Using Pulse Sequences to Characterize and Robustly Mitigate Qubit Noise

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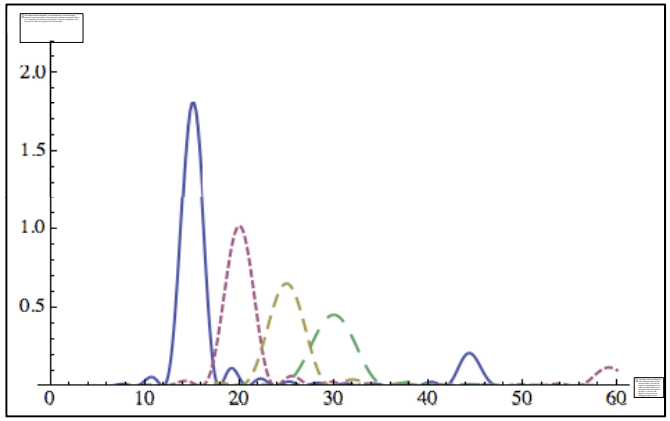
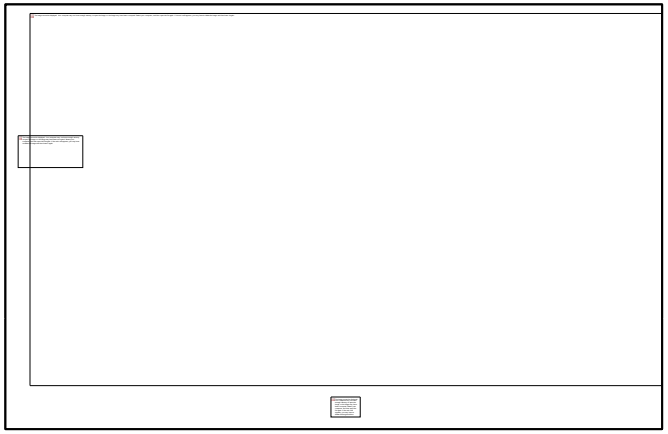
Outline

Characterizing Dephasing Noise

Example filter functions

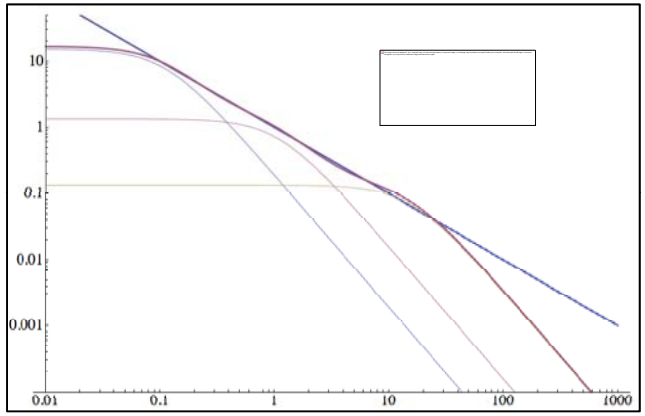


Simulated spectral density regression

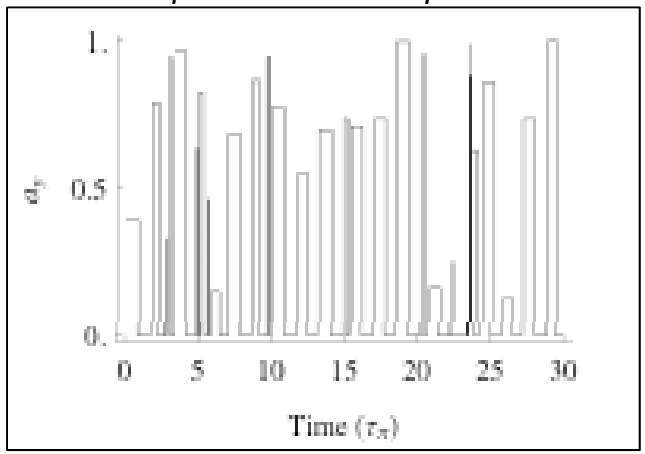


Simulating Noise for Robust Mitigation

Simulated noise spectral density



Optimized control pulse



Dephasing Noise

A single qubit under a semiclassical dephasing model

$$H = \frac{1}{2}a(t)\sigma_x + \frac{1}{2}\eta(t)\sigma_z$$

With this single qubit as a resource, what can we learn about the statistics of the noise, $\eta(t)$?

$$C(t) = \langle \eta(t)\eta(0) \rangle$$

Short-time correlations
Dynamical Decoupling

Long-time correlations
Free Induction Decays

Short-time Correlations I: Filter Functions

Only control is pi-pulse train,

$$H_T = \frac{1}{2} y(t)\eta(t)\sigma_z$$

$$y(t) = (-1)^{\int_0^t a(t')dt'/\pi}$$
$$= \begin{cases} 1 & \text{after even number of } \pi\text{-pulses,} \\ -1 & \text{after odd number of } \pi\text{-pulses.} \end{cases}$$

The coherence decays as,

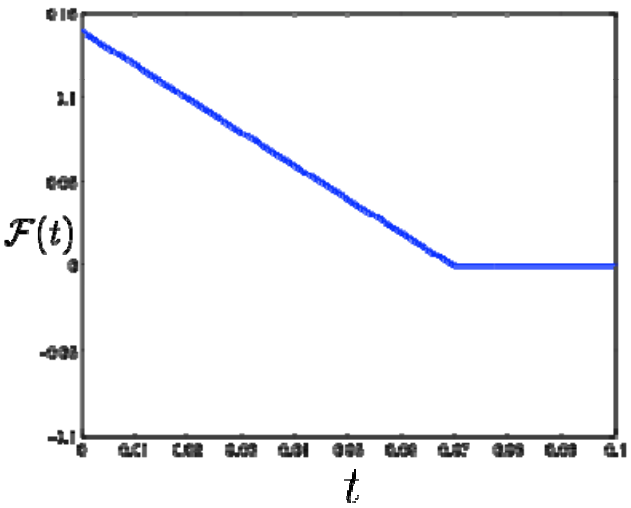
$$\langle\langle \sigma_+(t) \rangle\rangle = \left\langle \mathrm{Tr}\left(e^{i\int_0^t H_T(s)ds}\, \sigma_+\, e^{-i\int_0^t H_T(s)ds}\, \rho_0 \right) \right\rangle$$
$$\simeq \exp\left(-\int_0^\tau C(t)\mathcal{F}(t)dt \right) \sigma_+(0)$$

Where, $C(t) = \langle \eta(t)\eta(0) \rangle$ is the **Correlation Function**

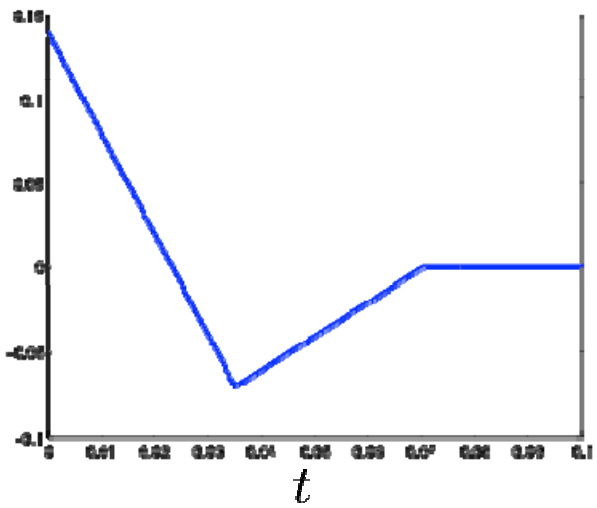
and $\mathcal{F}(t) = \int_t^{2\tau - t} du\, y\left(\frac{u+t}{2}\right) y\left(\frac{u-t}{2}\right)$ is the **Filter Function**

Short-time Correlations II: Filter Function Examples

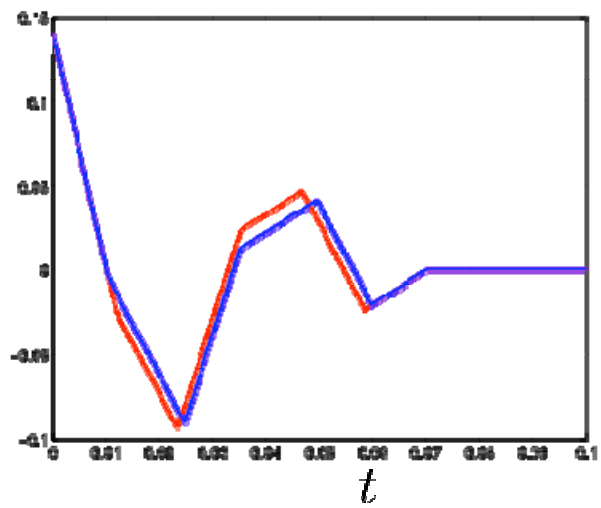
Free Induction Decay



Hahn Echo



Carr-Purcell (3)
Uhrig DD (3)



Filter functions calculated for $\tau = 0.7s$

Short-time Correlations III: Inversion of Dynamical Decoupling Eqn.

Dynamical decoupling theory says,

$$\langle\langle \sigma_+(t) \rangle\rangle \simeq \exp\left(-\int_0^\tau C(t)\mathcal{F}(t)dt\right)\sigma_+(0)$$

For a given pulse sequences, we can measure the integral,

$$\chi_i = \int_0^{\tau_i} C(t)\mathcal{F}_i(t)dt$$

Repeating for a large number of pulse sequences, we can use the theory of **underdetermined least squares**

$$C(t) \simeq \sum_{i,j} \chi_i \, \mathbb{F}^+_{ij} \, \mathcal{F}_j(t) \qquad \mathbb{F}_{ij} = \int_0^\infty \mathcal{F}_i(t)\mathcal{F}_j(t)dt$$

Short-time Correlations IV: Choosing Pulse Sequences

We hope to find a set of filter functions that well spans the space.
Corresponds to maximizing the **quality function** over the time range of interest.

$$Q(t) = \sum_{i,j} \int_0^\infty \mathcal{F}_i(t) \mathbb{F}^+_{ij} \mathcal{F}_j(s)\, ds$$

Must be careful that qubit **does not decohere too much**.

$$\langle\langle \sigma_+(t) \rangle\rangle \simeq \exp\left(- \int_0^\tau C(t) \mathcal{F}(t) dt \right) \sigma_+(0)$$

A good choice is the Uhrig DD sequences run for different N and τ .

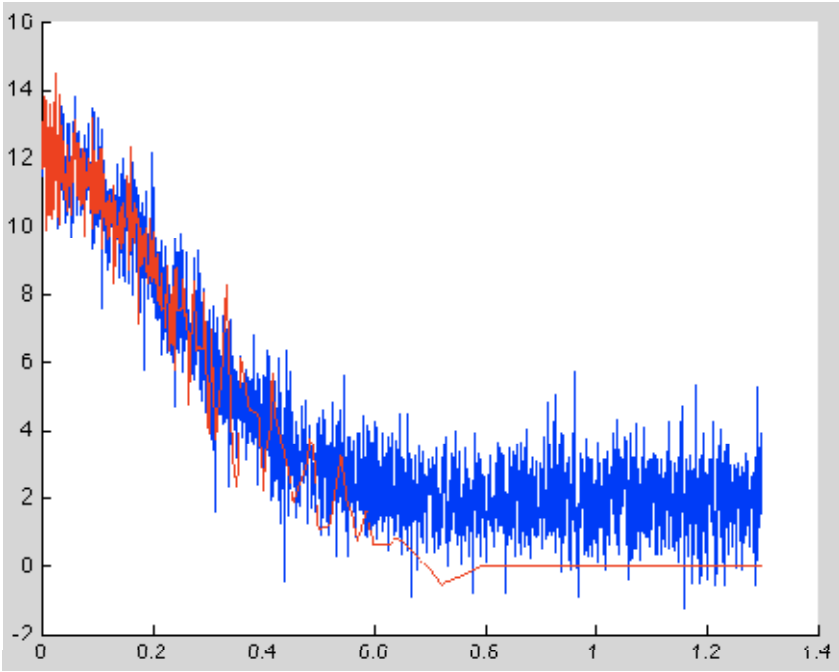
Free induction decay

$$t \simeq T_2^*$$

Uhrig's dynamical decoupling

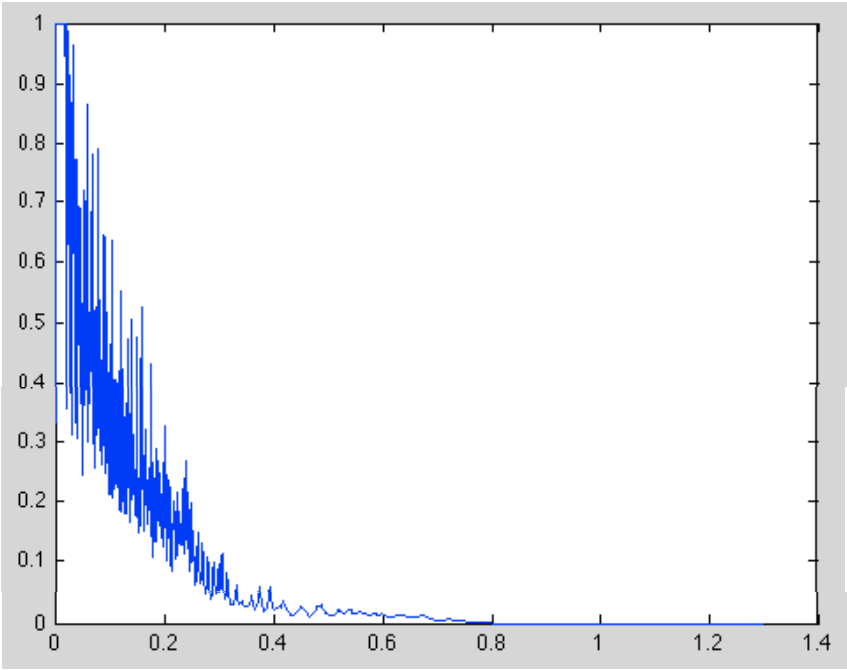
$$t \simeq T_2$$

Short-time Correlations II: Example Reconstruction



Reconstruction of sample correlation function using 10 FID and 50 UDD3 sequences.

Quality function for set of 10 FID and 50 UDD3 sequences used in reconstruction.



Long-time Correlations: Free Induction Decays

In order to learn about high-frequency (short-time) noise, **we echoed away** the effects of low frequency noise using dynamical decoupling.

To learn about low-frequency (long-time) noise, we **cannot use decoupling pulse sequences**.

Instead, repeat the following experiments,

1) Prepare an x-eigenstate
2) Allow to evolve
3) Measure in basis of σ_y

This will yield a set of results, +/- 1, and measurement times, the correlation function of the results is related to the correlation function of the noise:

$$\langle \eta(k\Delta_t)\eta(0)\rangle \simeq \frac{1}{\tau^2}\langle r_k r_0\rangle = \frac{1}{\tau^2}\frac{1}{N-k}\sum_{j=1}^{k} r_j r_{j+k}$$

Comment:
Noise Power Spectral Density

Our procedure fits naturally with the **correlation function** picture

$$C(t) = \langle \eta(t)\eta(0) \rangle$$

Noise statistics are usually thought of in terms of the **power spectral density**

$$S(\omega) = \int_{-\infty}^{\infty} C(t) e^{-i\omega t} dt$$

However, our procedure generates **irregular sampling** of the correlation function with **unequal error bars**

Use short time correlations to calculate the high-frequency components of the PSD, long-time correlations to calculate the low-frequency components.

Mitigating Effects of Noise Through Pulses Sequences

How can we best eliminate the effects of dephasing noise?

How can we perform **high-fidelity gates** in the presence of noise?

How do we **simulate** evolution under arbitrary noise?

Optimal Control

Allow more complicated controls, $a_x(t), a_y(t)$

$$H(t) = \frac{1}{2}\left(a_x(t)\sigma_x + a_y(t)\sigma_y + \eta(t)\sigma_z\right)$$

Use **gradient ascent pulse engineering (GRAPE)**[1] methods to find control fields that maximize the fidelity,

$$f = \max_{\{a(t)\}} \min_{\rho_0} \mathrm{Tr}\left(\rho(t_f)\rho_T\right)$$

To simulate the evolution of a quantum system under noise, use a **multistate Markovian fluctator**

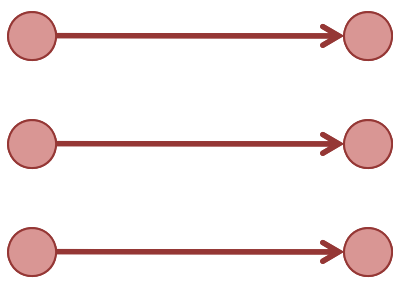
$$\eta(t) \in (\eta_1, \eta_2, \ldots, \eta_N)$$

$$\frac{d\vec{p}(t)}{dt} - \Gamma\,\vec{p}(t)$$

[1] N. Khaneja, et al., J. Mag. Res., 172 **2**, (2005)

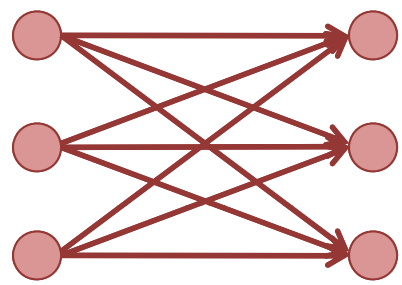
Simulating Noisy Evolution I: Deterministic Master Equation

Unitary Evolution



$$\dot{\rho}(t) = -i\hbar\,[H_0 + H_k, \rho_k(t)] = \tilde{M}_k\,\rho_k(t)$$

Noise Switching



$$\dot{\rho}_k(t) = \sum_j \Gamma_{jk}\rho_j(t)$$

Time-evolution of **conditional density matrix**

$$\dot{\rho}_k(t) - M_k\,\rho_k(t) + \sum_j \Gamma_{kj}\,\rho_j(t)$$

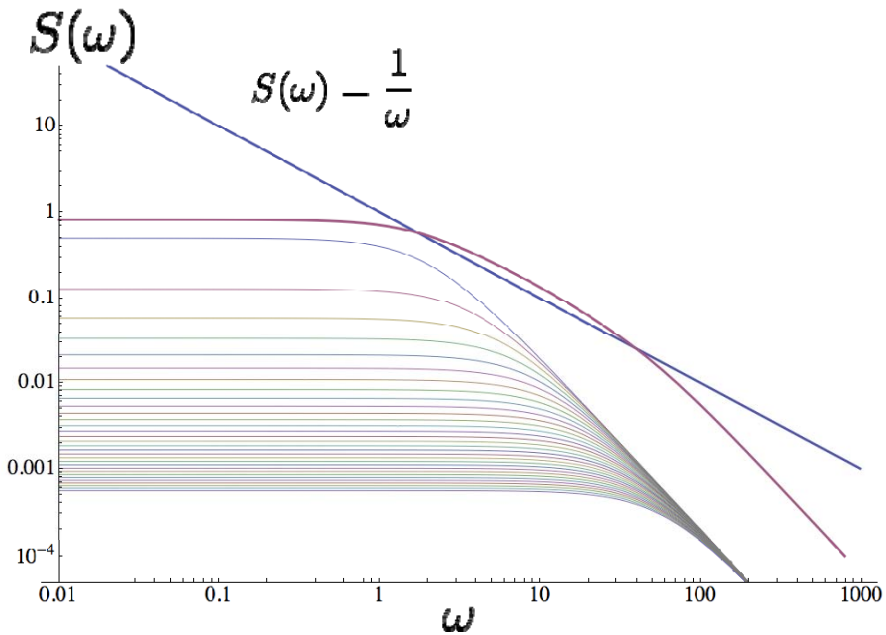
Time-evolution of **full density matrix**

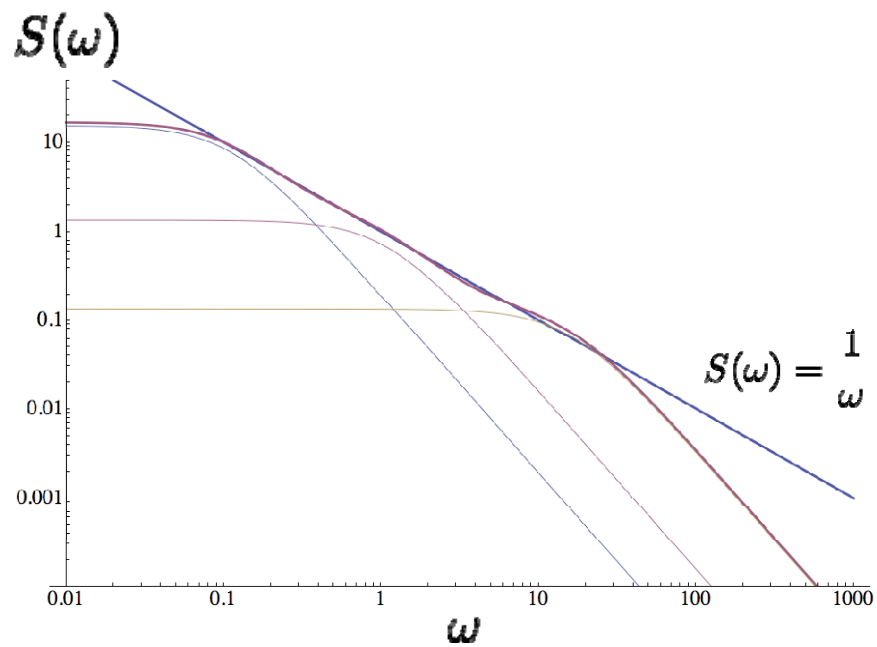
$$\rho(t) = \frac{1}{N}\,(I \otimes I \cdots \otimes I) \cdot \exp\left(\left(\bigoplus_k M_k + I \otimes \mathrm{I}'\right) t\right) \cdot (I \otimes I \cdots \otimes I)^{\mathsf{T}} \cdot \rho_0$$

P. Koupanportti, et al., Phys. Rev. A 77, 032334 (2008)

Simulating Noisy Evolution II: Multistate Markovian Fluctuators

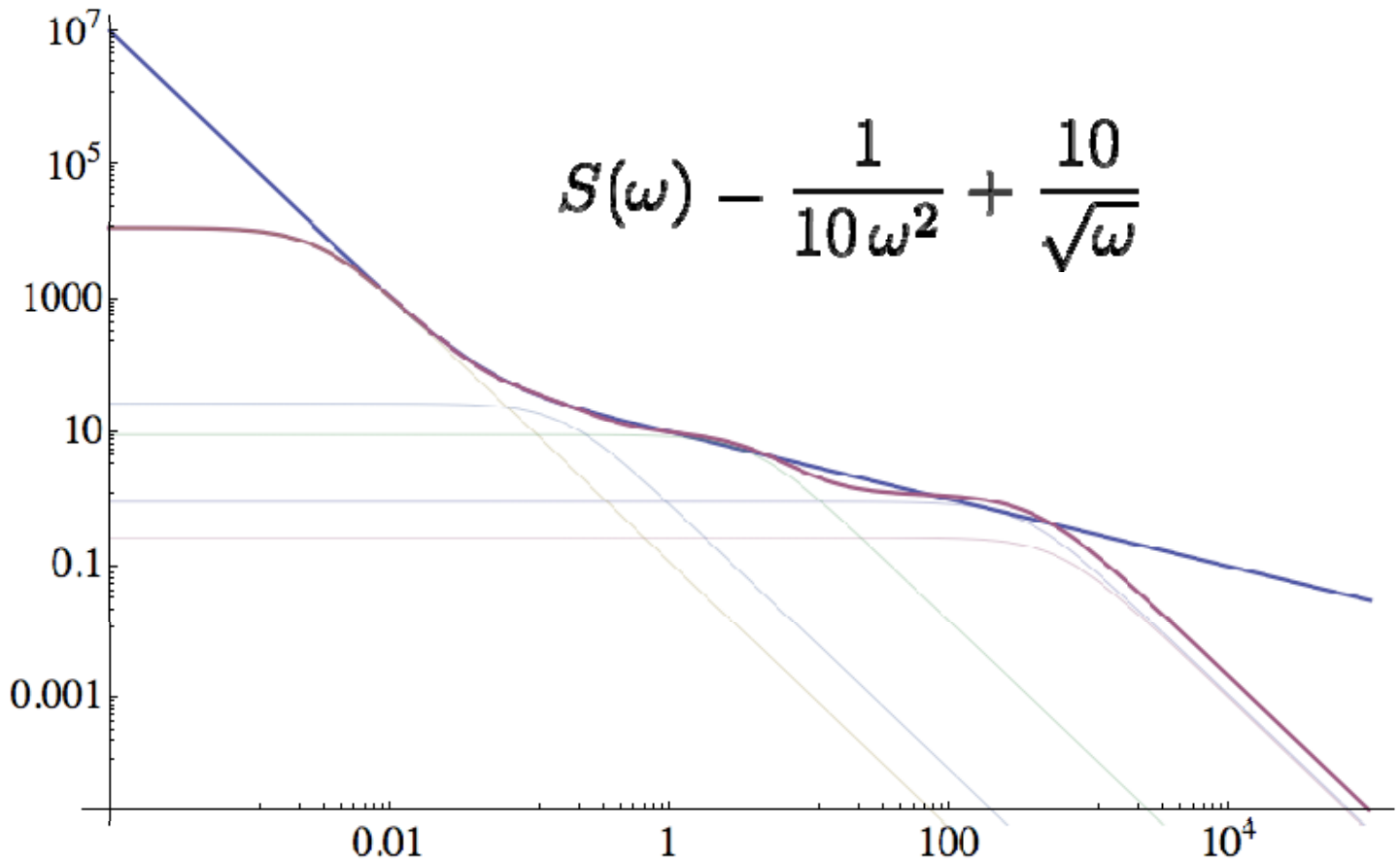
By choosing the transition rate matrix, Γ, and the noise states, $\{\eta\}$ match the fluctuator's noise spectrum to a that of a wide range of Markovian noise sources.





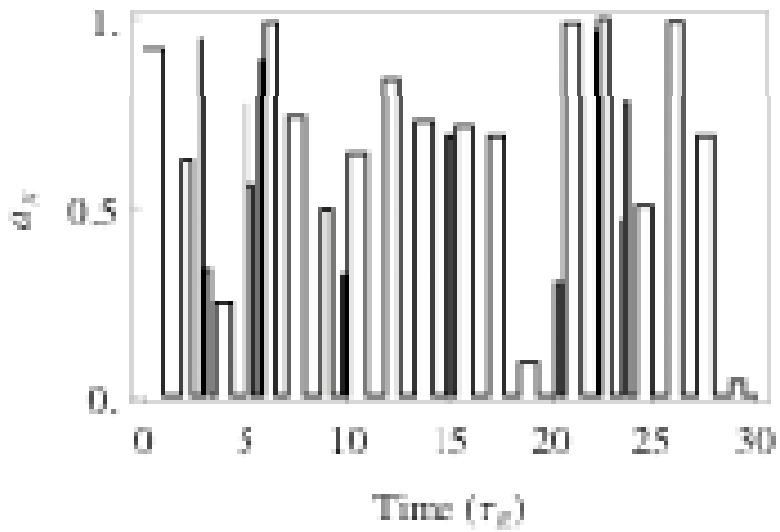
$$S(\omega) = \mathcal{F}\left(\frac{1}{N}\sum_{ij}\eta_i\left[e^{\Gamma|t|}\right]_{ij}\eta_j\right) = \sum_i \frac{\alpha_i}{\omega^2+\beta_i}$$

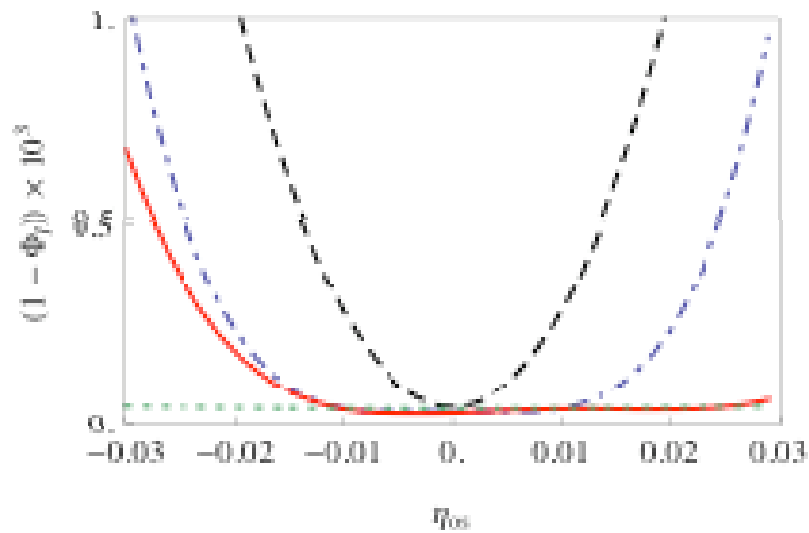
Simulating Noisy Evolution III: Multistate Markovian Fluctuators

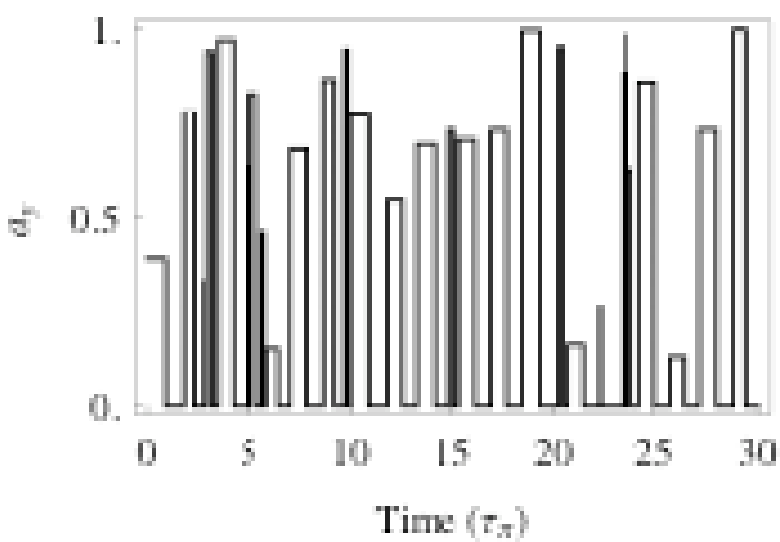


$$S(\omega) - \frac{1}{10\,\omega^2} + \frac{10}{\sqrt{\omega}}$$

Single Qubit Gates: Identity Gate



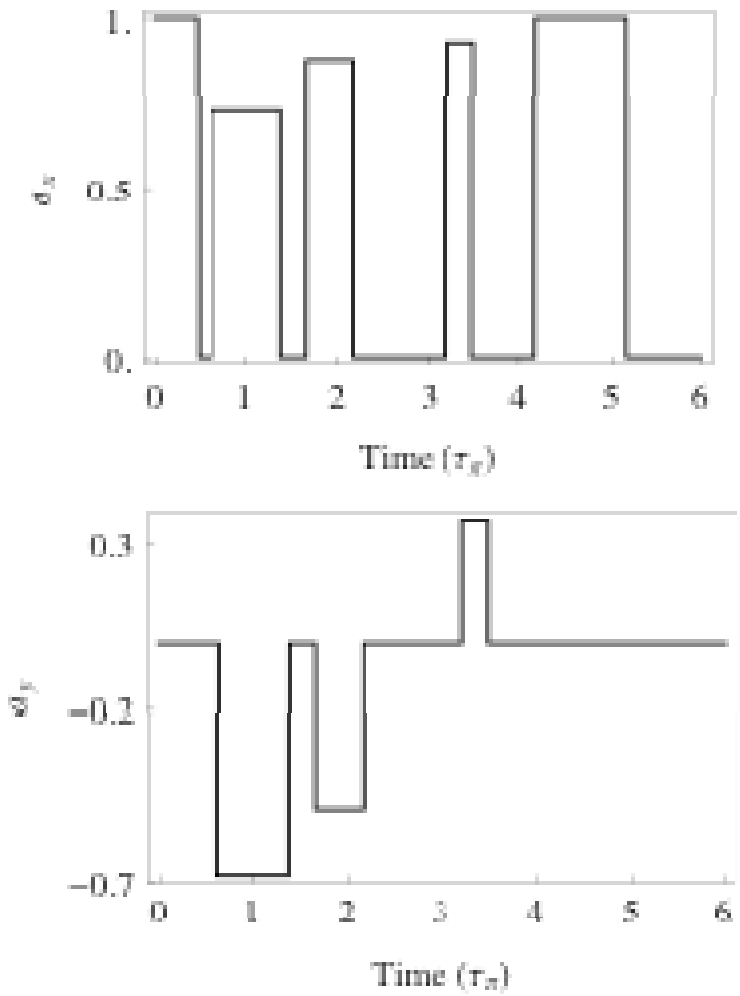


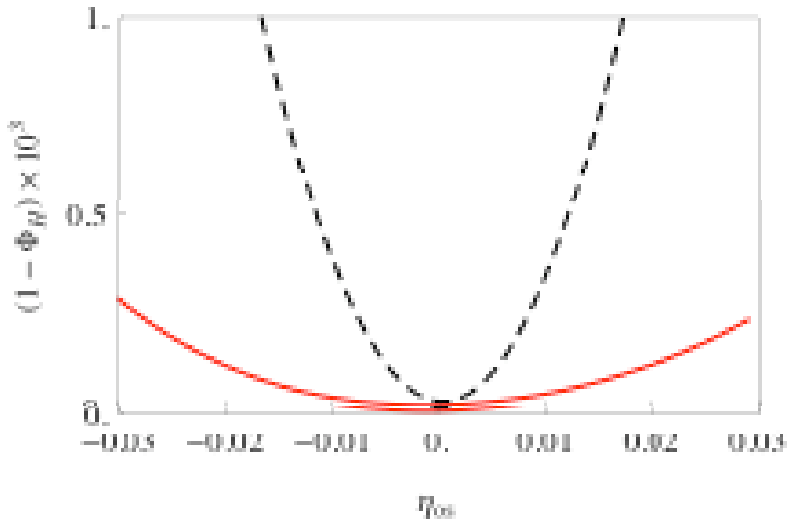


Robust to zero frequency noise

Compares well against similar Carr-Purcell sequence

Single Qubit Gates: Hadamard Gate





Robust to zero frequency noise

Still to do: Compare to DCGs

Two Qubits I: Model

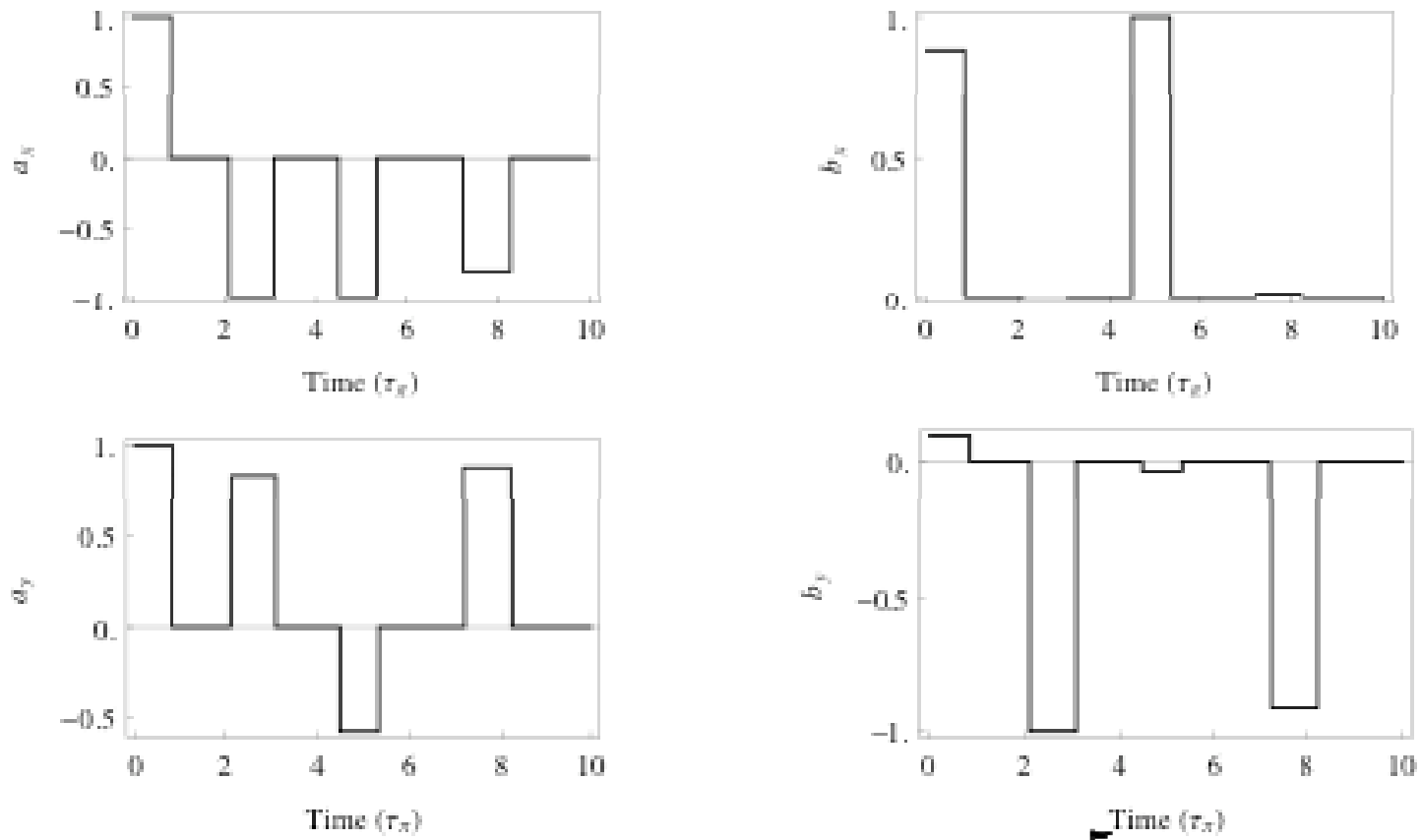
J-coupled qubits, correlated noise, independent *x, y* control on both qubits.

$$H(t) = \frac{1}{2}\left(\vec{a}(t)\cdot\vec{\sigma}^{(1)} + \vec{b}(t)\cdot\vec{\sigma}^{(2)}\right) + \frac{1}{2}\eta(t)\left(\sigma_z^{(1)} + \sigma_z^{(2)}\right) + \left(J + \delta J(t)\right)\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)}$$

Noise is assumed to be approximately 1/f

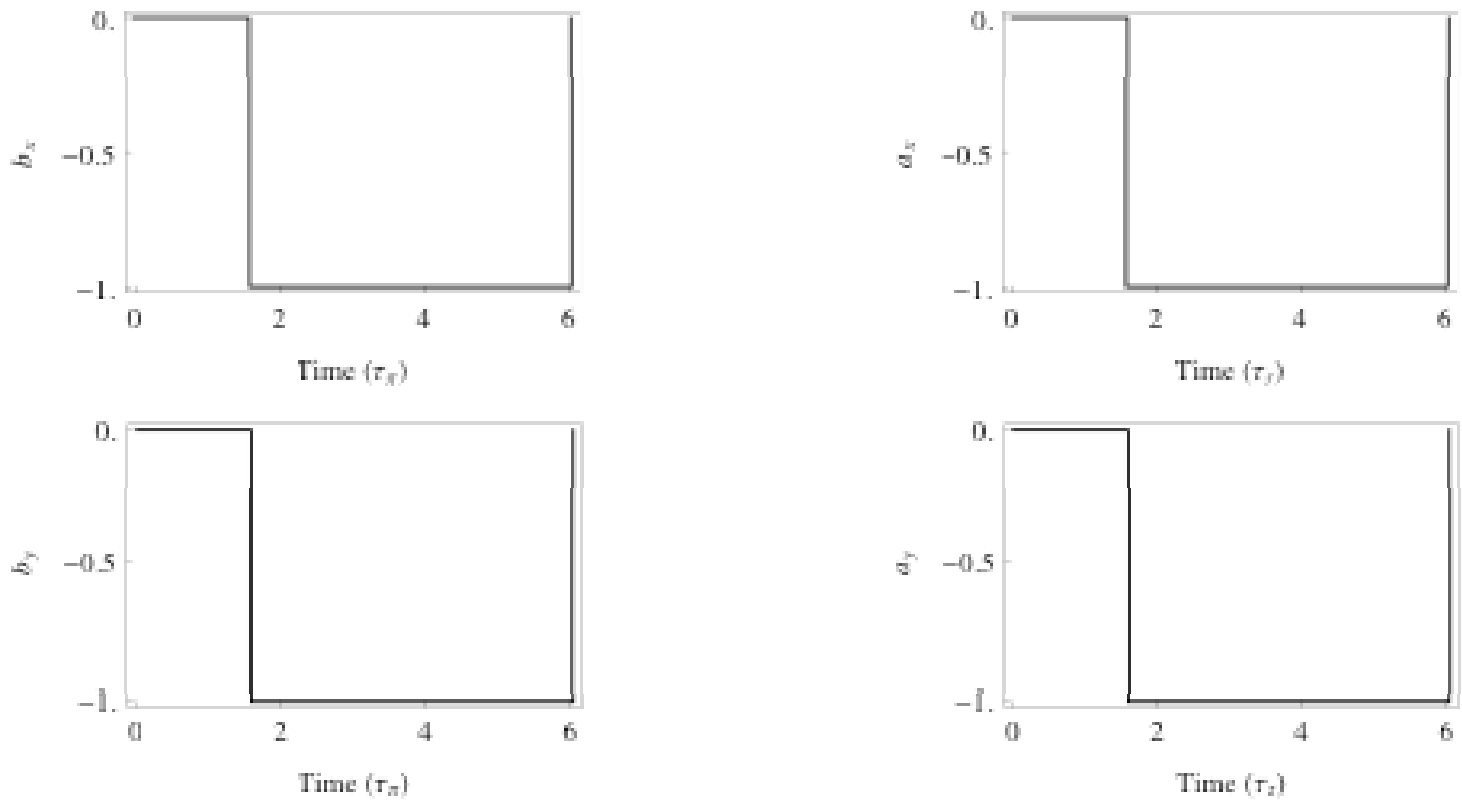
$$\begin{aligned} J &\simeq 4 \\ \delta J/J &\simeq 10^{-4} \\ \eta &\simeq 10^{-4} \end{aligned}$$

Two Qubits II: Identity Gate



$$\Phi = 1 - 8.66 \times 10^{-5}$$

Two Qubits II: $\sqrt{\mathrm{SWAP}}$



$$\Phi -1 \; - 5.59 \times 10^{-5}$$