A Sample Challenge

GO FIGURE 2000 Mathematical Challenge

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Show your work. You can receive partial credit for partial solutions.
Please write all solutions clearly, and legibly.

The positive integers are the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 ,10, 11, 12, 13, 14, 15, 16, 17, 18,...

1. (a.) What is the smallest number among the five sums
   1 + 36, 2 + 18, 3 + 12, 4 + 9, 6 + 6?

   (b.) What is the smallest number among the six sums
   1+ 72, 2 + 36, 3 + 24, 4 + 18, 6 + 12, 8 + 9?

2. Find the sixth term *a* and the seventh term *b* of the geometric progression 18, 12, 8, 16/3, 32/9, *a*, *b*.
   [One multiplies a term by a fixed number to get the next term.]

3. Find integers *c* and *d*, each greater than 1, such that 2491 = *c* x *d*.
   [Hint: Find a pattern in 91 = 7 x 13, 391 = 17 x 23, 891 = 27 x 33, 1591 = 37 x 43.]

4. In an arithmetic progression, one adds a fixed number (which may be positive, negative, or zero) to any term to get the next term.
   (a) Find the 70th term of the unending arithmetic progression 5, 10, 15, 20...
   (b) Find the 70th term of the unending arithmetic progression 2, 7, 12, 17...
   [Advice: Compare terms with those in part (a).]
   (c) How many terms are there in the arithmetic progression 14, 19, 24,...., 994?
   (d) What is the average (that is, arithmetic mean) of the terms of the progression in part (c)?
   (e) Find the sum 14 + 19 + 24 + ... + 994 of the terms of the progression in (c).

5. Powers b^e are defined for $e = 1, 2, 3, \ldots$ by $b^1 = b$, $b^2 = b \times b$, $b^3 = b \times b \times b$, $b^4 = b \times b \times b \times b$, and so on.
   (a.) Find an *r* in {0, 1, 2, 3} and a positive integer *q* such that $b^{123} = (b^4)^q \times b^r$.
   (b.) Find the units digit (that is, rightmost digit) of 7^{123}

6. Here each letter represents one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. For example, if *F* = 4 then 2*F* stands for 24 and *FF*5 stands for 445. Find digits *A*, *B*, and *C* such that *A*2*B* x 683 = 86*BCA*.

7. (a) What is the smallest value of *f* + *g* = *h* for positive integers *f*, *g*, and *h* such that *f* x *g* x *h* = 792?
   (b) What is the smallest sum of (more than three) positive integers whose product is 792?

PROBLEM 8 WILL BE USED ONLY TO BREAK TIES

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