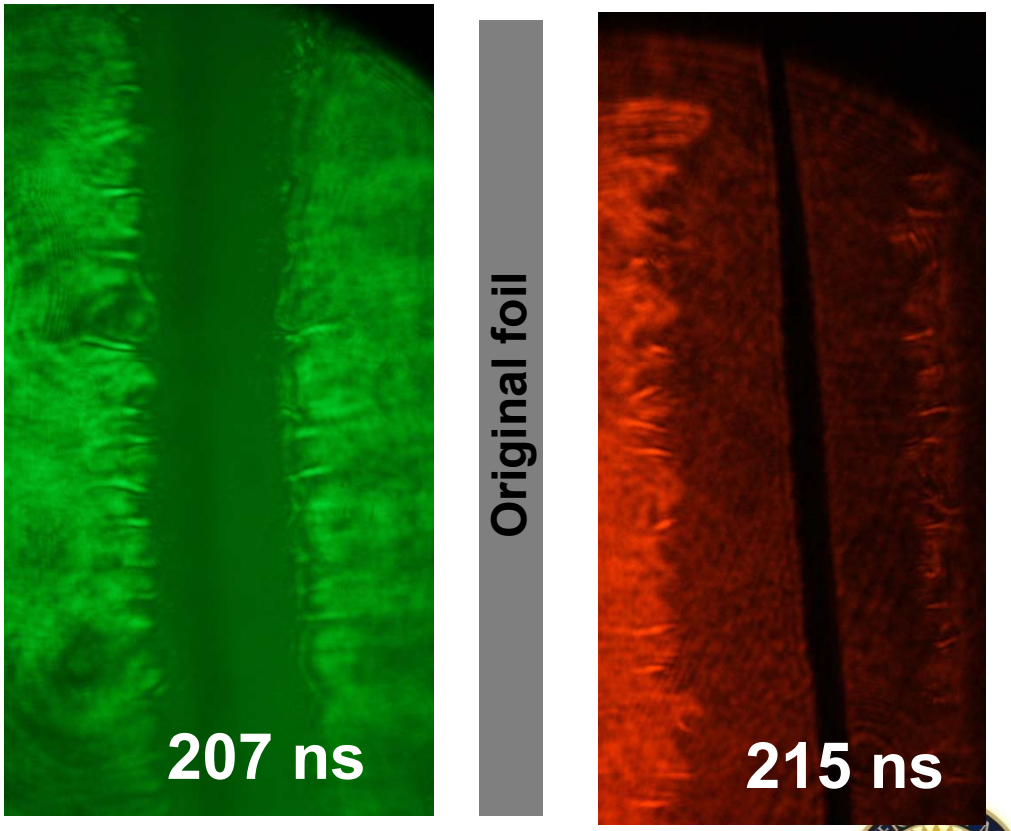
Seeded vs. unseeded – clear indications of expansion perturbation near seeded wavelength

Shot 441 – seeded at ~2 mm, new 150 fs, 775 nm laser



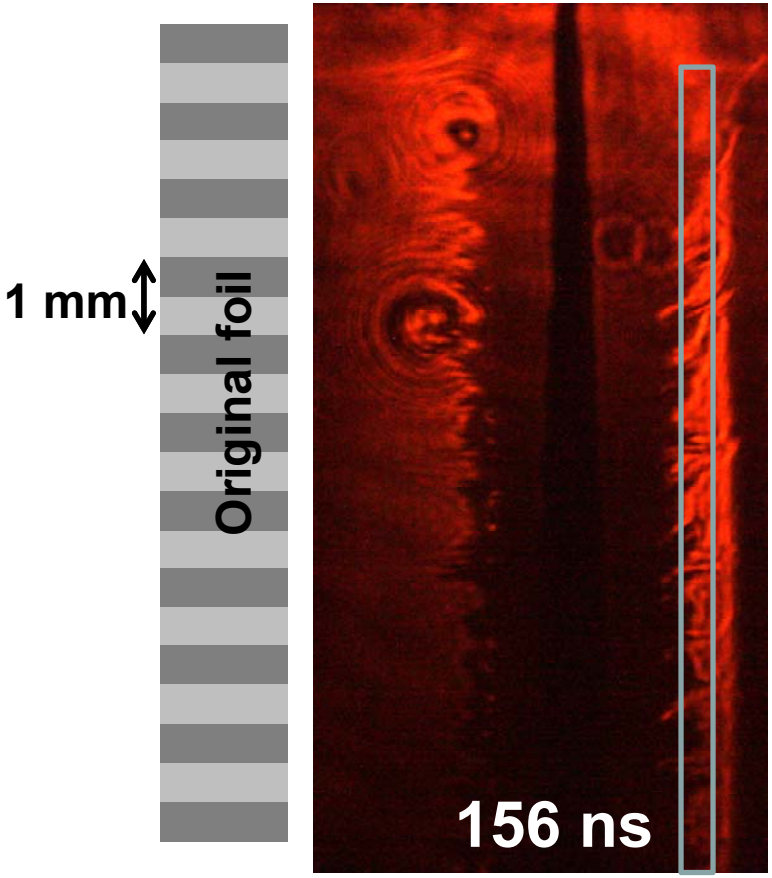
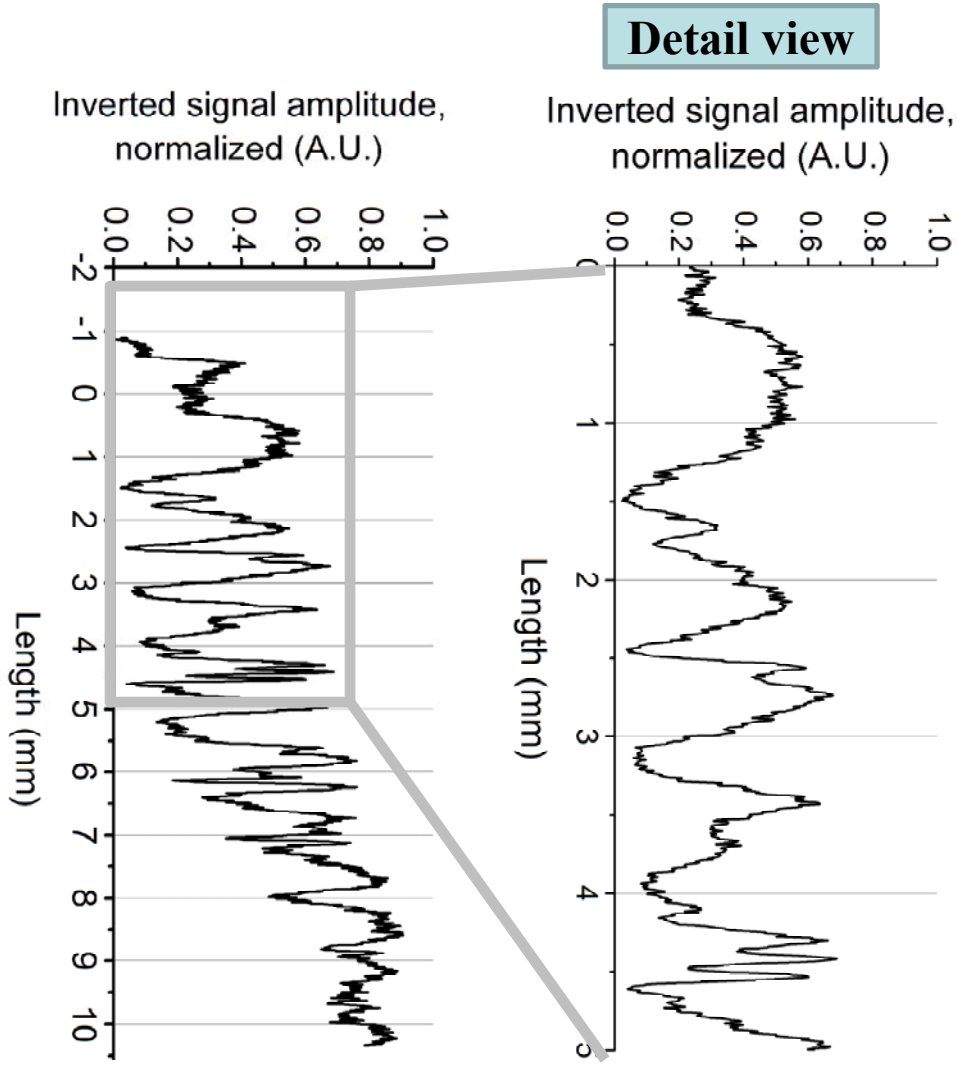
Peak I: 467 kA @ 154 ns

Shot 325 (old Nd:YAG) and 439 (new laser) – no seeding



Peak I: 464 kA @ 206 ns **Peak I: 535 kA @ 169**

Shot 443 – Initial tests on 1 mm wavelength - Effect of seeding seems to weaken



Max I: 600 kA @ 161 ns