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Go Figure 1999

For Students in grades $7, 8, 9, 10, 11$, and 12

Show your work. You can receive partial credit for partial solutions. Please write all solutions clearly, concisely, and legibly.

The positive integers are the numbers of the unending sequence

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, \ldots.$$

1. Find numbers a and b given that every term after the first term in the sequence

   $$a, b, 6, 18, 54, 162$$

   is a fixed number times the term just before it.

2. In an artithmetic progression, one adds a fixed number to a term to get the term that just follows it. Find the 100th term of each of the following unending arithmetic progressions:

   (a) $7, 14, 21, 28, 35, \ldots$

   (b) $5, 12, 19, 26, 33, \ldots$

   (c) $18, 25, 32, 39, 46, \ldots$

3. Find the ten positive integral factors of 48. [The positive integral factors of a positive integer m are the integers d such that $m \div d$ is a positive integer]. For example, the positive integral factors of 12 are $1, 2, 3, 4, 6, 12$.

4. What is the largest product of positive integers whose sum is 8? [The possible products are $7 \times 1,\ 6 \times 2,\ 6 \times 1 \times 1,\ 5 \times 3,\ 5 \times 2 \times 1,\ 5 \times 1 \times 1 \times 1,\ 4 \times 4,\ 4 \times 3 \times 1,\ 4 \times 2 \times 2,$ $4 \times 2 \times 1 \times 1,\ 4 \times 1 \times 1 \times 1 \times 1,\ 3 \times 3 \times 2,\ 3 \times 3 \times 1 \times 1,\ 3 \times 2 \times 2 \times 1,$ $3 \times 2 \times 1 \times 1 \times 1,\ 3 \times 1 \times 1 \times 1 \times 1 \times 1,\ 2 \times 2 \times 2 \times 2,\ 2 \times 2 \times 2 \times 1 \times 1,$ $2 \times 2 \times 1 \times 1 \times 1 \times 1,\ 2 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1,\ 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1.$]

5. Consider the five fractions

   $$\frac{5}{7}, \frac{12}{17}, \frac{20}{27}, \frac{30}{41}, \frac{60}{83}.$$

   (a) Which is the smallest number among these fractions?

   (b) Which is the largest number among these fractions?

6. In this problem, each letter represents one of the digits $1, 2, 3, \ldots, 9$. For example, if $D = 9$, then $2D$ is the number 29 and $DDD4$ is 9994.
   Find the digits A, B, and C such that

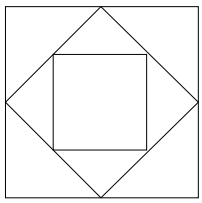
   $$A3 \times 2B = CCC8.$$

7. What is the largest product of positive integers whose sum is 29? [HINT: See Problem 4. The answer is larger than $5 \times 5 \times 5 \times 5 \times 5 \times 4$.]

PROBLEM 8 WILL BE USED ONLY TO BREAK TIES

8. As clearly as you can, describe a way to get the answer to problem 7 without unnecessary calculations.

9. Here A, B, C, D, E, F is a sequence of six squares. Each side of A has length 8 units. Except for A, the vertices of each square in this sequence are the midpoints of the sides of the square just before it in the sequence. The figure pictures any three consecutive squares of the sequence.



   (a) What is the area of the square C?

   (b) What is the shortest distance from a vertex of A to a vertex of F?

   (c) What is the largest distance from a vertex of A to a vertex of F?

10. In the listing for a multiset, some of the entries may be repeated. A multiset of numbers generates a value g if g is one of the entries or g is a sum of entries with no term repeated more times in the sum than in the multiset. For example, the multiset $\{1, 1, 1, 5\}$ generates the numbers 1, $1+1 = 2$, $1+1+1 = 3$, 5, $1+5 = 6$, $1+1+5 = 7$, and $1 + 1 + 1 + 5 = 8$. Also the multiset $\{3, 5, 10, 12\}$ generates fourteen numbers; they are $3, 5, 8, 10, 12, 13, 15, 17, 18, 20, 22, 25, 27, 30$. Note that 15 is generated both as $3 + 12$ and as $5 + 10$.

   (a) Give a multiset with three entries that generates $1, 2, 3, 4, 5, 6, 7$.

   (b) Give a multiset with five entries that generates the 31 numbers $1, 2, 3, \ldots, 31$.

   (c) How many numbers are generated by the multiset {1,1,1,5,7}?

   (d) How many numbers are generated by {1,1,1,5,7,25,25,125,625,3125}?