Peridynamic Modeling of the Failure of Heterogeneous Solids

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Need for a new theory of solid mechanics

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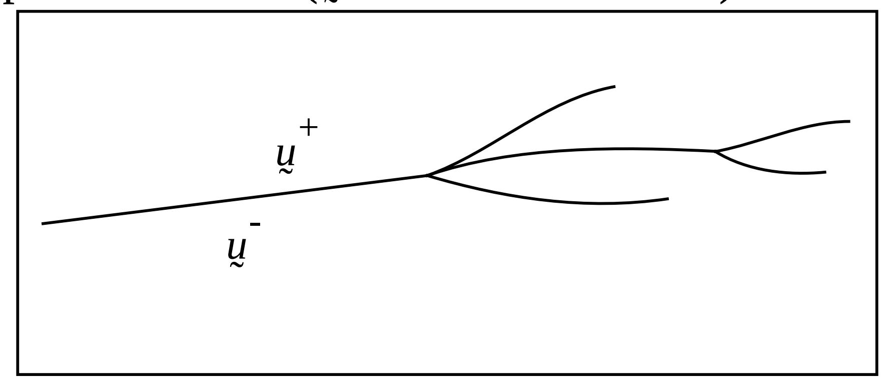
- Classical formulation uses partial differential equations.
- The necessary spatial derivatives <u>may not exist</u> everywhere in the body.
- Example: Fracture ($\underset{\sim}{u}$ is discontinuous)



- Special techniques (of which there are many) are needed to model cracks in the classical theory.

Goal

Develop a model in which exactly the same equations hold everywhere, regardless of any discontinuities.

- To do this, get rid of spatial derivatives.

Basic idea of the peridynamic theory

- Equation of motion:

$$\rho \ddot{\underset{\sim}{u}} = \underset{\sim}{L}_u + \underset{\sim}{b}$$

where $\underset{\sim}{L}_u$ is a functional.

- A useful special case:

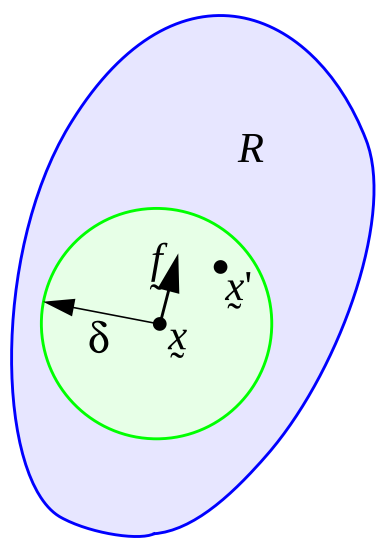
$$\underset{\sim}{L}_u(\underset{\sim}{x}, t) = \int_R \underset{\sim}{f}(\underset{\sim}{u}(\underset{\sim}{x}', t) - \underset{\sim}{u}(x, t), \underset{\sim}{x}' - \underset{\sim}{x})dV_{\underset{\sim}{x}'}.$$

where $\underset{\sim}{x}$ is any point in the reference configuration, and $\underset{\sim}{f}$ is a vector-valued function.

More concisely:

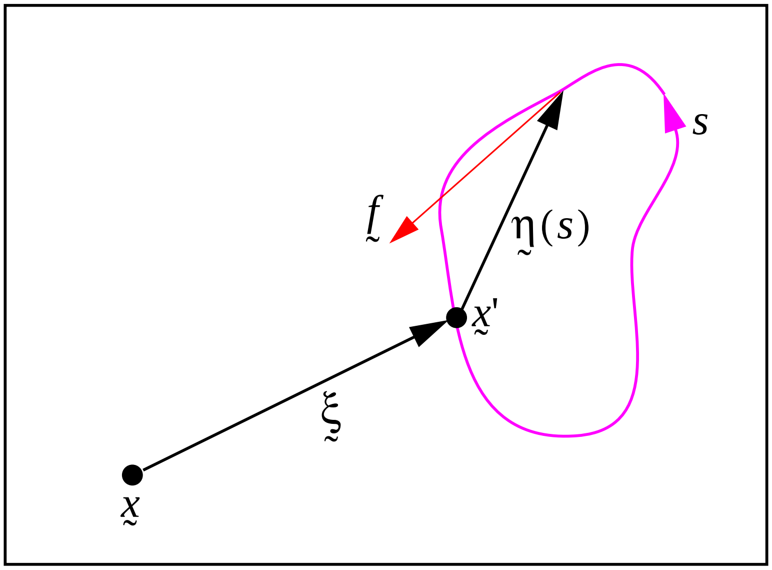
$$\underset{\sim}{L} = \int_R \underset{\sim}{f}(\underset{\sim}{u}' - \underset{\sim}{u}, \underset{\sim}{x}' - \underset{\sim}{x})dV'.$$

- $\underset{\sim}{f}$ is the <u>pairwise force function</u>. It contains all constitutive information.
- It is convenient to assume that $\underset{\sim}{f}$ vanishes outside some <u>horizon</u> δ.



Microelastic materials

A material is <u>microelastic</u> if, holding any $\underset{\sim}{x}$ fixed, the work done by $\underset{\sim}{f}$ in moving any $\underset{\sim}{x}'$ around a closed path is 0.



In this case, Stokes' Theorem implies:

- There exists a scalar-valued function w, called the <u>micropotential</u>, such that

$$\underset{\sim}{f}(\underset{\sim}{\eta}, \underset{\sim}{\xi}) = \frac{\partial w}{\partial \underset{\sim}{\eta}}(\underset{\sim}{\eta}, \underset{\sim}{\xi})$$

where

$$\underset{\sim}{\xi} = \underset{\sim}{x}' - \underset{\sim}{x}$$

$$\underset{\sim}{\eta} = \underset{\sim}{u}' - \underset{\sim}{u}$$

Can further show:

- There exists a scalar function H such that

$$\underset{\sim}{f}(\underset{\sim}{\eta}, \underset{\sim}{\xi}) = (\underset{\sim}{\xi} + \underset{\sim}{\eta})\, H(|\underset{\sim}{\xi} + \underset{\sim}{\eta}|, \underset{\sim}{\xi})$$

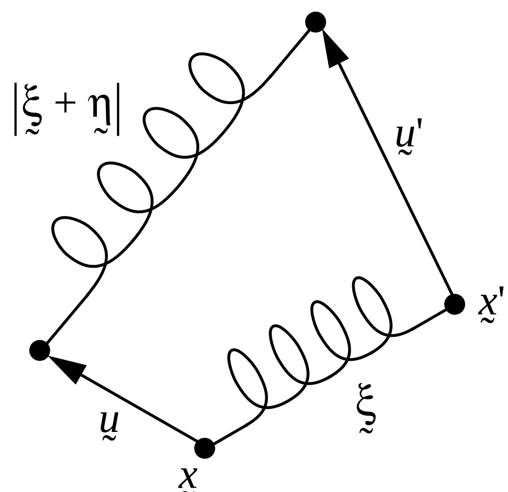
Interpretation of microelasticity

So the micropotential can depend only on:

- the current separation distance $\left|\underset{\sim}{\xi} + \underset{\sim}{\eta}\right|$
- the reference separation vector $\underset{\sim}{\xi}$.

Meaning: any two points $\underset{\sim}{x}$ and $\underset{\sim}{x}'$ are connected by a (possibly nonlinear) spring.

The spring properties can depend on the reference separation vector.



- Can prove: “microelastic implies macroelastic”:
    - Work done by external forces is stored in a recoverable form

Some material models

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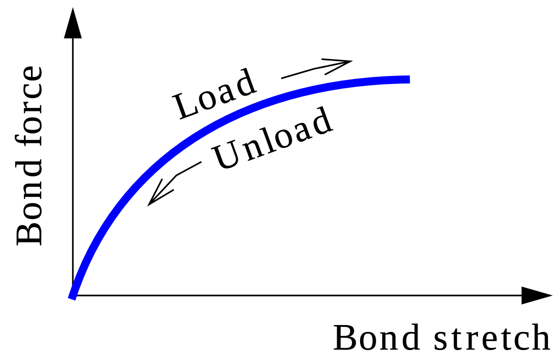
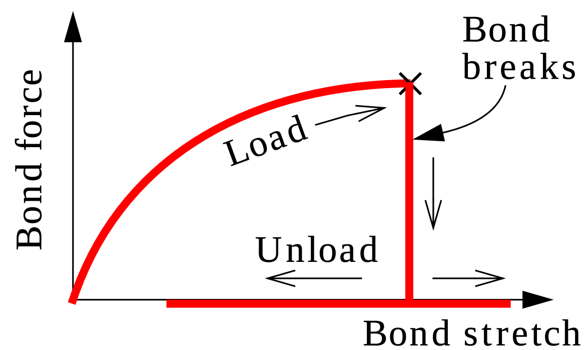
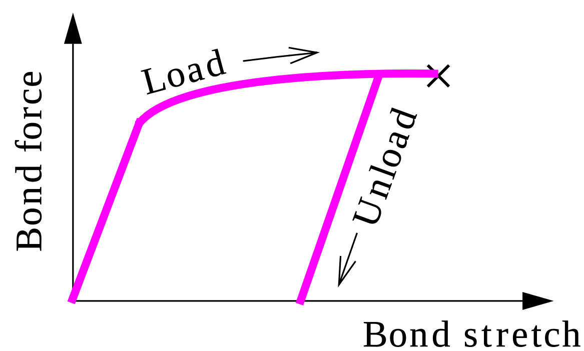
- Microelastic
  - Each pair of particles is connected by a spring.
  - Linear
  - Bilinear
  - etc.



- Brittle microelastic
  - Springs break irreversibly



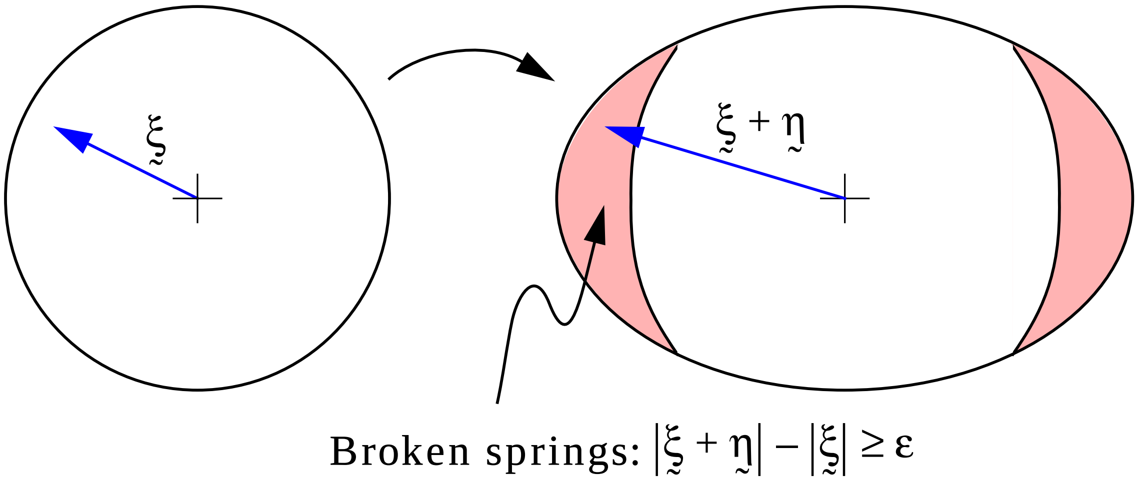
- Microplastic
  - Permanent bond deformation upon unloading.



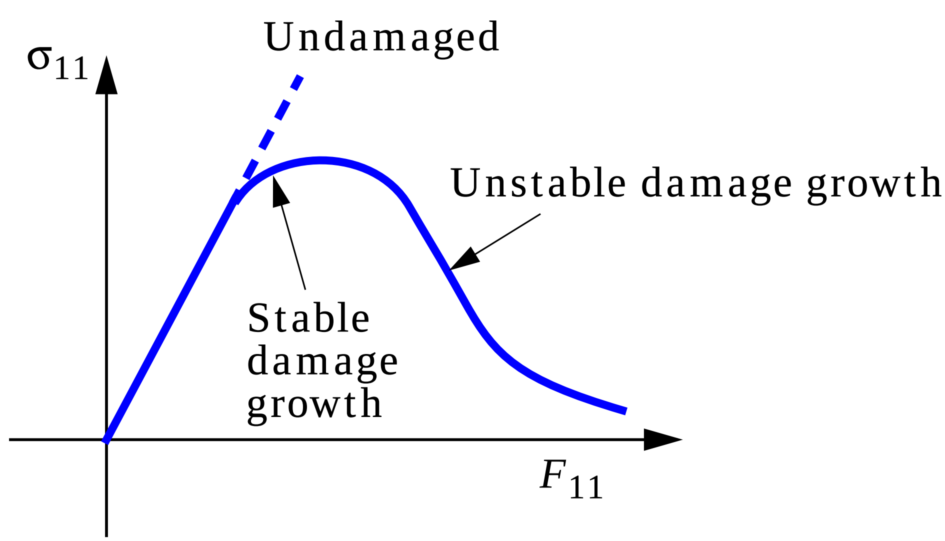
- All of the above can have explicit rate dependence.

How bond breakage leads to material fracture

- Continuum damage is caused by deformation:



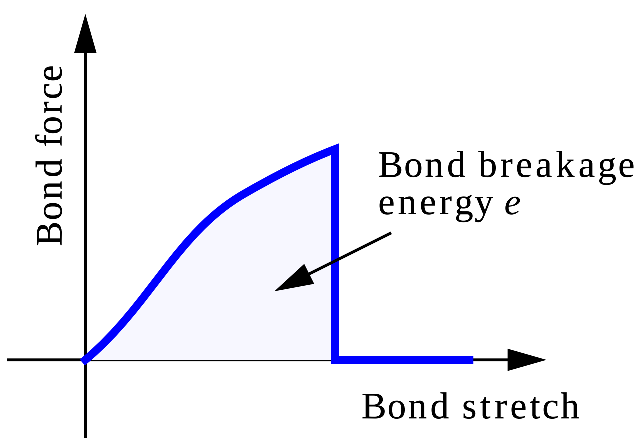
- This causes a change in the "stress-strain" curve:

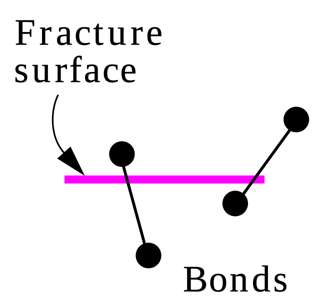


- Need to understand the mathematical conditions under which discontinuities can emerge.

Determination of constitutive parameters

- Linear microelastic:
  - Basic:
    - Spring constant is fit to wave speed data.
  - Advanced:
    - Can fit wave dispersion data if available.
    - Bond properties can depend on initial bond length.
- Microplastic:
  - Fit to uniaxial stress-strain curve.
- Bond breakage properties:
  - Fit breakage stretch to fracture toughness data.



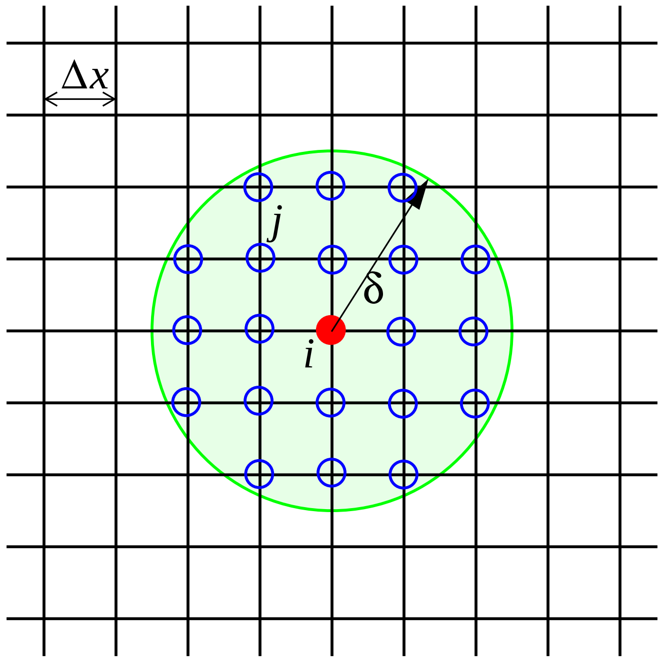


Sum over bonds that are broken by the fracture:

$$\text{Fracture energy} = \sum \text{bond breakage energy}$$

Numerical solution method for dynamic problems

- Theory lends itself to <u>mesh-free</u> numerical methods.
  - No elements.
  - Changing connectivity.
- Brute-force integration in space.



$$\rho \ddot{\underset{\sim}{u}}^i = \sum_{\left|\underset{\sim}{x}^j - \underset{\sim}{x}^i\right| < \delta} \underset{\sim}{f}(\underset{\sim}{u}^j - \underset{\sim}{u}^i, \underset{\sim}{x}^j - \underset{\sim}{x}^i)(\Delta x)^3$$

- Solution method has been found to scale well (almost linear speedup) when run on the Intel Teraflops computer at Sandia.
- Stable time step does not depend on mesh spacing (!)

Peridynamic fracture model is “autonomous”

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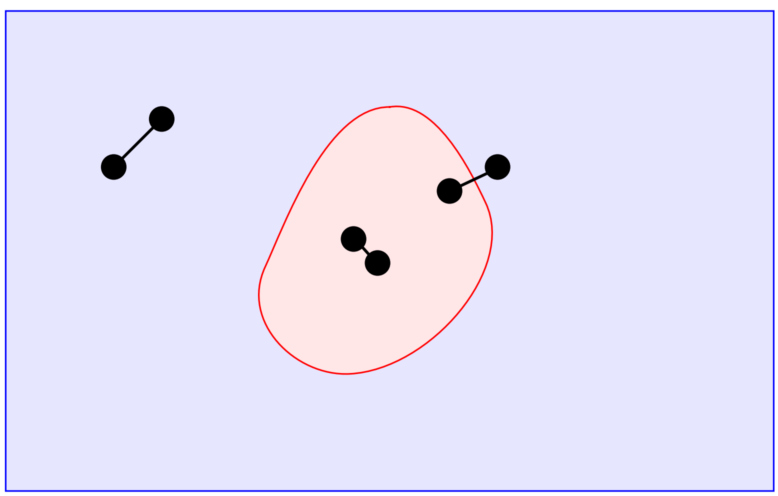
- Cracks grow when and where it is energetically favorable for them to do so.
- Path, growth rate, arrest, branching, mutual interaction are predicted by the constitutive model and equation of motion (alone).
    - No need for any externally supplied relation controlling these things.
- Any number of cracks can occur and interact.
- Interfaces between materials have their own bond properties.



Mode-III crack tip field

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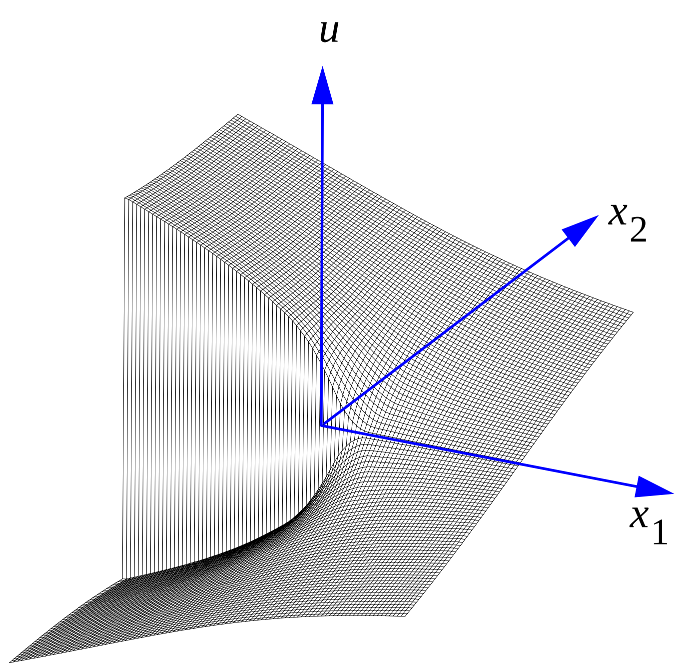
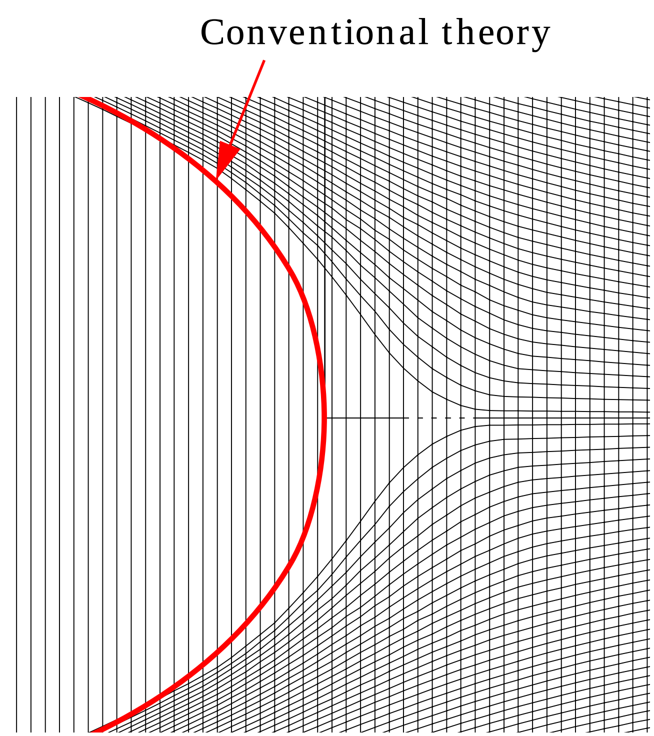
- Same equations are applied everywhere.
- Crack faces have cusp shape near tip.
  - No need for additional hypotheses (e.g. Barenblatt).



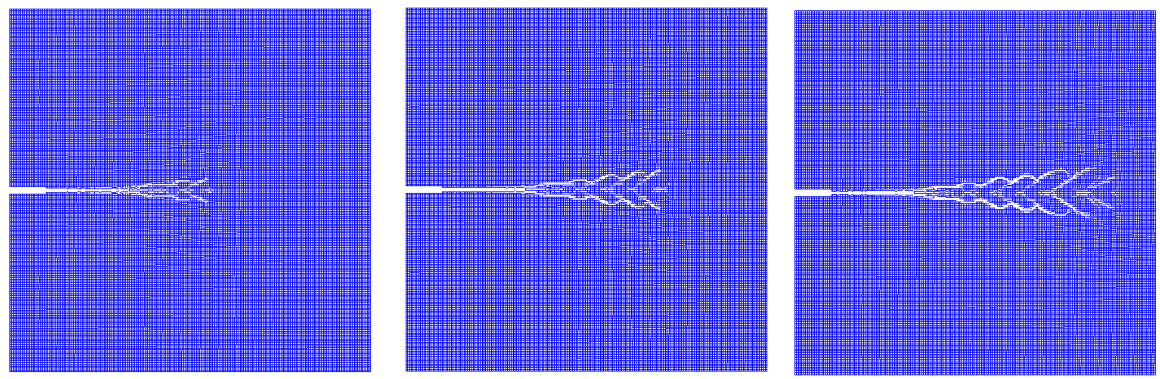
Surface into which a plane deforms

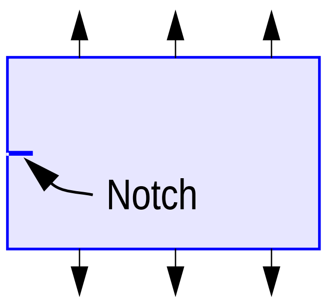


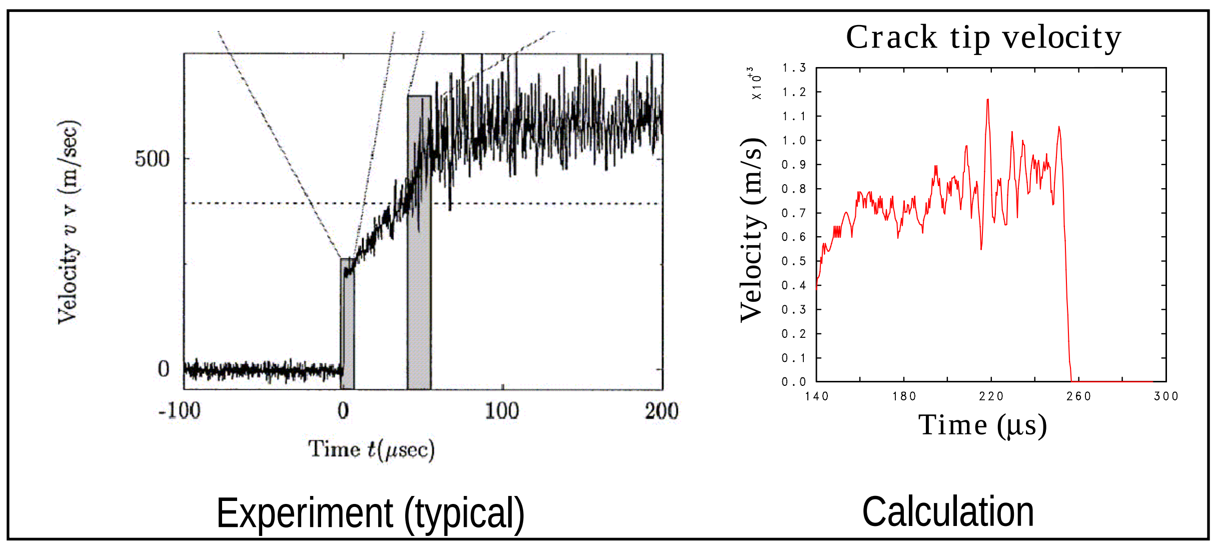
Enlarged view of crack tip

Dynamic brittle fracture

- Stretching of a PMMA plate[1]

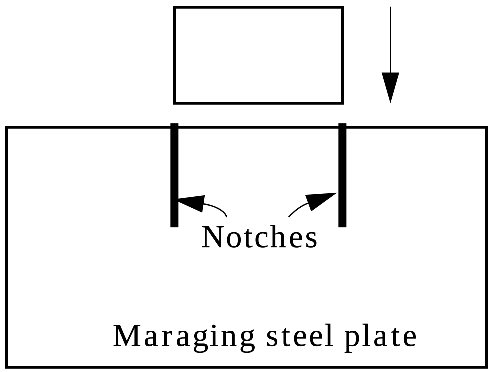




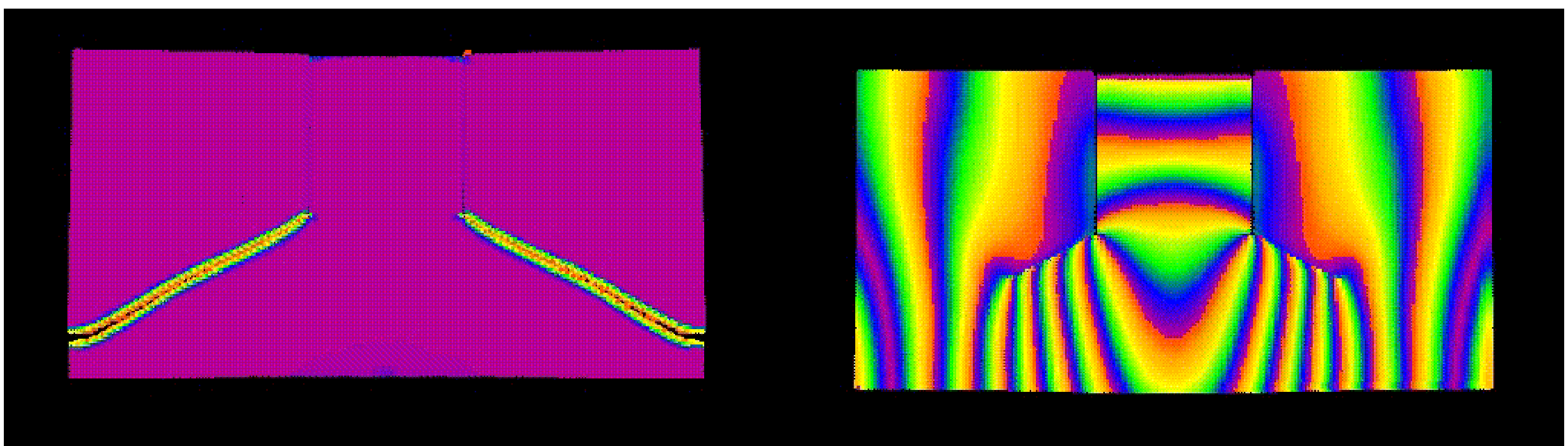


1. J. Fineberg and M. Marder, *Physics Reports* 313 (1999) 1-108

Dynamic fracture in a tough steel: Kalthoff-Winkler experiment



- Code predicts correct crack angles.



Crack paths

Simulated Moire fringes

Perforation of thin ductile targets

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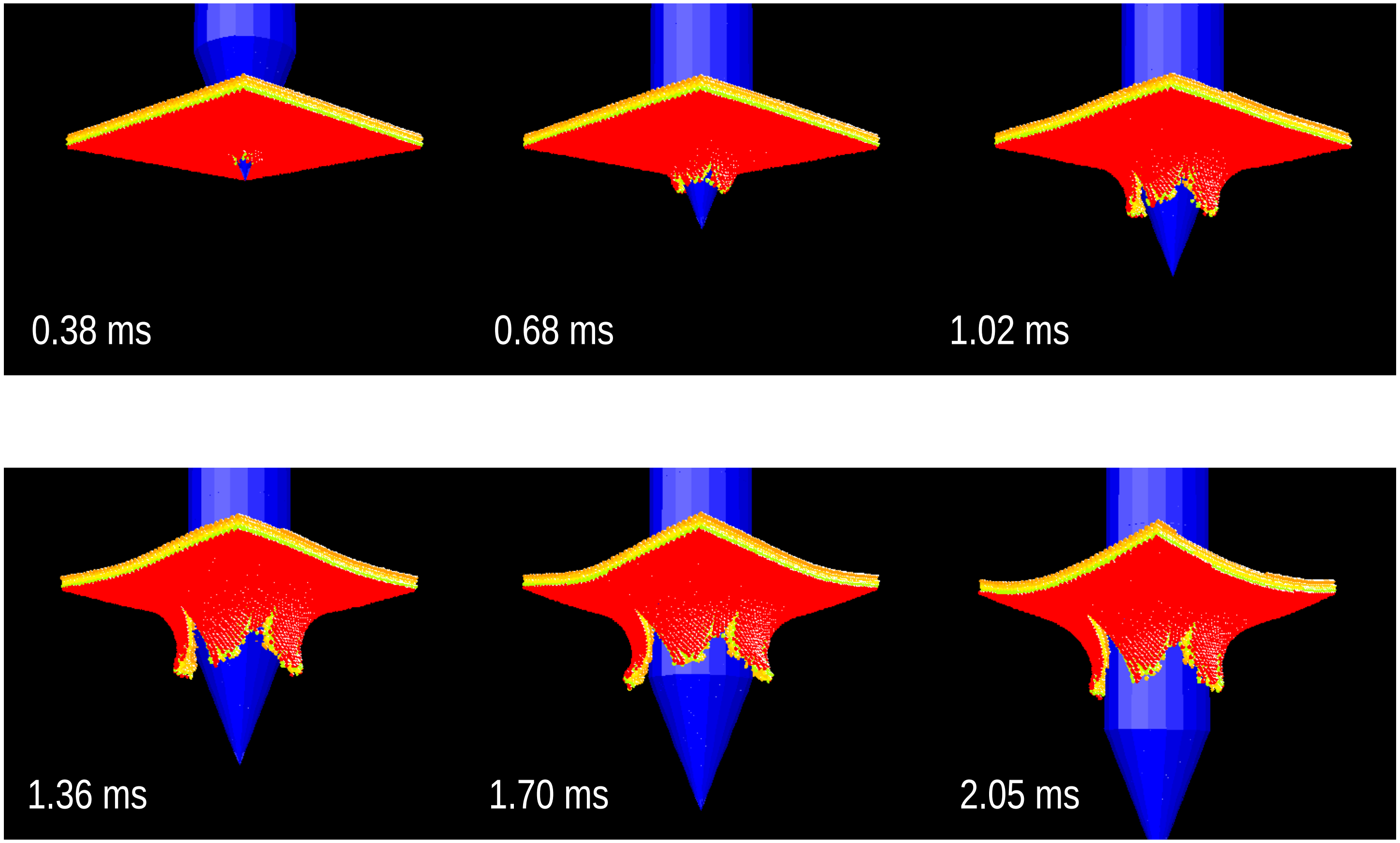
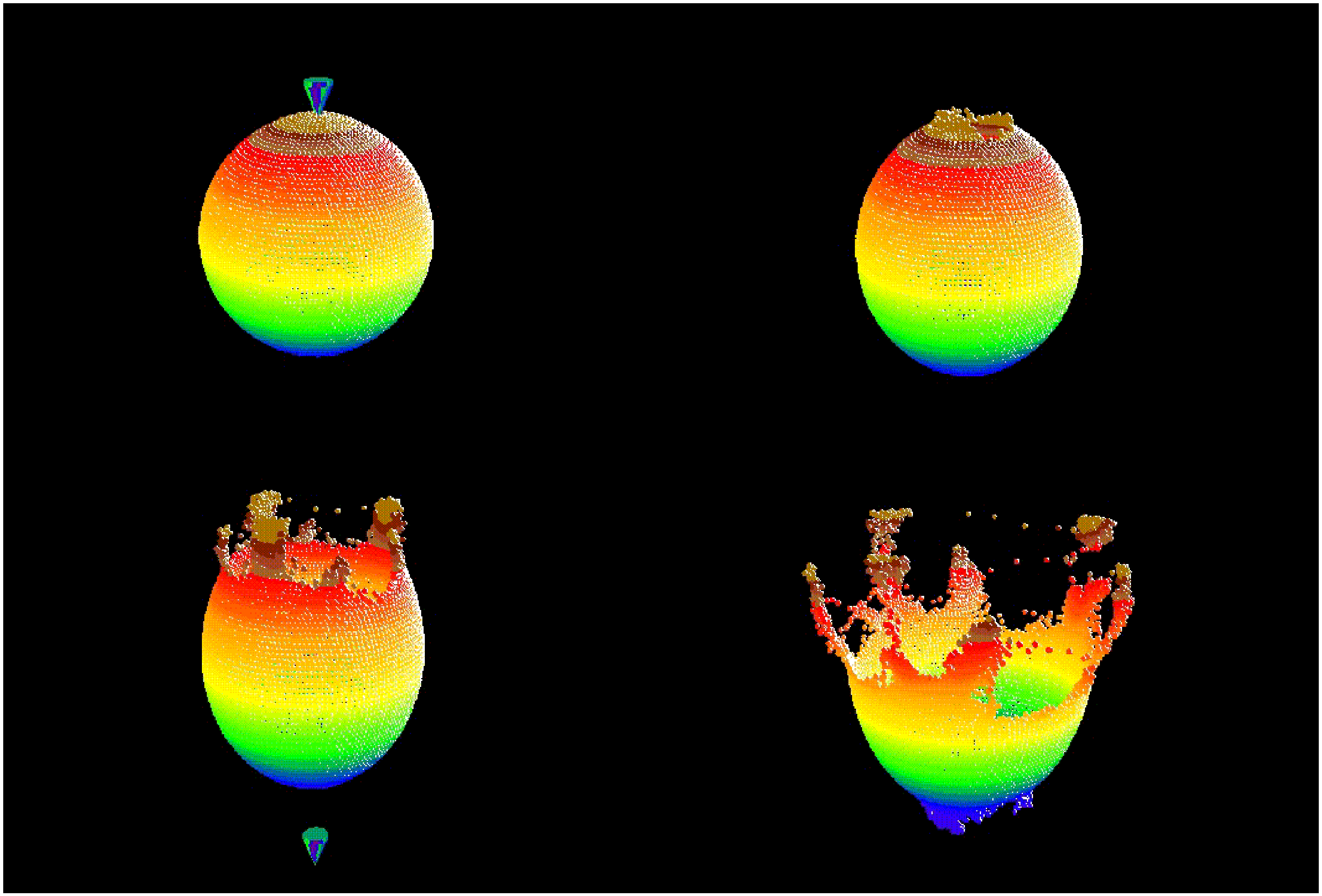
- Peak force occurs at about 0.4ms (end of drilling phase):[1]



1. Not all of the target is shown.

Dynamic fracture in a balloon



Nonhomogeneous materials: Perforation of reinforced concrete

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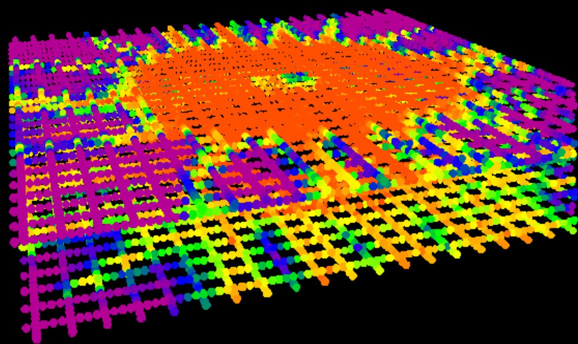
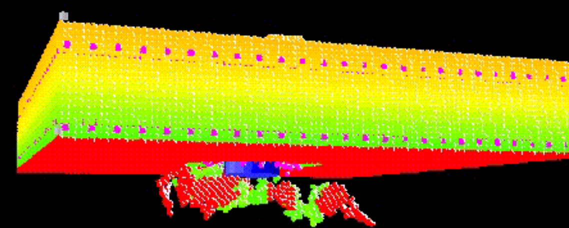
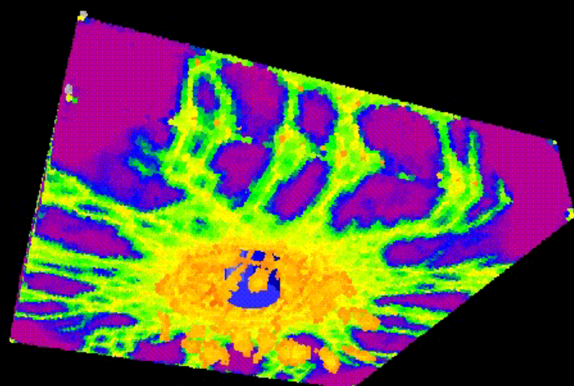


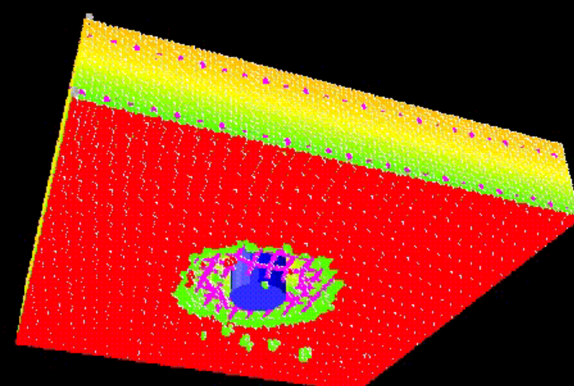
Pullout damage



Exit debris



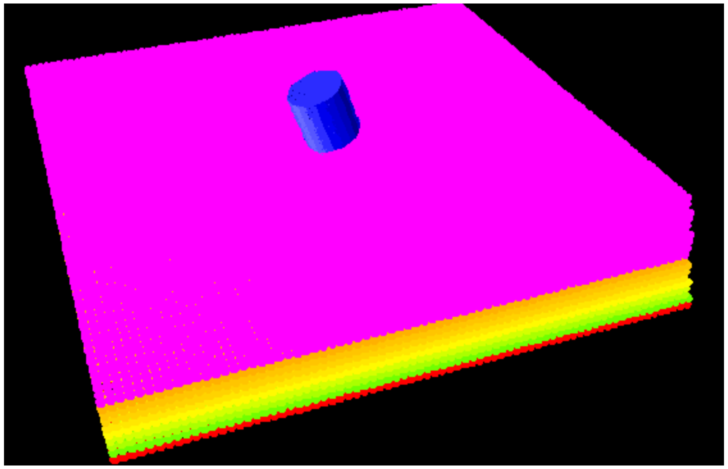
Damage on surface

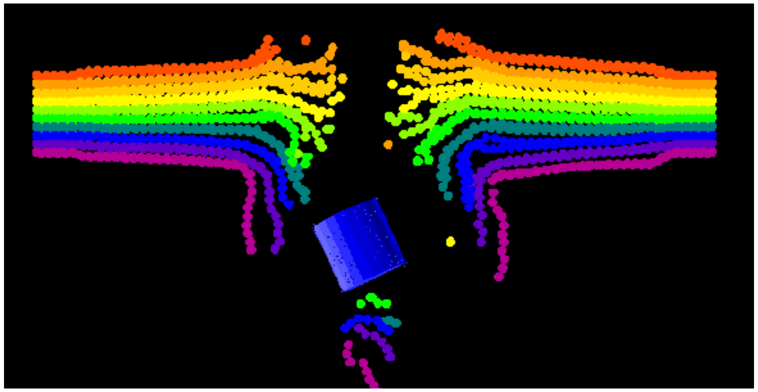


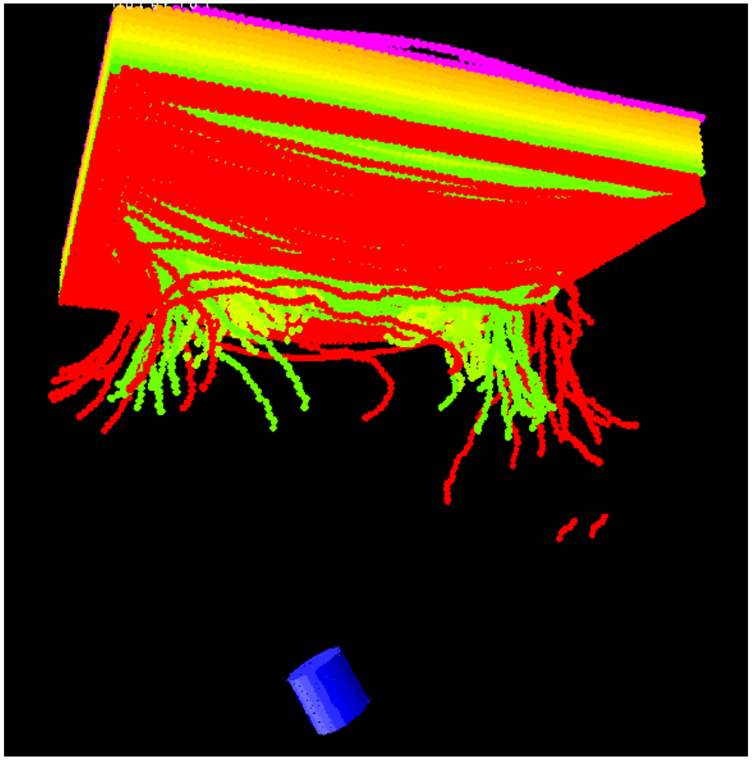
Exit crater

Nonhomogeneous materials: Perforation of a fabric

- Model includes fiber breakage, contact, and adhesion:

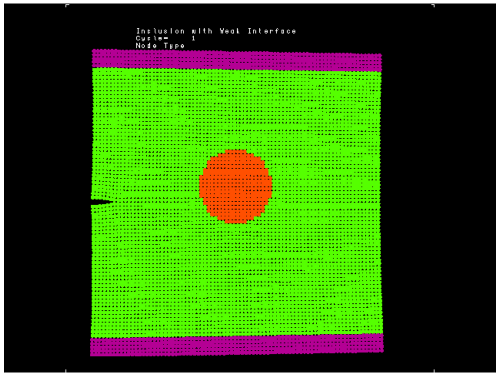




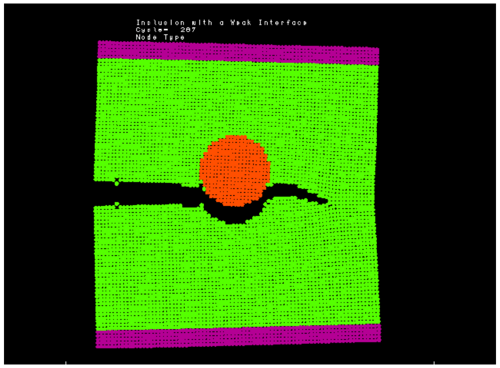


Nonhomogeneous materials: Fracture in a composite unit cell

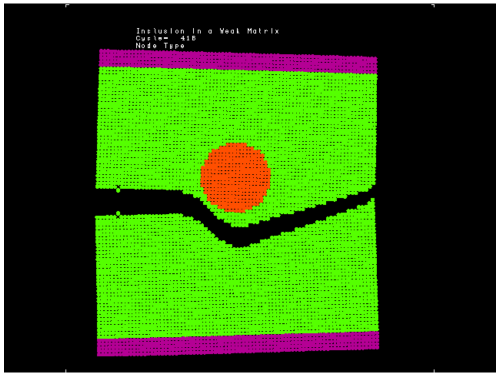
- Crack path, growth, and stability depend only on material properties.
- No need for separate laws governing crack growth.



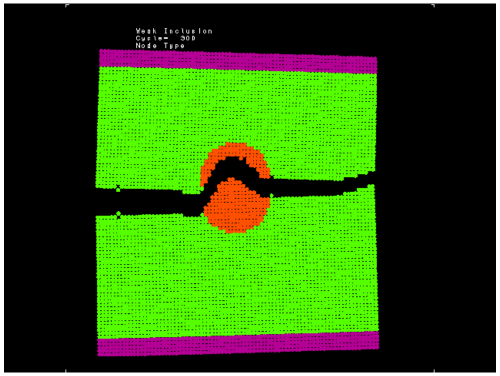
Initial condition



Weak interface



Weak matrix



Weak fiber

Layered material example: Where does failure first occur?

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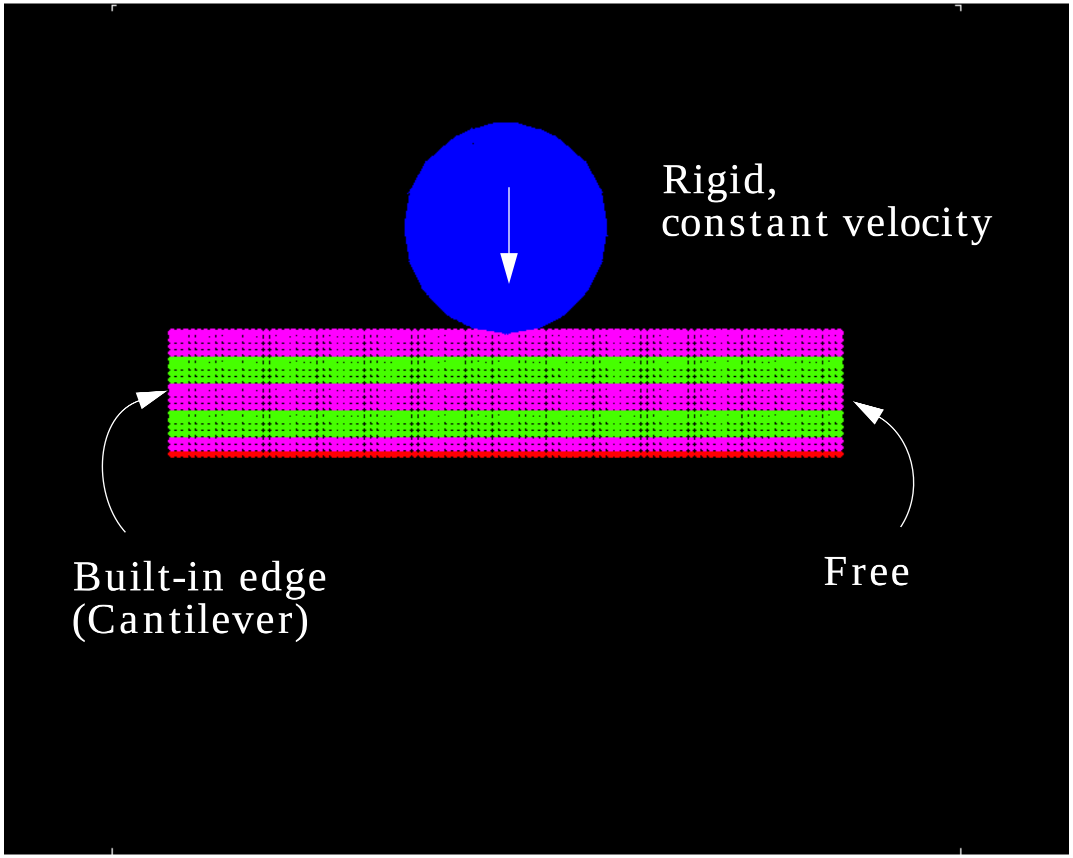
- Layers have identical properties.
- Interfaces have half the strength of the layers.



Initial failure site and mode depends on loading rate

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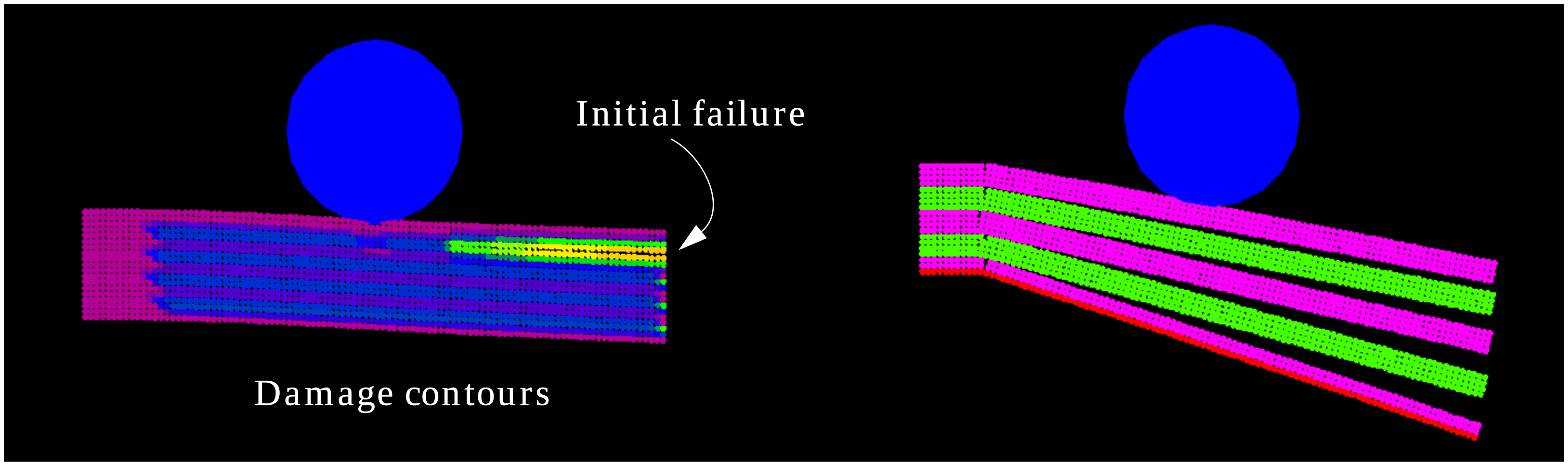
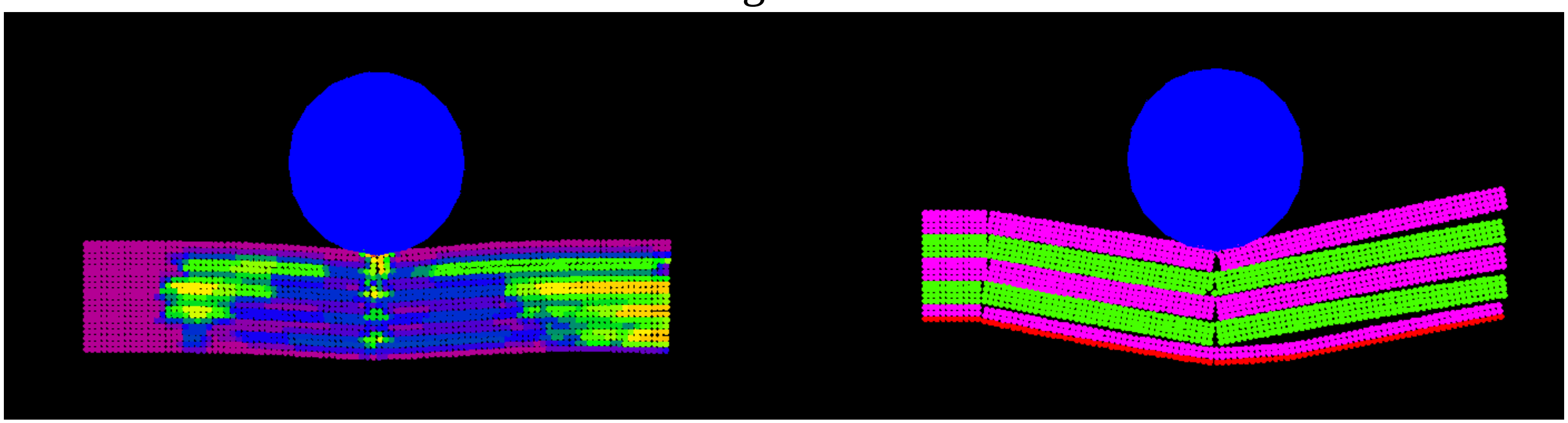
Low rate



High rate



Correspondence with atomic-scale physics

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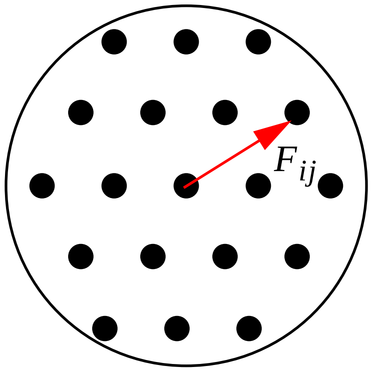
- Can a constitutive model be derived rigorously from an atomic-scale physical description?
  - Classical theory: people have been trying for a long time.
  - Peridynamic theory may be a more natural way to do this because of its similarity to molecular dynamics.
    - ♦ This is currently being attempted by Bhattacharya (Caltech) and Abeyaratne (MIT).
  - May lead to a good way to do multiscale modeling.



Conclusions

- Method appears to have the potential to model:
  - Heterogeneous materials of great complexity.
  - Complex fracture systems without the need to keep track of each crack.

Possible research directions

- Mechanics of heterogeneous materials
  - Understand how failure progresses from one material to another.
  - Improved material models.
  - Validation against interface crack data.
  - Fatigue cracks.
  - Multiscale modeling.
  - Learn how to do complex material systems.
- Theory and numerical solutions
  - Improved solvers.
    - Multigrid, iterative, implicit, etc.