Sponsored by Sandia National Laboratories **October 19, 2002**

Go Figure 2002

For Students in grades 7, 8, 9, 10, 11, and 12

Show your work. You can receive partial credit for partial solutions. Please write all solutions clearly, concisely, and legibly.

The positive integers are the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 \ldots$

Percentage is a shorter, more convenient way to refer to a fraction with a denominator (bottom) of 100. For example, the following are all the same thing: 42% (42 percent), $\frac{42}{100}$, and .42. When we say 42 percent of a set of marbles is red, we mean that 42 out of every 100 marbles is red (if we could mix them perfectly and pick 100). The number of red marbles divided by the total number of marbles is .42.

1. Let B be 1000 increased by 15%. That is, B is 15% larger than 1000. Let C be B decreased by 15%. What is C?

2. A jar contains 600 coins and a coffee can contains 150 coins. 30% of the coins in the jar are pennies. 20% of the coins in the coffee can are pennies. The jar and coffee can are emptied into an initially-empty bucket. What percentage of the coins in the bucket are pennies?

3. Put the following fractions in order from largest to smallest: $\frac{7}{9}, \frac{12}{13}, \frac{21}{31}, \frac{42}{55}, \frac{84}{97}$.

4. In an arithmetic progression, the difference $s - t$ of adjacent terms t, s is fixed. For example, the arithmetic progression $4, 8, 12, 16 \ldots$ has $(8 - 4) = (12 - 8) = (16 - 12) = 4$ as the fixed difference.

   (a) Find the 103rd term of the unending arithmetic progression $3, 6, 9, 12 \ldots$ [The three dots after 12 are read as "and so on" to indicate that this unending progression continues with the pattern required of an arithmetic progression.]

   (b) How many terms are in the arithmetic progression $6, 11, 16, \ldots, 441$?

   (c) What is the sum of the first and last terms of the arithmetic progression in part (b)?

   (d) What is the sum of the second term and the next-to-last term of this progression?

   (e) What is the sum of all the terms of the arithmetic progression in part (b)? Hint: consider your answers to parts (b), (c), and (d).

5. In this problem, each of A, B and C stands for a different digit chosen from $\{0, 1, 2, 3, 6, 7\}$ and each is fixed throughout the problem. For example, if F stood for 6 then $3F$ would represent 36 and $F4F$ would stand for 646. Find digits A, B, C that make the following a correct multiplication statement:

$$5A4 \times BCC = 9AC48.$$

6. An *interval* is a set of consecutive positive integers. For example $1, 2, 3, 4, 5$ is an interval with 5 terms and $12, 13, 14, 15, 16, 17, 18, 19, 20$ is an interval with 9 terms. We can represent an interval as the first term in the interval, followed by two dots, followed by the final term in the interval. For example, the interval $1, 2, 3, 4, 5$ may be written as 1..5.

   (a) Find an interval with 10 terms that contains the fewest possible numbers divisible by 7. (There are many possible solutions)

   (b) Find an interval with 10 terms that contains the most possible numbers divisible by 7. (There are many solutions).

   (c) How many numbers in the interval 2028..5048 are divisible by 7?

7. The 3-*score* of a number n is the number of times 3 repeatedly divides n. Alternatively, it is the power (exponent) of 3 in the prime factorization for n. For example, the 3-score of 27 is 3 because $27 = 3\times3\times3 = 3^3$ divides 27. The 3-score of 21 is 1 because $3^1 = 3$ divides 21 but $3^2 = 9$ does not. The 3-score of 17 is 0 because it is not divisible by 3 at all.

   (a) What number in the interval 1..250 has the highest 3-score? What is that 3-score?

   (b) If the 3-score in part (a) was s, how many numbers in the interval 1..250 have 3-score at least $s-1$?

   (c) What is the sum of the 3-scores of all the numbers in the interval 1..250?

   (d) What is the sum of the 3-scores of all the numbers in the interval 244..493?

8. A multiset is a set of integers in which some entries may be repeated. The sum of a multiset is the sum of the elements (with repetition). For example, the sum of $\{1, 1, 1, 5\}$ is $1+1+1+5 = 8$ and the sum of $\{2, 2, 8, 8\}$ is $2+2+8+8 = 20$. A multiset is generated from a base set by selecting each element in the base set zero or more times. For example, base set $\{1, 5\}$ generates multisets $\{1\}, \{5\}, \{1, 5\}, \{1, 1\}, \{5, 5\}, \{1, 1, 5\}, \{1, 5, 5\}$, and so on.

   (a) How many multisets generated from base set $\{1, 2\}$ have a sum of 10?

   (b) How many multisets generated from base set $\{1, 2, 6\}$ have a sum of 12?

   (c) How many multisets generated from base set $\{1, 2, 6\}$ have a sum of 5994?

9. Suppose you plant a vegetable garden every year, always with the same kinds of vegetables. Your friend gives you 5 new types of seeds so you can add new plants to your garden next year if you want to. There are 3 types of flowers and 2 types of herbs (such as oregano and cilantro).

   (a) How many different types of garden (mixes of new plants with the old) can you plant next year if you don't plant any herbs? Remember that leaving the garden as it was (planting nothing new) is an option. Hint: you must make an add/not-add decision for each type of flower.

   (b) How many different types of garden can you plant next year if you consider all 5 types of seed?

   (c) How many different types of garden can you plant if you will consider adding either flowers or herbs, but you will not mix flowers with herbs?

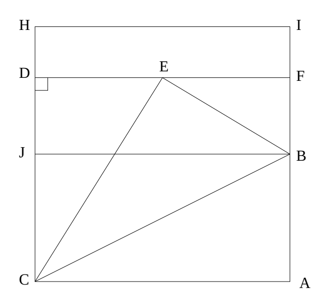
   (d) How many ways can you plant exactly 3 new plants?

10. The factorial of a positive integer n is denoted $n!$ and read "n factorial". It is defined as $n! = n \times (n-1) \times (n-2) \ldots \times 2 \times 1$. For example, $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. The number of trailing zeros in a positive integer is the number of decimal places, starting with the units place, that are zero. For example, 100 has 2 trailing zeros. How many trailing zeros does 2002! have? Hint: define the 5-score of a number n the same way 3-score is defined, but using 5 instead of 3 as the base of the exponent, and consider problem 7.

11. As clearly as you can, justify your answer for Problem 10.

    PROBLEMS 12 AND 13 ARE OPTIONAL FOR STUDENTS IN GRADES 7, 8, AND 9.

12. $HIAC$ is a square with side length 1. The square is folded in half along the line segment JB. That is, B is the midpoint of segment IA and J is the midpoint of segment HC. The square is then folded upward along segment BC. Point E is the place where point A lies after this folding. Therefore triangle ECB is the a reflection of triangle ABC about the line BC. Let x be the length of segment BF. What is the length of segment DE in terms of x?



13. What is the value of x in problem 12?