Project 3: Interface Reduction on Hurty/Craig-Bampton Substructures with Mechanical Joints

Students: Patrick Hughes (UC San Diego), Wesley Scott (UW Madison), Wensi Wu (Cornell)
Mentors: Rob Kuether (SNL), Matt Allen (UW Madison), Paolo Tiso (ETH Zurich)

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Agenda

- Background & motivation

- Theory Review
  - Hurty/Craig-Bampton substructuring (HCB method)
  - System-level characteristic constraint mode interface reduction (S_CC method)
  - Normal contact
  - Friction

- Selection of interface reduction basis

- Results

- Conclusions & future research
**Goal:** add nonlinear elements here & apply interface reduction

**Interface DOF**
reduced by characteristic constraint (CC) mode methods

**Interior DOF**
reduced by component mode synthesis (CMS) methods

Image: http://www.ssanalysis.co.uk/blog/2012/10/29/bolted-joints-in-finite-element-models-our-first-training-webinar
Prototype C-beam assembly ("S4 beam")

- **Analysis Overview**
  - Full FEA Model (94,000 DOF) → HCB Model (3,700 DOF)
  - Define contact areas between surfaces with penalty spring elements
  - HCB Model (3,700 DOF) → SCC Model (50 DOF)
  - Use normal contact to define friction in contact plane
  - Simulate reduced order model and observe response
Review of Craig-Bampton Substructuring

- Equations of motion for an arbitrary dynamical system with localized nonlinearities
  \[
  [M]\{\ddot{u}\} + [K]\{u\} + \{f_{NL}(u, \dot{u})\} = \{f_{ext}\}
  \]

- Apply Hurty/Craig-Bampton method to reduce interior (non-interface) degrees of freedom with \([M_{ii}] - \omega^2[K_{ii}]\) \(\Phi_{FI} = \{0\}\)
  \[
  \begin{align*}
  \{u_i\} &= \begin{bmatrix}
  \Phi_{FI} & -K_{ii}^{-1}K_{ij} \\
  0 & I
  \end{bmatrix}
  \{q_i\} = [T_{HCB}]{\{q\}}
  \end{align*}
  \]
  Where \(n_{ui} \gg n_{qi}\)

- Transform equations of motion:
  \[
  [T_{HCB}]^T[M][T_{HCB}]{\ddot{q}} + [T_{HCB}]^T[K][T_{HCB}]{q} + [T_{HCB}]^T\{f_{NL}(u, \dot{u})\} = [T_{HCB}]^T\{f_{ext}\}
  \]
  \[
  [M_{HCB}]{\ddot{q}} + [K_{HCB}]{q} + \{f_{NL}^{HCB}(u, \dot{u})\} = \{f_{ext}^{HCB}\}
  \]

Model size can still be unacceptably large because of the number of DOF at substructure interfaces
Review of System Characteristic Constraint Interface Reduction

- Reduction method requires all subcomponents to be assembled together first (CMS). Then, can keep interior modal DOFs and reduce physical interface DOFs using the S_CC method:

\[
\{q\} = \begin{bmatrix} q_i \\ u_j \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \Psi \end{bmatrix} \{q_i\} = [T_{SCC}]\{s\}
\]

with \((M_{jj} - \omega^2 K_{jj})\Psi = \{0\}\)

- Apply Transformation:

\[
[T_{SCCe}]^T [M_{HCB}] [T_{SCCe}] \{\ddot{s}\} + [T_{SCCe}]^T [K_{HCB}] [T_{SCCe}] \{s\} + [T_{SCCe}]^T \{f_{NL}^{HCB}(u, \dot{u})\} = [T_{SCCe}]^T \{f_{ext}^{HCB}\}
\]

\[
[M_{SCCe}] \{\ddot{s}\} + [K_{SCCe}] \{s\} + \{f_{NL}^{SCCe}(u, \dot{u})\} = \{f_{ext}^{SCCe}\}
\]

**Converts all remaining physical DOF to modal DOF**
S_CC does not retain physical DOF

- Need physical DOF onto which we can apply preload:

\[ \begin{bmatrix} q_i \\ u_j \end{bmatrix} \rightarrow \begin{bmatrix} q_i \\ u_r \\ u_b \end{bmatrix} \rightarrow \begin{bmatrix} q_i \\ q_r \\ u_b \end{bmatrix} \]

Want to maintain physical bolt DOF:

And reduce such that: \( n_{ur} \gg n_{qr} \)
System Level Constraint Modes Expansion (SCCe)

\[ M_{CB} = \begin{bmatrix} M_{CB_{ii}} & M_{CB_{ir}} & M_{CB_{ib}} \\ M_{CB_{ri}} & M_{CB_{rr}} & M_{CB_{rb}} \\ M_{CB_{bi}} & M_{CB_{br}} & M_{CB_{bb}} \end{bmatrix} \quad \text{K}_{CB} = \begin{bmatrix} \Omega_{FI}^2 & 0 & 0 \\ 0 & K_{CB_{rr}} & K_{CB_{rb}} \\ 0 & K_{CB_{br}} & K_{CB_{bb}} \end{bmatrix} \]

\((M_{CB_{rr}} \omega^2 - K_{CB_{rr}}) \psi_{SCC_{rr}} = 0\)

These modes aren't enough by themselves to correctly constrain the bolt and patch interfaces:

Augment system with constraint modes similar to the HCB method.

\[ \Psi_{SCC} = \begin{bmatrix} \psi'_{SCC_{rr}} & \Phi_{CM} \end{bmatrix} \quad \Phi_{CM} = \begin{bmatrix} -K_{rr}^{-1}K_{rb} \\ I_{nb} \end{bmatrix} \]

Then the transformation:

\[ T_{SCC} = \begin{bmatrix} I_{ni} & 0 \\ 0 & \psi_{SCC_{ce}} \end{bmatrix} = \begin{bmatrix} I_{ni} & 0 & 0 & 0 \\ 0 & \psi_{SCC_{rr}} & -K_{rr}^{-1}K_{rb} & I_{nb} \end{bmatrix} \quad \begin{bmatrix} q_i \\ u_r \\ u_b \end{bmatrix} = T_{SCC} \begin{bmatrix} q_i \\ q_r \\ u_b \end{bmatrix} \]
Expansion to S_CC Theory: SCCe

- Typically interface reduction means ALL of interface must be reduced.
- Can’t just multiply a partition of DOF by identity.
  - Causes reduced interface set to act like fixed interface modes.
  - Alleviate with constraint modes

\[
T_{SCCe} = \begin{bmatrix} I_{ni} & 0 \\ 0 & \Psi_{SCCe} \end{bmatrix} = \begin{bmatrix} I_{ni} & 0 & 0 \\ 0 & \psi_{SCCr} & 0 \\ 0 & 0 & I_{nb} \end{bmatrix} - K_{rr}^{-1} K_{rb}
\]

Physical DOF retained, 42 DOF model provides accuracy of <1% error for modes under 1kHz.
Normal contact model – penalty method

Don’t want interfaces to be able to penetrate

Preload

Compressive only springs

Force

Overlap
Sensitivity to penalty stiffness, $k_{pen}$

- Excessive nodal interpenetration
- "sweet spot"
- Potential numerical ill-conditioning/instability
Sensitivity to penalty stiffness, $k_{pen}$

$k_{pen}/k_{bolt} = 0.1$

$k_{pen}/k_{bolt} = 1$

$k_{pen}/k_{bolt} = 10$

$k_{pen}/k_{bolt} = 100$
Preload-induced deformation \((k_{pen} = 100 \cdot k_{bolt})\)
Friction models

Coulomb’s Law:

\[
\mu(V_r) = \begin{cases} 
-\mu_s \text{ sign}(V_r) & \text{if } V_r \neq 0 \text{ (slip)} \\
\mu_0 \text{ with } |\mu_0| \leq \mu_s, & \text{if } V_r = 0 \text{ (stick)}
\end{cases}
\]

\(\mu_s\) = static friction coefficient

\(\mu_d\) = dynamic friction coefficient

\(V_r\) = relative velocity of contacting surfaces

Regularized Friction Model:

\[
\mu_n(V_r) = g(nV_r) = \frac{-\mu_d V_r \sqrt{V_r^2 + \frac{\varepsilon}{n^2} - 2\frac{\alpha}{n} V_r}}{V_r^2 + \frac{1}{n^2}}
\]

\(\alpha = \sqrt{\mu_s(\mu_s - \mu_d)}\)

\(\varepsilon\) = model parameter (usually a small number ~ \(10^{-4}\))

\(n\) = model parameter controlling stiffness of governing ODE (high \(n\) \(\rightarrow\) stiffer system)

Perfectly vertical slope (multi-valued \(\mu\) at \(V_r = 0\))

Non-vertical slope (unique \(\mu\) for every \(V_r\))
Verification: Regularized Coulomb friction models

Verification: Regularized Coulomb friction models

HCB Results

Impulse load
HCB Results – Full-field Deformation History
Problem: how do we choose the “right” characteristic constraint (CC) modes to capture the local dynamics at the interfaces?
Selection of Interface Reduction Basis

- Essence of the problem:
  - Change in system stiffness is governed by change in interface contact area (*nodes free to connect & disconnect*)
  - Interface-substructure force interaction is controlled by contacting nodes (*nodes constrained together*)
  - Need mode shapes that represent BOTH free-interface and constrained-interface motion

- Solution: constraining/unconstraining process to build mode shapes
Constrained/Unconstrained Mode Shapes

Perform preload analysis and determine:
- set of nodes in contact \( \Gamma_c \)
- vector of nodal displacements \( \{x_p\} \)

\( \approx 66\% \) of patch in contact after preload

Build transformation matrix \([L]\) that constrains node pairs in \( \Gamma_c \) to have the same y-displacement

\( \{u_{HCB}\}_u = [L]\{u_{HCB}\}_c \)

**Nodes in \( \Gamma_c \)**
- partially constrained

**Nodes free to move independently**

\[
[M_c] = [L]^T[M_{HCB}][L]
\]

\[
[K_c] = [L]^T[K_{HCB}][L]
\]

Now have constrained \( M \) and \( K \)
Constrained/Unconstrained Mode Shapes

Build $[T_c]$ using the SCCe method on **constrained system**

$M_c = \begin{bmatrix}
M_{ci} & M_{ci} & M_{cib} \\
M_{cri} & M_{cri} & M_{cbr} \\
M_{cbi} & M_{cbr} & M_{cbb}
\end{bmatrix}$

$K_c = \begin{bmatrix}
\Omega_F^2 & 0 & 0 \\
0 & K_{crr} & K_{crb} \\
0 & K_{cbr} & K_{cbb}
\end{bmatrix}$

$\left(M_{crr} \omega^2 - K_{crr}\right) \psi_{SCcrr} = 0$

$T_c = \begin{bmatrix}
I_n & 0 & 0 \\
0 & \psi_{SCcrr} & -K_{crr}^{-1}K_{crb} \\
0 & 0 & I_{nb}
\end{bmatrix}$

Transform $[T_c]$ back to unconstrained coordinates using $[L]$ & augment with preloaded nodal displacements $\{x_p\}$

$[T_u] = \left[ [L][T_c] \quad \{x_p\} \right]$

$\{u_{SCCe}\} = [T_u]\{u_{HCB}\}_u$

$[M_{SCCe}] = [T_u]^T[M_{HCB}][T_u]$

$[K_{SCCe}] = [T_u]^T[K_{HCB}][T_u]$

**Can now use $M_{SCCe}$ and $K_{SCCe}$ to run dynamic analysis**
Results – 10 SCCe modes

System-level displacement OK, but contact area not captured well
Results – 10+ SCCe modes

Slowly converges to HCB “truth” solution, but still doesn’t allow for loss of contact
Idea: augment with interface RBMs

\[ [T_{\text{new}}] = [ [T_{\text{old}}] [\Psi_{\text{RBM}}]] \]

Allows for loss of contact, but still need many CC modes
Comparison of Analysis Run Times

### Run Time for 10 ms of Simulation Time (Explicit)

<table>
<thead>
<tr>
<th></th>
<th>Hours</th>
<th>Minutes</th>
<th>Seconds</th>
<th>% of HCB Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCB</td>
<td>4</td>
<td>20</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>SCCe - 10 CC modes</td>
<td>0</td>
<td>38</td>
<td>8</td>
<td>15%</td>
</tr>
<tr>
<td>SCCe - 50 CC modes</td>
<td>0</td>
<td>42</td>
<td>8</td>
<td>16%</td>
</tr>
<tr>
<td>SCCe - 100 CC modes</td>
<td>1</td>
<td>1</td>
<td>16</td>
<td>24%</td>
</tr>
</tbody>
</table>

### Run Time for 10 ms of Simulation Time (Implicit)

<table>
<thead>
<tr>
<th></th>
<th>Hours</th>
<th>Minutes</th>
<th>Seconds</th>
<th>% of HCB Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCB</td>
<td>0</td>
<td>4</td>
<td>42</td>
<td>-</td>
</tr>
<tr>
<td>SCCe - 10 CC modes</td>
<td>0</td>
<td>6</td>
<td>4</td>
<td>129%</td>
</tr>
<tr>
<td>SCCe - 50 CC modes</td>
<td>0</td>
<td>8</td>
<td>23</td>
<td>178%</td>
</tr>
<tr>
<td>SCCe - 100 CC modes</td>
<td>0</td>
<td>15</td>
<td>20</td>
<td>326%</td>
</tr>
</tbody>
</table>

*Interface reduction here is valuable if you must use explicit methods, but not if implicit methods are available*
Preliminary results for friction implementation

Front view
Friction Model Results

Without friction (impulse amplitude = 1,000N)

With friction (impulse amplitude = 1,000N)
Friction Model Results

Displacements in the x-direction (impluse: 1,000N)

- Blue line: without friction
- Red line: with friction
Friction Model Results

Without friction (impulse amplitude = 10,000N)  

With friction (impulse amplitude = 10,000N)
Friction Model Results

Displacements in the x-direction (impulse: 10,000N)

- Blue line: without friction
- Red line: with friction

x displacements

0 1 2 3 4 5 \times 10^{-3}

0 0.01 0.02 0.03 0.04 0.05
Conclusions

- System-level displacement shows agreement between the HCB model and the SCCe model
  - However, SCCe models exhibit difficulties in capturing the contact area
- Interface reduction provides cost savings for explicit time integration scheme (but not implicit)
  - reduces computational cost substantially in dynamic simulations

Next Step

- Incorporate regularized friction elements into the C-Beam model to gain insight into the significance of friction in structural dynamics
Acknowledgments

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Reference


Appendix
Violation of the contact condition (nodes do not overlap) is “penalized” by adding energy to the system that is proportional to the non-physical overlap \( (E_{\text{pen}} = \frac{1}{2} k_{\text{pen}} \Delta_{\text{gap}}^2) \).
What’s the correct preload to apply?

\[ F_{\text{ext}} = EA\varepsilon \]

Impose \( \varepsilon \)

Apply preload

\[ L_1 = (1 + \varepsilon)L_0 \]

\[ u_T \]

Solve for displacement field

\[ L_2 = L_0 + (u_T - u_B) \]

\[ \Delta L = L_2 - L_1 \]

Update \( \varepsilon \)

\[ F_{\text{int}} = \frac{EA}{L_0} \Delta L \approx F_{\text{tr}}? \]
What’s the correct preload to apply?

- Given bolt torque from experimental group (Project #5), compute transmitted axial force
- Use equation from [1] to do conversion:

\[ F_{tr} = \frac{T}{0.159P + 0.578d_2\mu_T + 0.5D_f\mu_H} \]

- \( F_{tr} \) = transmitted axial force, \( T \) = applied bolt torque, \( P \) = bolt pitch, \( d_2 \) = nominal bolt diameter, \( D_f \) = average contact diameter, \( \mu_T \) = thread friction coeff., \( \mu_H \) = head friction coeff.