

# Multi-Length Scale Algorithms for Failure Modeling in Solid Mechanics



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## Introduction

- Explicit dynamics is widely used in solid mechanics, especially for modeling failure
  - Smallest elements in model control critical time step for stability
  - Explicit dynamics does not work well with models that span length scales because the time step is too small
- A multi-length scale technique has been developed for transient dynamics
  - A coarse mesh is used in conjunction with the fine mesh of the actual model.
  - Coarse mesh controls time step
  - Fine mesh models geometry in detail

## Methodology

- Dynamic equilibrium ( $\mathbf{a}$  is acceleration,  $\mathbf{M}$  is mass,  $\mathbf{R}$  is residual):

$$\mathbf{M}\mathbf{a} = \mathbf{R}$$

- Decompose  $\mathbf{a}$  into high and low frequency parts  $\mathbf{a}_{hf}$  and  $\mathbf{a}_{lf}$  represented on fine and coarse meshes where  $\Phi$  is an interpolation matrix:

$$\mathbf{a} = \mathbf{a}_{hf} + \Phi \mathbf{a}_{lf}$$

- Orthogonal decomposition in  $\mathbf{M}$  yields equation for acceleration from high, low frequency components:

$$\mathbf{a} = \Phi \mathbf{M}_c^{-1} \Phi^T \mathbf{R} + \mathbf{M}^{-1} (\mathbf{R} - \mathbf{M} \Phi \mathbf{M}_c^{-1} \Phi^T \mathbf{R})$$

where  $\mathbf{M}_c$  is the mass on the coarse mesh, and is computed as:

$$\mathbf{M}_c = \Phi^T \mathbf{M} \Phi$$

- Scale mass only on high frequency response using diagonal scaling matrix,  $\alpha$ , to allow integration at time step of coarse mesh

- This yields the following dynamic equilibrium equation:

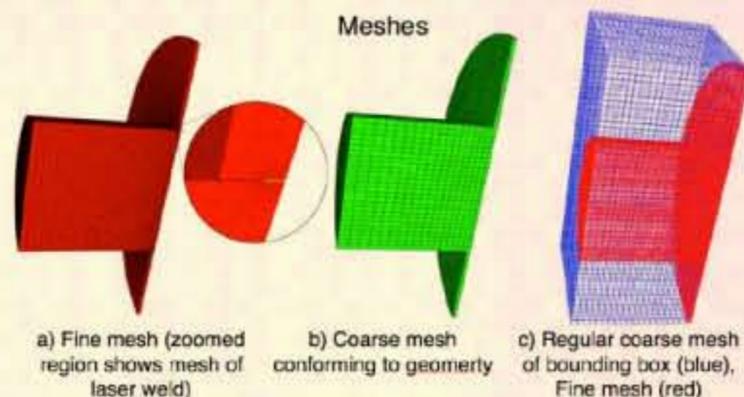
$$\mathbf{a} = \Phi \mathbf{M}_{lf}^{-1} \Phi^T \mathbf{R} + (\alpha \mathbf{M})^{-1} (\mathbf{R} - \mathbf{M} \Phi \mathbf{M}_{lf}^{-1} \Phi^T \mathbf{R})$$



## Applications

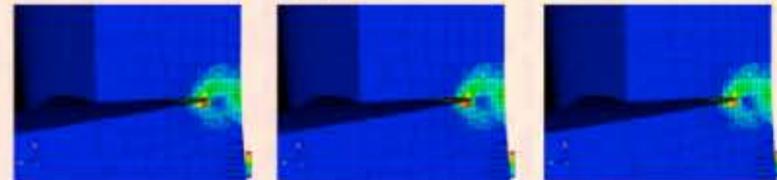
### Laser Weld Test

- This explicit multi-length scale technique has been used to model an experimental test of laser welds on a weapon component
- The test consists of a "plug" laser welded to a base plate. The base plate is bolted to an assembly that is dropped. When the assembly hits the bottom, the inertia of the plug loads the laser weld.
- An extreme amount of mesh refinement is required to model failure in the welds



## Applications

### Results (Equivalent Plastic Strain)

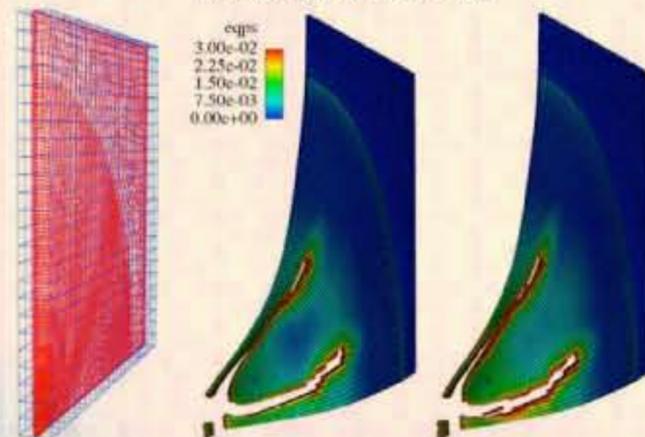


- a) Baseline model (a) critical  $dt = 3.6e-9s$ , run time = 205m (200 processors)
- b) Multi-length scale model with conforming coarse mesh (b) critical  $dt = 1.2e-7s$ , run time = 8m (200 processors) 24x speedup
- c) Multi-length scale model with regular coarse mesh (c) critical  $dt = 1.8e-7s$ , run time = 6m (200 processors) 36x speedup

### Plate Blast Test

- A plate is subjected to a blast load at the center
- This test demonstrates that the explicit multi-length scale technique can model problems with large deformations and failure (element death)

### Model and Results



- a) Fine mesh of plate (red) with coarse mesh (blue)
- b) Baseline model (colored by equivalent plastic strain)
- c) Multi-length scale model (4.2x increase in critical time step, 2.2x speed-up)

## Summary

- Multi-length scale technique for explicit dynamics permits the use of fine meshes that capture multiple length scales with reasonable time steps
- Accuracy and efficiency of this technique has been demonstrated
- Fine mesh is chosen to adequately capture geometry and solution gradients of interest without concerns for the effect of small elements on time step
- Coarse mesh is chosen to allow integration at a tractable time step. Dynamics of interest can be captured with a time step much greater than that dictated by stability considerations.